"The Use of Coleman's Power Indices to Inform the Choice of Voting Rule with Reference to the IMF Governing Body and the EU Council of Ministers"

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Abstract

In his well known 1971 paper the mathematical sociologist James S. Coleman, proposed three measures of voting power: (1) "the power of a collectivity to act", (2) "the power to prevent action" and (3) "the power to initiate action". (1) is a measure of the overall decisiveness of a voting body taking into account its size, decision rule and the weights of its members, while (2) and (3) are separate indices of the power of individual members, in being able to block or achieve decisions. These measures seem to have been little used for a variety of reasons, although the paper itself is widely cited. First, much of the power indices literature has focused on normalised indices which gives no role to (1) and means that (2) and (3) are identical. Second, Coleman's coalition model is different from that of Shapley and Shubik which has sometimes tended to dominate in discussions of voting power. Third, (2) and (3) are indistinguishable when the decision quota is a simple majority, the distinction becoming important in other voting situations. In this paper I propose that these indices, which are based on a fundamentally different notion of power than that assumed by game-theoretic approaches, have a useful role in aiding a better understanding of collective institutions in which decisions are taken by voting. I use them to illustrate different aspects of the design of a weighted voting system such as the governing body of the IMF or World Bank, or the system of QMV in the European Council.

The article by James S. Coleman "Control of Collectivities and the power of a collectivity to act", published in 1971, is widely cited by writers on the measurement of voting power but its full significance – and the work of Coleman on voting power more generally – has not been fully appreciated by many. Very often, the power measures proposed in that paper by Coleman are mentioned in a footnote or throwaway remark in the discussion of the Banzhaf index along with those by Penrose, Rae, Dahl, Chow and others (see Dubey and Shapley, 1979), which all use the same coalition model as their basis. (We can also include the Cubbin and Leech index in this list. See Cubbin and Leech (1983).) Some authors describe Coleman's indices in detail and then quickly go on to show how they are related to those of Banzhaf and therefore need no further discussion. Some combine the two approaches to the measurement of power and simply refer to the Banzhaf-Coleman index. Not all authors are so quick to marginalize Coleman's measures, at least not in all their writings, however, such exceptions including Felsenthal and Machover (1998), Nurmi (1997) and Brams (1975 and 1976).

In this paper I want to discuss some aspects of Coleman's work on power and to argue that, far from being just an alternative statement of that of Banzhaf (which was really a rediscovery of that of Penrose (1946) as Felsenthal and Machover have pointed out), it embodies a significantly different conception of power measurement. Moreover it is one which offers the promise of an approach to better understanding the decision-making systems like those of the EU, the Bretton Woods institutions, the World Trade Organisation and the United Nations, and one that more closely engages with the concerns of political actors than the mainstream power indices literature does.

Coleman's Contribution to Voting Power Theory

Coleman's 1971 paper argued strongly against the use of game theory in general and the Shapley-Shubik power index in particular. In fact his paper contains a fundamental theoretical critique of that index based, first, on its use of orderings of members to give different weight to coalitions of different sizes and, second, its characterisation of voting as a group of rivals bargaining among themselves over a fixed payoff in a game. There does not seem ever to have been a proper reply to Coleman's arguments but the game-theoretic approach and Shapley-Shubik index continue to be taken seriously.

Coleman's approach was based on the dynamic idea of power in relation to action and not the static idea of power as division of spoils. This allowed the relaxation of some of the analytical constraints that came from game theory, such as the requirement that the power indices of the different players should add up to a constant (an idea often referred to as the "efficiency axiom") and the restriction that the quota has to be at least half the total number of votes (the restriction to "proper games"). This meant that voting power, in Coleman's sense, was conceived in absolute not relative terms. It shifted the focus of the analysis from the powers of the members in relation to each other to the relationship between the powers of individual members and that of the collectivity, which relationship is where much of the real concern lies in discussing institutions. Coleman made a distinction between the negative power to (in his terminology) prevent action and the positive power to initiate action, which again is a key distinction of much practical value. His perspective also was ideal as a basis for considering how power changes as a result of members participating in coalitions, something for which game theory is ill suited, and this was a central concern in his work.

Coleman's Rejection of Models Derived from Game Theory

Coleman expressly dismissed game theory as a suitable method for studying collective decisions and proposed instead an approach based on probability theory. He argued that game-theoretic approaches such as that of Shapley and Shubik are inappropriate because they are ultimately derived from a value concept such as the Shapley value where there is a quantity to be divided among the players. Power in such games is seen as being about bargaining over payoffs that are realised by the winning coalition. The Shapley value here (which is the Shapley-Shubik index) is the expectation to each player of playing the game where the payoff to a winning coalition is equal to 1 unit of success.

Coleman argues that decisions taken by collective bodies are normally quite different, and cannot be modelled in this way. Decisions are about actions to be taken by the collectivity - such as the provision of a public good or the enactment of a law of which the consequences for every member are fixed exogenously and cannot be bargained over. This makes the discussion of the questions of the power of one member relative to another quite irrelevant since there can be no bargaining over the division of spoils in such a situation.

He did concede, however, that there may be circumstances where such a division of spoils might be thought to occur. In a party convention, for example, those delegations that support the winning candidate agree to divide up the spoils of office in such things as other associated offices which can be filled through patronage. Coleman suggest this is an exceptional case where there is a winning nominee and there are spoils to be divided up. The usual case he envisages is where voting leads to collective action. In his 1973 paper "Loss of Power" he applied the approach in a much more general framework. These two types of models of power have been labelled, by Felsenthal and Machover, I-power (power as influence) and P-power (power as a prize), and correspondingly the most appropriate modes of analysis those of Coleman or Banzhaf, using probability theory, or Shapley and Shubik using game theory.

Instead of a constant-sum co-operative game among n players, bargaining over fixed payoffs, Coleman defines power in terms of different kinds of action. There are different kinds of actors - members of the collectivity versus the collectivity itself – and different kinds of power – power to prevent action and power to initiate action as well as collective power to act. All of these distinctions have a relevance to the discussions surrounding the institutions of the European Union.

Coleman's Measures of Power

Coleman defined three measures of power on the basis of effectively the same probabilistic coalition model as Penrose and Banzhaf. A collectivity is assumed to comprise n members, labelled by integers, and represented by the set $N=\{1,2,\ldots,n\}$ and has a decision rule in terms of a quota representing the minimum number of votes required for action, q. In his original presentation of the indices Coleman borrowed some of his notation from game theory, in particular making use of characteristic functions to represent winning and losing coalitions in the manner of Shapley and Shubik. It is not necessary to do that here and I will avoid it.

For each voting outcome in the collectivity let there be s members who vote for a particular action and n-s voting against. Those voting for action are represented by a subset S, $S \subseteq N$, and cast a total number of votes equal to w(S), while the number of votes against action is w(N\S). The collectivity will take action if $w(S) \geq q$. Every voting outcome is treated equally on the basis that each member has the right to vote either for or against action. Each subset S can be thought of as being randomly selected with equal probability.

This is equivalent to assuming a random -voting model in which each member votes for or against with equal probability independently of all other members. This is not a behavioural assumption – that people actually cast their votes indifferently – but a reflection of the rights of individuals – they *can* vote either way. (The use of probabilities is not to imply random behaviour but as a method of analysis.) This assumption means the analysis is of *a priori* power which is able to provide insight into the power implications of the decision rule itself. This probabilistic-voting assumption can be varied later to introduce more realism but then the nature of the analysis changes from the *a priori* to the actual.

Let the number of winning subsets, that is outcomes that lead to action, be denoted by ω. A member, i, can swing a vote if there is a subset of members, S, such that $q - w_i \leq w(S) < q$. The number of such subsets, the number of swings for member i, is denoted η_i.

(1) The Power of the Collectivity to Act, A, is defined as the relative number of voting outcomes that lead to action:

$$A = \frac{\omega}{2^n} \qquad (1.)$$

There are 2^n possible outcomes and in ω of these there is a majority vote leading to action. A is the probability of a winning vote occurring.

Where there is a unanimity decision rule, for example, all members must vote for action to occur and therefore $\omega = 1$, so $A = 2^{-n}$. If n is large then the power to act is very small. On the other hand, a simple majority rule, where $q = w(N)/2$, gives the maximum value of $A = 1/2$, since then exactly half the voting outcomes lead to action, $\omega = 2^{n-1}$.

Coleman proposed two indices of the power of individual:

(2) The Power to Prevent Action, P_i, is the ability of member i to prevent action by witholding his vote from a group which would win to one which loses. The denominator is the number of winning subsets, and the numerator is the number of swings for i.

$$P_i = \frac{\eta_i}{\omega} \qquad i=1,\ldots,n \qquad (2.)$$

(3) The Power to Initiate Action, I_i, is the ability of member i to swing a vote that would fail to produce a majority for action without him, to one which wins with him. The numerator is the same, the number of swings, but the denominator is the number of losing subsets, the complement of that in (2.).

$$P_i = \frac{\eta_i}{2^n - \omega} \qquad i=1,\ldots,n \qquad (3.)$$

Each of these indices is a non-negative fraction because it is a probability. A member whose weight is large enough to block action, with $w_i > w(N) - q$, has power to prevent action which is total, $P_i = 1$ because then $\eta_i = \omega$ and all winning votes are swings for i. That does not mean that member i is a dictator, however, because his power to initiate action is not necessarily total: $I_i = \frac{\eta}{2^n - \omega}$ need not be large even though P_i is. A prominent example of this is the IMF where the voting system is designed to guarantee the USA a veto over certain important decisions. A dictator has $w_i \geq q$ so that $\omega = \eta_i = 2^{n-1}$ and therefore $I_i = P_i = 1$, and the power to act is at its maximum, $A = \frac{1}{2}$. If the decision rule is a simple majority, then A is $\frac{1}{2}$, and again, $\omega = 2^{n-1}$ so $P_i = \frac{\eta_i}{2^{n-1}}$ and

$$I_i = \frac{\eta_i}{(2^n - 2^{n-1})} = \frac{\eta_i}{2^{n-1}} = P_i.$$

The Decision Rule in Terms of the Quota

One of the most important consequences of Coleman's defining power in terms of action and his rejection of the game-theoretic perspective, is that any decision rule is allowable. In the traditional cooperative game model, it is an absolute condition there can only be one prize to be divided up among the members of the winning side. This has come across to voting games as the idea that there can only be one decision in the sense that the decision rule can allow only one group of voters to be said to win. Therefore the quota must always be such as to prevent two groups from both winning simultaneously. Accordingly it is customary to insist that the quota is more than half the total weight,

$q > \frac{w(N)}{2}$, and the game is a "proper game". The tendency to impose this condition is not confined to applications which use the explicitly game-theory-based Shapley-Shubik index, but also the Banzhaf index as well.

This tendency is little more than a bad habit and contributes nothing to the analysis. There are many voting bodies, or situations which can be characterised as such, where decisions which commit the collectivity to action are made with a quota equal to less than half the total weight, and where there is no ambiguity. This is relevant to the study of power. Coleman gave numerous examples for which his power measures are natural tools of analysis. He referred to symmetric voting rules – simple majority – and asymmetric rules – supermajority rules where $q > w(N)/2$, or rules where collective action could be taken on the basis of fewer than half the votes.

A decision to raise the fire alarm can be taken by one member of the collectivity acting alone. Here the decision rule can be thought of to be a quota $q = \min(w_1, w_2, \ldots, w_n)$, the number of voting outcomes that lead to action is, $\omega = 2^n - 1$, and therefore the power of the collectivity to act is, $A = 1 - 2^{-n}$. For any individual, i, the number of swings is $\eta_i = 1$ and therefore the power to initiate action is, $I_i = \frac{\eta_i}{2^n - \omega} = \frac{1}{2^n - (2^n - 1)} = 1$,

and the power to prevent action is, $P_i = \frac{\eta_i}{\omega} = \frac{1}{(2^n - 1)}$. Another, similar example is that of a decision by a group of farmers to pollute a stream adjoining their land: any one of them can take the decision unilaterally and therefore each has a power to initiate action equal to unity and very little power to prevent action, while the power to act is very large. Coleman contrasts this with the power analysis involved in any action that might be taken to prevent the pollution, noting that any collective action to clear up or stop such pollution would be likely to require a majority vote.

The Relationships between Coleman's Indices and the Banzhaf Indices

Coleman's power to initiate action and power to prevent action have often been described in the literature as merely different transformations of the Banzhaf index, and therefore equivalent. Some authors even refer simply to the Banzhaf-Coleman index. The equivalences can be demonstrated as follows.

There is a slight difference in the probability models assumed by Banzhaf and Coleman. Banzhaf's main interest was in the power of a member within the collectivity, and therefore he used a probability model that assumed that the i^{th} voter voted strategically while the remaining n-1 members's votes were random. On the other hand Coleman used a model in which the assumption of random voting was applied to all the members including i, thereby enabling him to define the collective power to act. It is necessary to allow for this in discussing the relationship between the indices.

The absolute Banzhaf index measures the probability of a swing for member i, and this reflects both the power of the member to prevent or initiate action and the power of the collectivity to act. But this is conditional on the behaviour of member i. Therefore, using Coleman's probability model, in which the number of random voting outcomes is 2^n not 2^{n-1}, it is necessary to adjust this by the probability that member i votes for action, which is 1/2. Therefore we can write the identity:

Pr (swing for i| i votes for action)xPr(i votes for action)□

$= \Pr(\text{swing for } i \mid \text{collective action}) \times \Pr(\text{collective action})$

$+ \Pr(\text{swing for } i \mid \text{no collective action}) \times \Pr(\text{no collective action})$

$$\beta_i \times \frac{1}{2} = \frac{\eta_i}{2^{n-1}} \times \frac{1}{2} = \frac{\eta_i}{\omega} . \frac{\omega}{2^n} = P_i \times A,$$

$$= \frac{\eta_i}{(2^n - \omega)} . \frac{(2^n - \omega)}{2^n} = I_i \times (1 - A). \quad (4.)$$

Hence, the Banzhaf index can be written as, $\beta'_i = 2P_iA = 2I_i(1-A)$.

Therefore Coleman's indices enable two different decompositions of the power of a member. A member's power, as the probability that he is the swing voter, can be written as the product of the conditional probability of a swing given that the collectivity acts, multiplied by the probability of collective action - that is, the power to prevent action times the power of the collective body to act. Alternatively it can be written as the conditional probability of a swing given no action (his power to initiate action) times the probability that there is no action. Coleman did not make this link, although he occasionally referred to the power of a member. He referred to the power to prevent action when it and the power to initiate action coincided.

Normalising either of these indices to make the power indices of all members together sum to 1, the equivalence with the normalised Banzhaf index is clear. They can both be considered as different ways of arriving at the normalised Banzhaf index.

$$\beta_i = \frac{\eta_i}{\sum \eta_i} = \frac{\beta'_i}{\sum \beta'_i} = \frac{P_i}{\sum P_i} = \frac{I_i}{\sum I_i}.$$

Many writers have taken these identities to mean that the Coleman indices tell us nothing that we cannot find out from the Banzhaf index.

That is not correct because, first, there is a fundamental difference between the absolute and the normalised Banzhaf indices. The former has a straightforward interpretation as the probability of a swing while the latter has been criticised (for example by Dubey and Shapley (1978)) as requiring a dubious normalising constant which does not have a straightforward meaning. This has led some to try and solve this particular problem by introducing new indices. (For example Johnston (1978) and Deegan dn Packel (1982)). Second, the absolute Banzhaf index cannot be regarded as equivalent to the Coleman indices in general. They are equal only in the special case where the decision rule is a simple majority and therefore A = 1/2, so that $P_i = I_i = \beta'_i$. It is important to bear this in mind when considering voting bodies that use supermajority decision rules.

A further important relationship between them is that there is a sense in which the Banzhaf index can be regarded as the average of the powers to prevent and to initiate action. The absolute Banzhaf index is their harmonic mean:

$$\frac{1}{\beta'_i} = \tfrac{1}{2}\left(\frac{1}{P_i} + \frac{1}{I_i}\right) = \tfrac{1}{2}\left(\frac{\omega}{\eta_i} + \frac{2^n - \omega}{\eta_i}\right) = \frac{2^{n-1}}{\eta_i}. \qquad (5.)$$

Power and Collective Institutions

Coleman (1973) presented a framework for analysing the power relationships surrounding institutions. This was intended to be the basis of a general theory applicable to all types of collective actors: firms, trade unions, churches, etc, as well as political organisations. His model assumes that all social actors belong to a larger collective body, in the decision-making system of which they have voting power. At its most general, this can be thought of as society. There are two types of social actors, individuals and corporate actors of which individuals are members. Thus we are envisaging two types of collectivities where voting takes place: the universal, representing society, and the collective institution which the individual may or may not join. The individual can gain the power of combined resources by joining the institution, through its greater voting power, by virtue of its bloc vote being larger than his individual vote. However he does not have total control over its use, since he becomes a member of a collectivity with limited voting power. It will be worth joining the collective institution if he can increase his voting power by doing so.

Let the voting power of an actor, labelled a, within the collectivity, labelled b, (continuing but slightly extending the notation above) be equal to $\square'_{ab}$. The institution can be considered either as a collectivity or as an actor. Thus an actor is either an individual, $a = i$, where i has the same meaning as before, or the collective institution, $a = C$. The collectivities are either the institution, $b = C$, or the universal society, $b = U$.

Then the power of the individual, if he stays outside the institution, is equal to $\square'_{iU}$. This reflects all aspects of the decision-making rule and the voters, taking account of the resources (voting weight) of the institution, without his contribution. If he joins the institution, then his power is equal to the product of his power in decision-making within it times the power of the institution within the universal society. Thus his power will be equal to $\square'_{iC}\square'_{CU}$. (Note that the decision-making systems C and U will change when i joins; the details of that need not detain us here but have to be allowed for in any numerical analysis.)

Coleman's work focuses on the relationship between these two powers: the power given up is $\square'_{iU}$ and the power acquired in exchange, through being a member of the collective institution, is $\square'_{iC}\square'_{CU}$. This is at least part of the framework that politicians can be thought to employ when considering whether their country might join the EU. Coleman (1973) derives the mathematics of the calculations for simple majorities and one-person-one-vote where the central question he considered is how the power of the individual is affected by the size of the institution; the analysis considers the tradeoffs involved as the number of members changes. Figure 1 illustrates the comparison.

I am not going to apply this formal model here because the universal collectivity is not sufficiently well defined for a general analysis. One application would be to investigate possible scenarios involving the EU voting as a bloc within the international economic institutions like the IMF. We could compare the power of a country in the IMF with its power over the IMF voting as a member of EU, taking into account the bloc vote of the latter and the vote of the country within it. That is work in progress that requires a fuller treatment than is possible in this paper.

Coleman's concern was how power changed as the size of the institution C changed. Since he was assuming simple majority voting, the power to prevent action was the same as the power as measured by the absolute Banzhaf or Penrose measure. In the context of the EU Council, however, the two are not the same because of the use of a supermajority threshold for qualified majority voting. His framework can be used to analyse how power changes when the decision rule changes for a given membership. This is done using the decomposition (4), and also the power index directly, which makes it possible to address the concerns of politicians with the question of how much blocking power they will have to give up in return for an increase in the power to act through majority decisions of the Council.

Coleman's Indices and the Choice of Decision Rule: the IMF

(This section is taken from Leech (2002)). This issue was actually discussed by John Maynard Keynes in the context of the construction of the Bretton Woods institutions that came into being just after the second world war, and it is of interest and relevant to consider the arguments. Keynes' perspective was remarkably prescient and can be thought to have anticipated in some degree, the arguments being used in the context of the EU. In the discussions that led to the creation of the IMF and World Bank, the design of the voting system was an important area of debate where there were significant differences between the British and Americans. The United States wanted to ensure it retained a veto for itself over the most important decisions while Keynes, leading the British delegation, preferred that the formal decision making system be based on simple majority voting in all matters, though every effort should be made to promote consensus and formal votes should always be avoided whenever possible. The Americans proposed that major decisions should require a special majority of four-fifths of the votes to pass, thereby ensuring that the USA, then in possession of 33 percent of the votes, would be able to block any proposals it did not like.

Keynes addressed the question in his maiden speech as a member of the House of Lords ". . . the requirement in the American plan for a four fifths majority will be found, if the paper is read carefully, to relate not to all matters by any means, but only to a few major issues. Whether on second thoughts any one would wish to allow a negative veto to any small group remains to be seen. For example, the American proposals might allow the gold-producing countries to prevent the United States from increasing the gold value of the dollar, even in circumstances where the deluge of gold was obviously becoming excessive; and in some ways, by reason of their greater rigidity, the American proposals would involve a somewhat greater surrender of national sovereignty than do our own." (Keynes (1943a)). He also wrote "I disagree strongly, on non-economic grounds, of the individual country veto-power unless it is granted to all countries regardless of their quotas . . . . the 80 percent majority rule would limit the power of the US with respect to changes it may desire in an existing status as much as it would increase its power to stop undesired changes." (Keynes (1943b))

We would say, in Coleman's terminology, that Keynes criticised the Americans for wanting the quota to be high enough to ensure they had total power to prevent action, when an implication of that was that they sacrificed some of their power to initiate action. Within the narrow context of the IMF and World Bank, the Americans were failing to maximise their own power. On the other hand, they wanted to keep a veto because they did not trust these new collective bodies. Their view prevailed and special supermajorities have been a fundamental feature of the IMF constitution ever since.

Figure 2 shows how the decision rule affects the powers of the top five members in the IMF board of governors. In every case, including the United States, power falls as the quota is increased. At the level required for special decisions, 85% of the votes, all the power measures, including those of the USA, are very close to zero except the USA's power to prevent action, which is total. Figure 3 shows the power to prevent action of the same countries against the power to act at various levels of the quota. This shows the tradeoffs involved between blocking power and the collective power to act. It shows the dominance the USA would still have even if the decision rule were changed to enable the IMF to become a more effective voting body.

Coleman's Indices and the Choice of Decision Rule: the EU

In the negotiations over the system of qualified majority voting to be used in an enlarged union , most recently at the Nice summit, member countries were concerned to a very extent with their blocking power. Considerations of this type were particularly important in determining how they thought about the QMV threshold, once the weights had been agreed. Thus, it is relevant to examine how members' powers to prevent action, and powers, vary with the quota.

The relevance of this is made clear by Galloway (2001) at the end of his chapter describing the discussions surrounding the weighting of votes in the Council. He sums up: "…it is difficult to see where any incentive for change will come from in the future.

Although three of the system's basic parameters (i.e. new weightings, the member state criterion and the population safety net) now appear to be written in stone, there is one crucial element that will remain negotiable: the level of the QMV threshold. This will become a key institutional battleground in each treaty of accession, as heralded in the declaration on the threshold. While it is impossible to speculate on how this declaration will be applied in future negotiations in changed political circumstances, what actually happens to the threshold in practice will depend to a large extent on the comfort level of member states in predicting negotiating outcomes in the new enlarge Union, and the level of security they accordingly feel they need to retain in terms of blocking power in the Council. This allows at least some cause for cautious optimism." (p.93)

This suggests that a key analysis which would inform this process would be to examine the trade-offs between the power of a member state to prevent action and the power of the Council to act. Figure 4 (taken from Leech (forthcoming)) shows how the powers to prevent action of individual countries, within the enlarged council, change as the threshold is varied over the range between 50% and 100%. It suggests that the relationship, while positive, is not dramatically sensitive even for the larger member states. Figure 5 shows the equivalent relationship between P_i and A for the member states, while Figure 6 shows how the threshold affects their overall powers.

Conclusion

This paper has argued that the power indices proposed by James Coleman, and the theory of voting power on which they are based, are a useful tool for discussing the voting arrangements in institutions. Their relevance to the constitution of the IMF and the system of QMV used by the EU Council has been demonstrated by examples taken both from the Nice treaty and the IMF board of governors.

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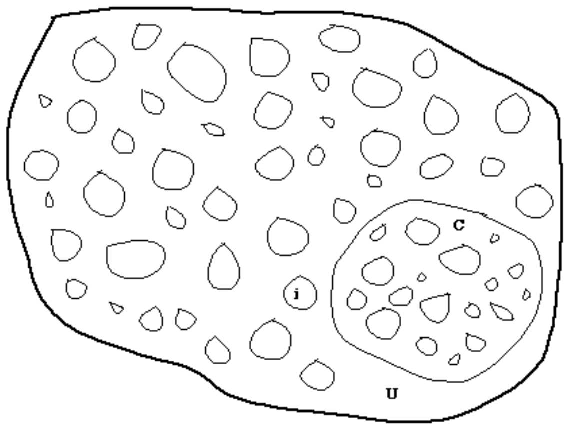
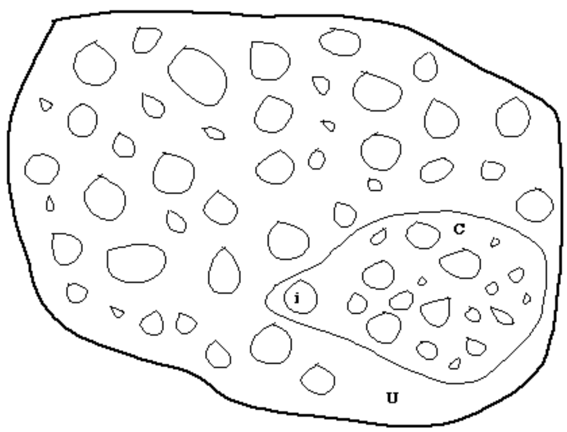
<u>Figure 1</u>



Power of i = $\square_{iU}'$



Power of i = $\square_{iC}'\square_{CU}'$

Figure 2: Effect of the Majority Requirement q on Coleman's and Banzhaf's Indices, Top Five Members of the IMF

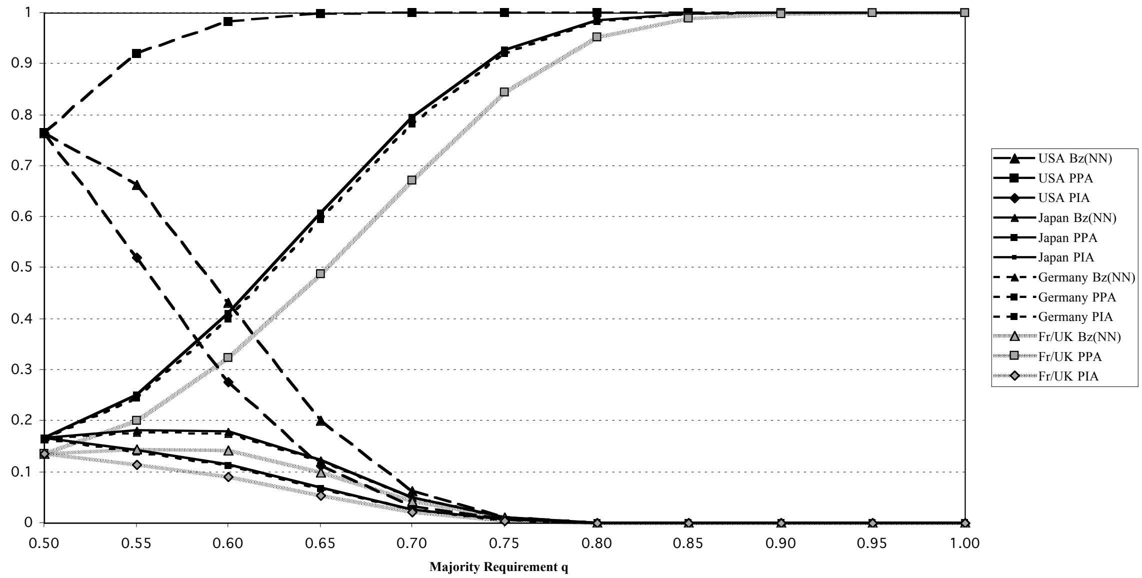


Figure 3: Power to Prevent Action Pi v. Power to Act A, Top Five IMF Members

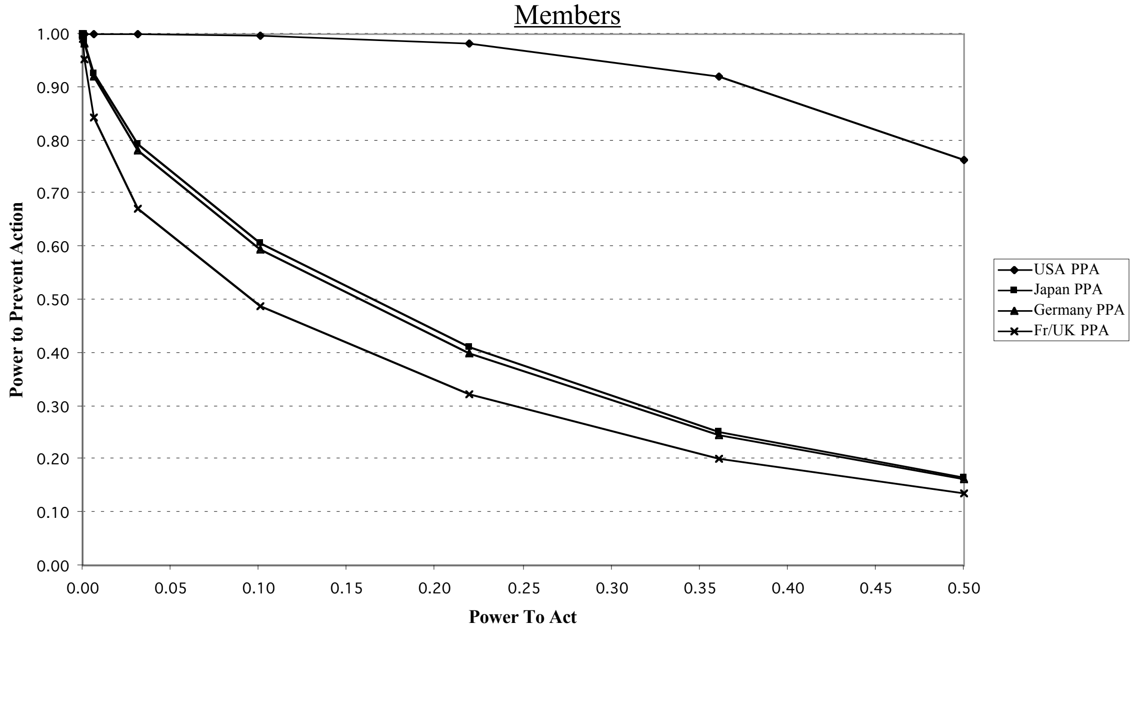


Figure 4: Power to Prevent Action P_i v. the Threshold q in the European Council EU27

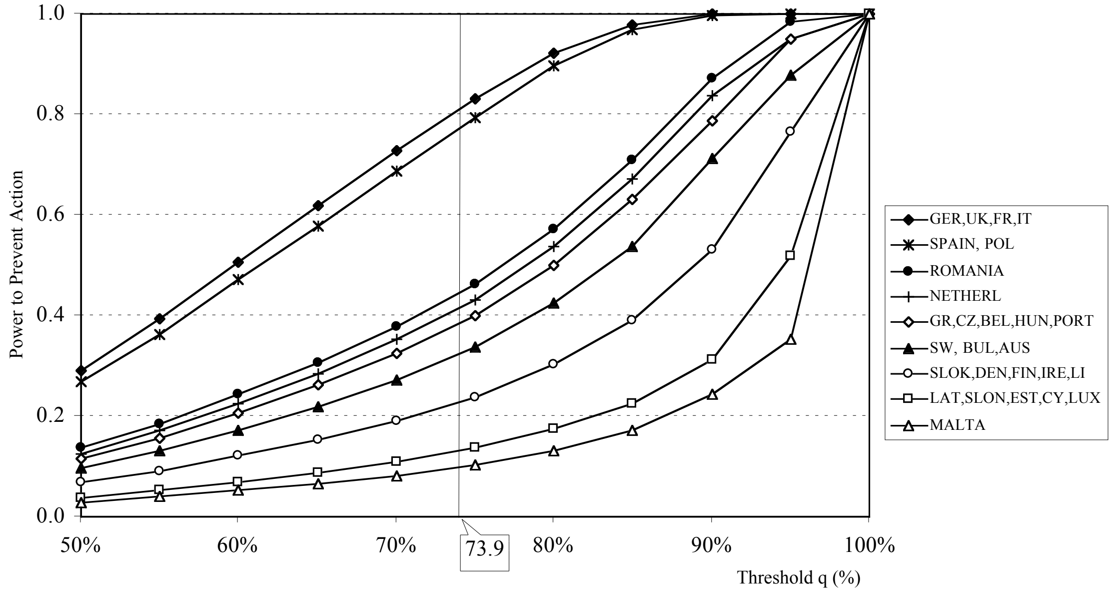


Figure 5: Power to Prevent Action P_i v. Power to Act A in the European Council EU27

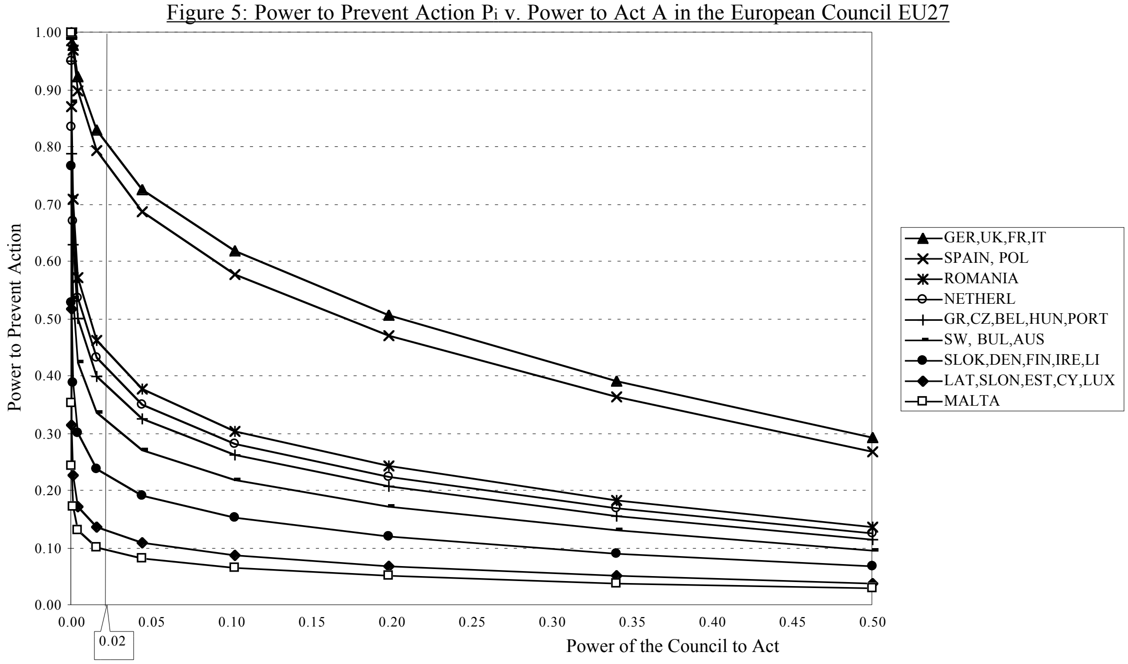


Figure 6: Voting Power 'i v. the Threshold q in the European Council EU27

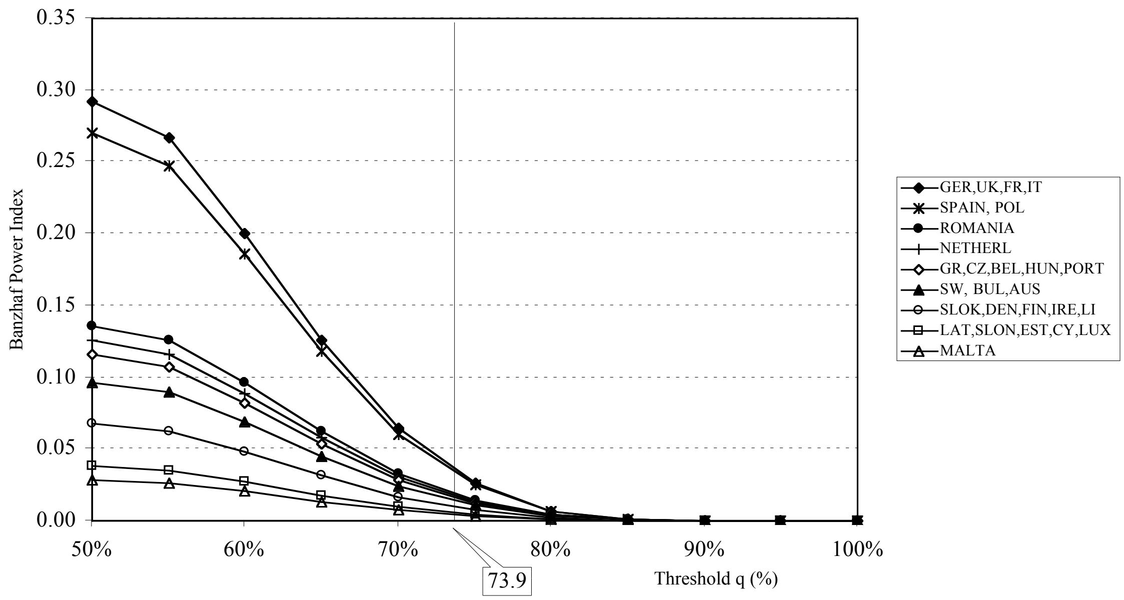


Figure 7: Voting Power i v. Power to Act A in the European Council EU27