Mathematics

Department of Mathematics public lecture

Non-Western Mathematics

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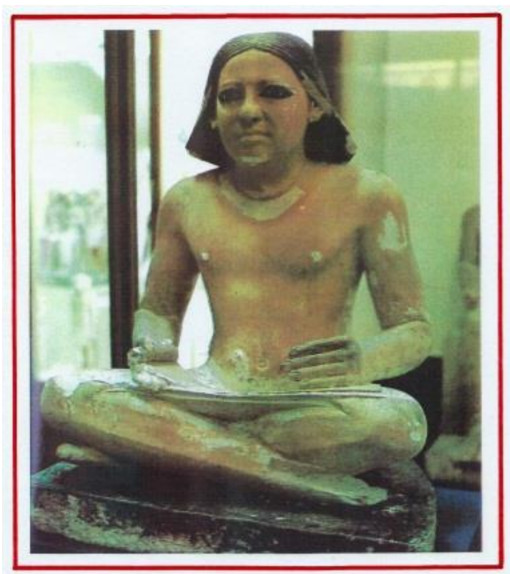
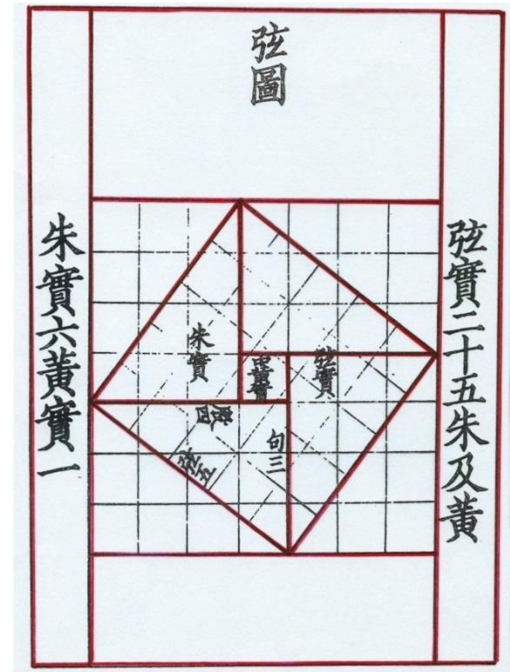
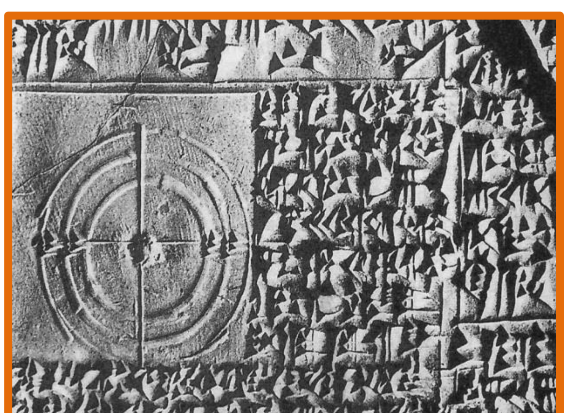
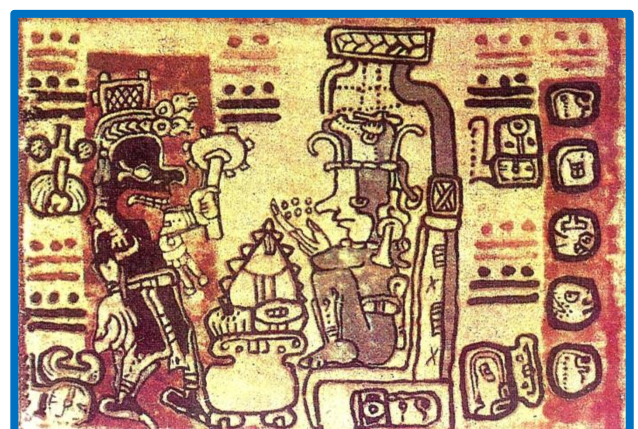
Non-Western mathematics

Robin Wilson (LSE)









Early Mathematics Time-line

- 2700 – 1600 BC : Egypt
- 2000 – 1600 BC : Mesopotamia (‘Babylonian’)
- 600 BC – AD 500 : Greece (three periods)

- 300 BC – AD 1400 : China
- AD 400 – 1200 : India
- AD 500 – 1000 : Mayan

- AD 750 – 1400 : Islamic / Arabic
- AD 1000 – . . . : Europe

Place-value number systems

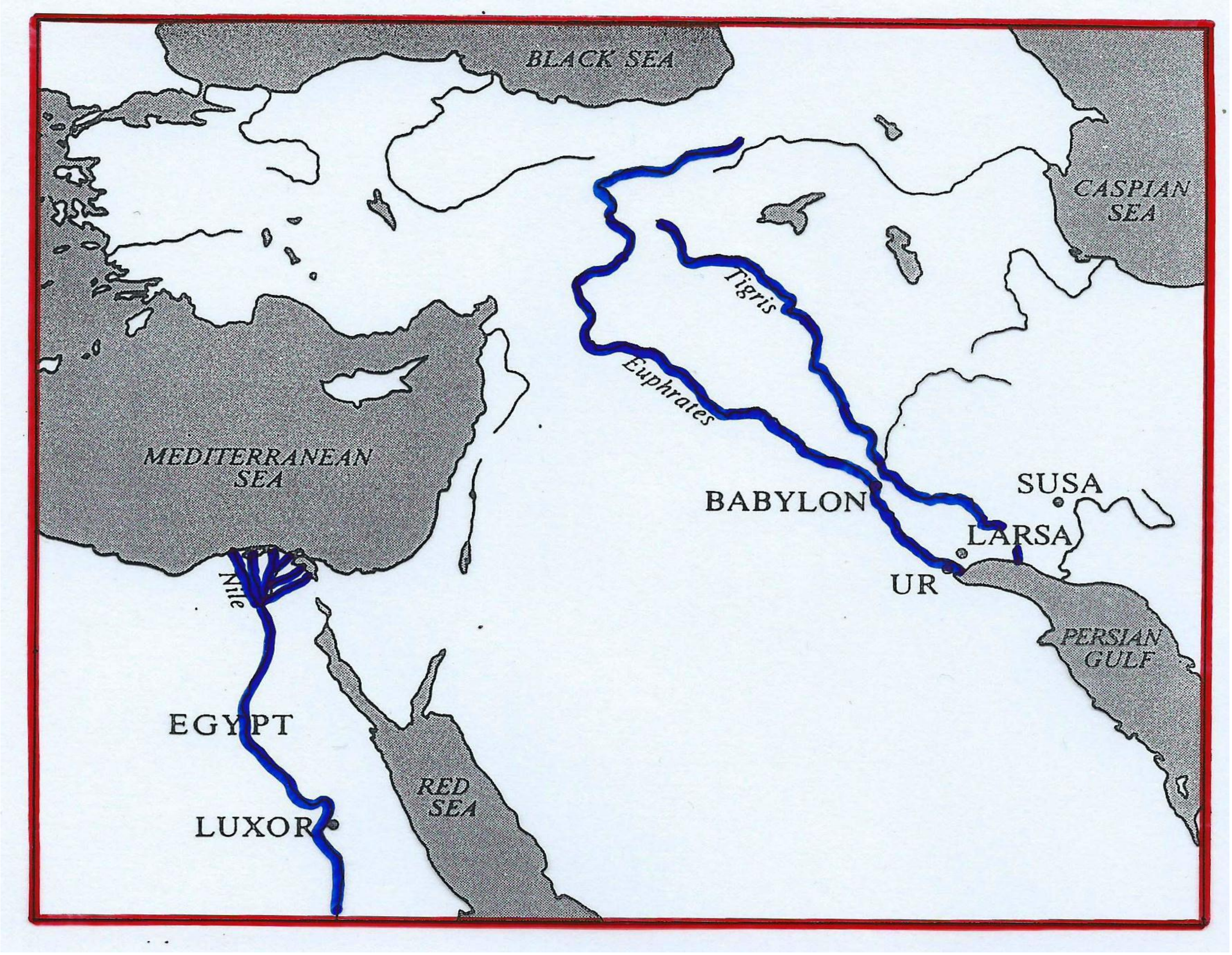
Our decimal place-value system uses
only the numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, and 0

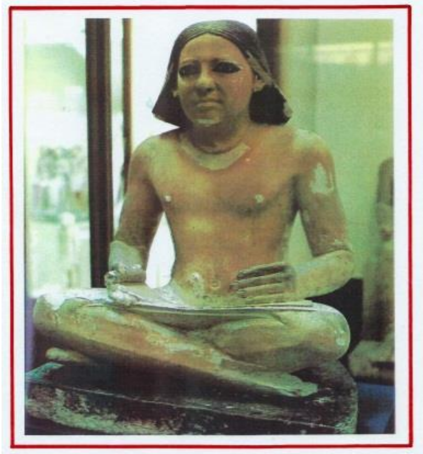
The same digit can represent different numbers
– for example, in the number 3139,
the first 3 represents 3000
and the second 3 represents 30.

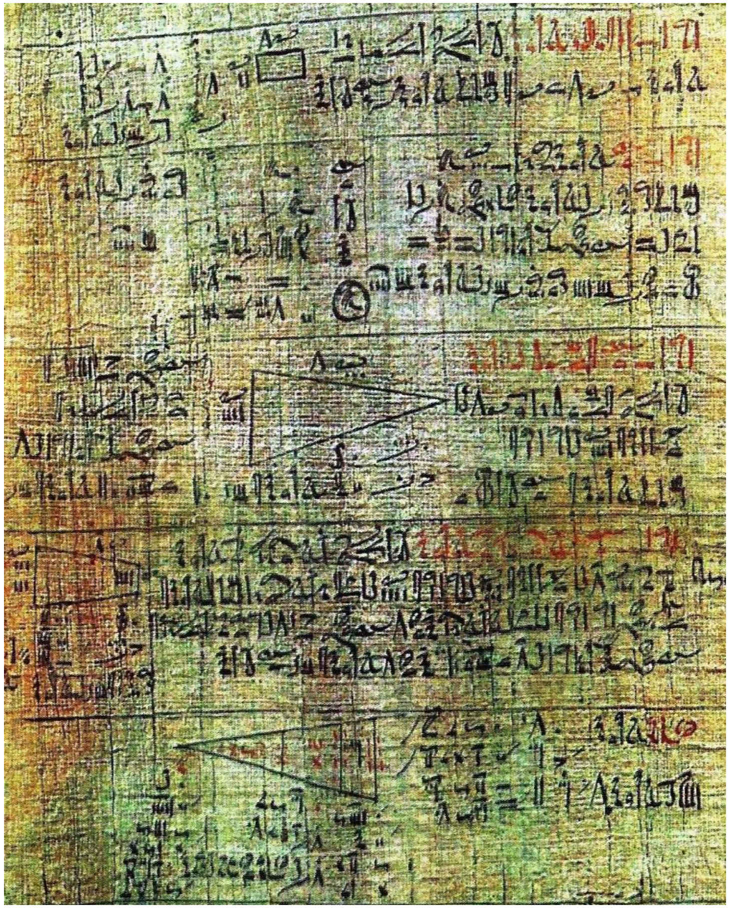
We can then carry our calculations in columns –
units, tens hundreds, etc.

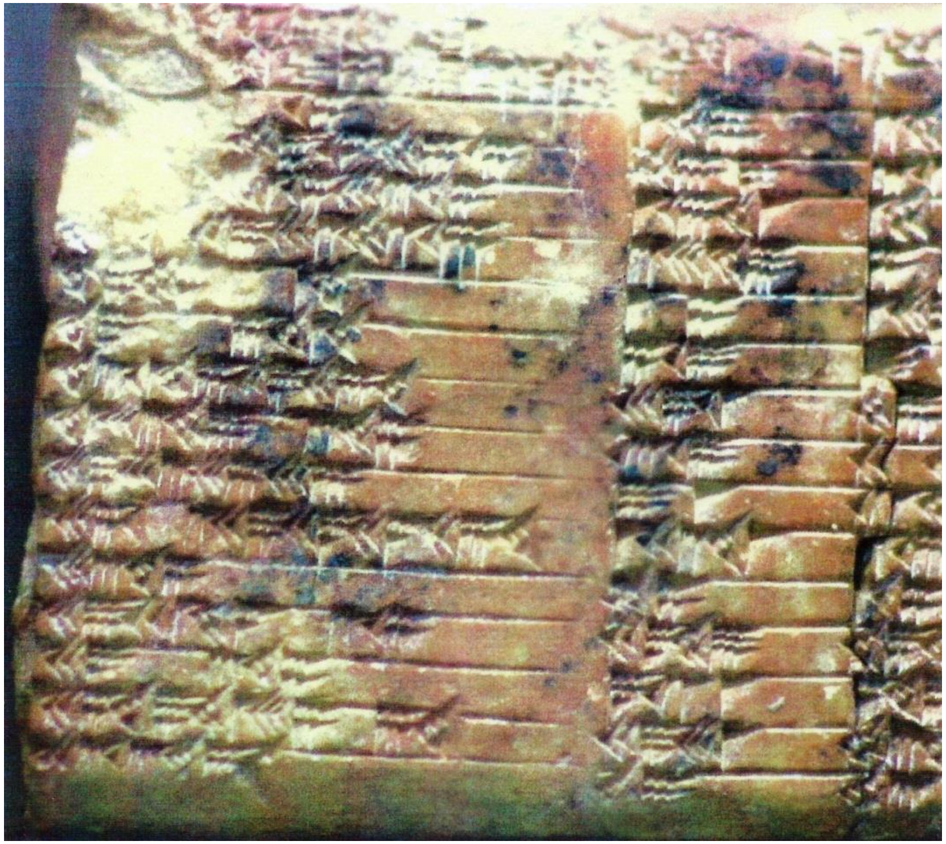
Egypt and Mesopotamia



Papyruses and clay tablets (c.1850–1650 BC)

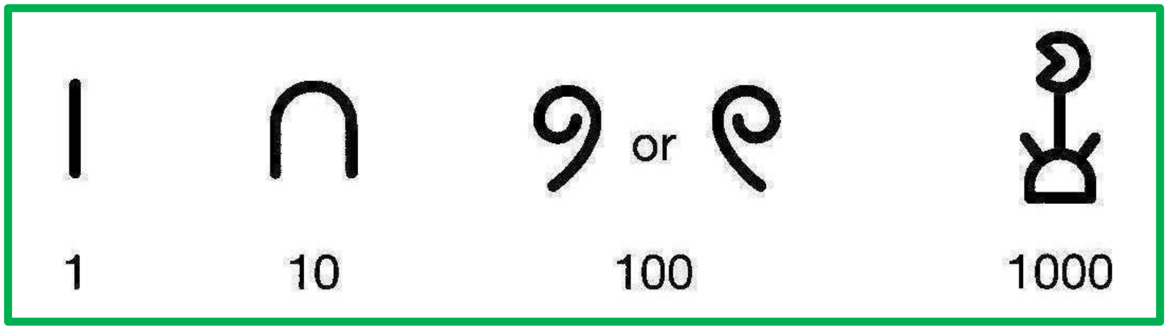






Egyptian counting

Decimal system, with different symbols for 1, 10, 100, etc.

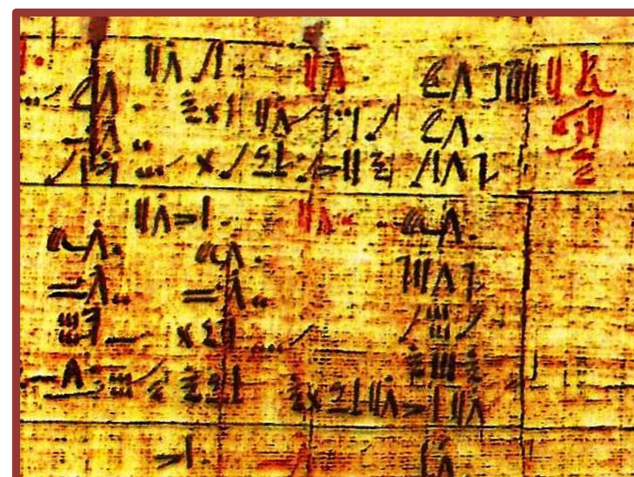


1 = rod

10 = heel bone

100 = coiled rope

1000 = lotus flower



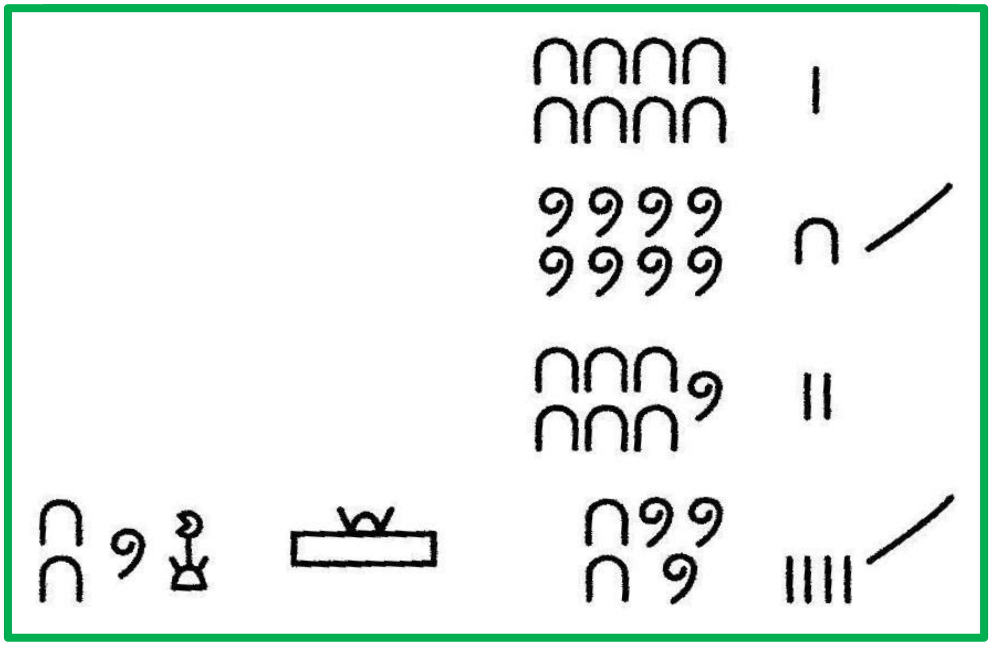
Fractions: reciprocals $^1/_n$ (or $^2/_3$)

for example: $^2/_{13} = {}^1/_8 \; {}^1/_{52} \; {}^1/_{104}$

Adding Egyptian numbers

367 + 756 = 1123

Egyptian multiplication



80	1
800	10 /
160	2
320	4 /

$80 \times 14 = 1120$

Rhind Papyrus – Problem 25

A quantity and its $^1/_2$ added together become 16. What is the quantity?

$[x + x/2 = 16]$

Answer: $10^2/_3$

Assume 2

method of false position

\	1	2
\	$\frac{1}{2}$	1
	Total	3

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

\	1	3	←
	2	6	
\	4	12	← 16
	$\frac{2}{3}$	2	
\	$\frac{1}{3}$	1	←
	Total	$5\frac{1}{3}$	

	1	$5\frac{1}{3}$
\	2	$10\frac{2}{3}$

Rhind Papyrus – Problem 31

A quantity, its $^2/_3$, its $^1/_2$, and its $^1/_7$, added together, become 33. What is the quantity?

[Solve $x + 2x/3 + x/2 + x/7 = 33$]

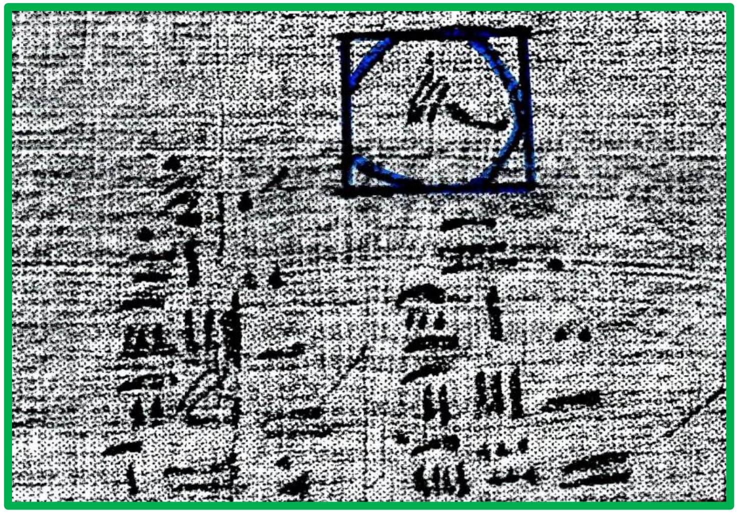
Solution: The total is

$$14\ \frac{1}{4}\ \frac{1}{56}\ \frac{1}{97}\ \frac{1}{194}\ \frac{1}{388}\ \frac{1}{679}\ \frac{1}{776},$$

$14\frac{28}{97}$

which multiplied by $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$ makes 33.

Problem 50



Example of a round field of diameter 9 khet. What is its area?

Answer

Take away $^1/_9$ of the diameter, namely 1.

The remainder is 8.

Multiply 8 times 8; it makes 64.

Therefore it contains 64 setat of land.

Problem 79

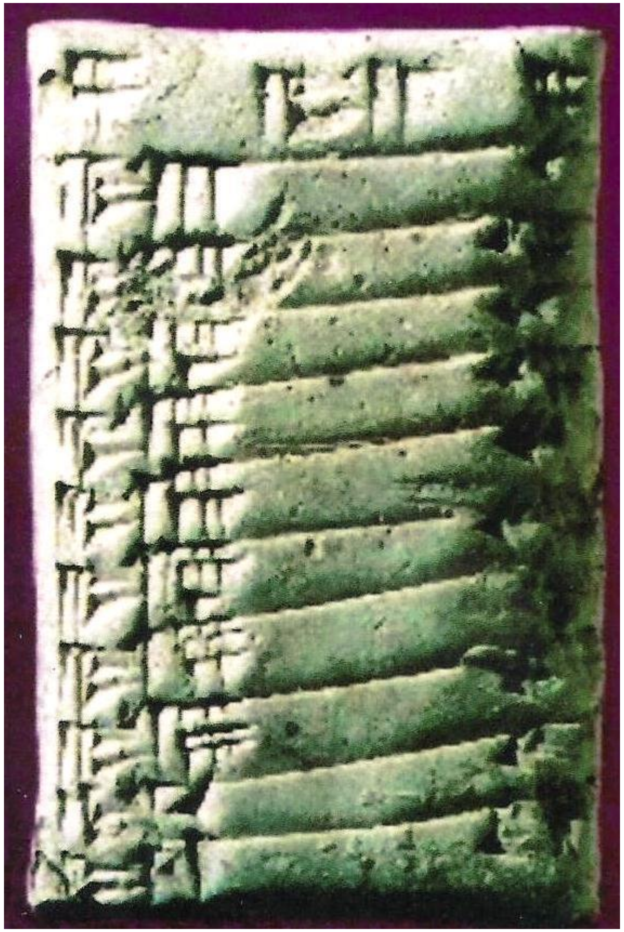
Houses	7
Cats	49
Mice	343
Spelt	2401
Hekat	16807
Total	**19607**

Mesopotamian counting

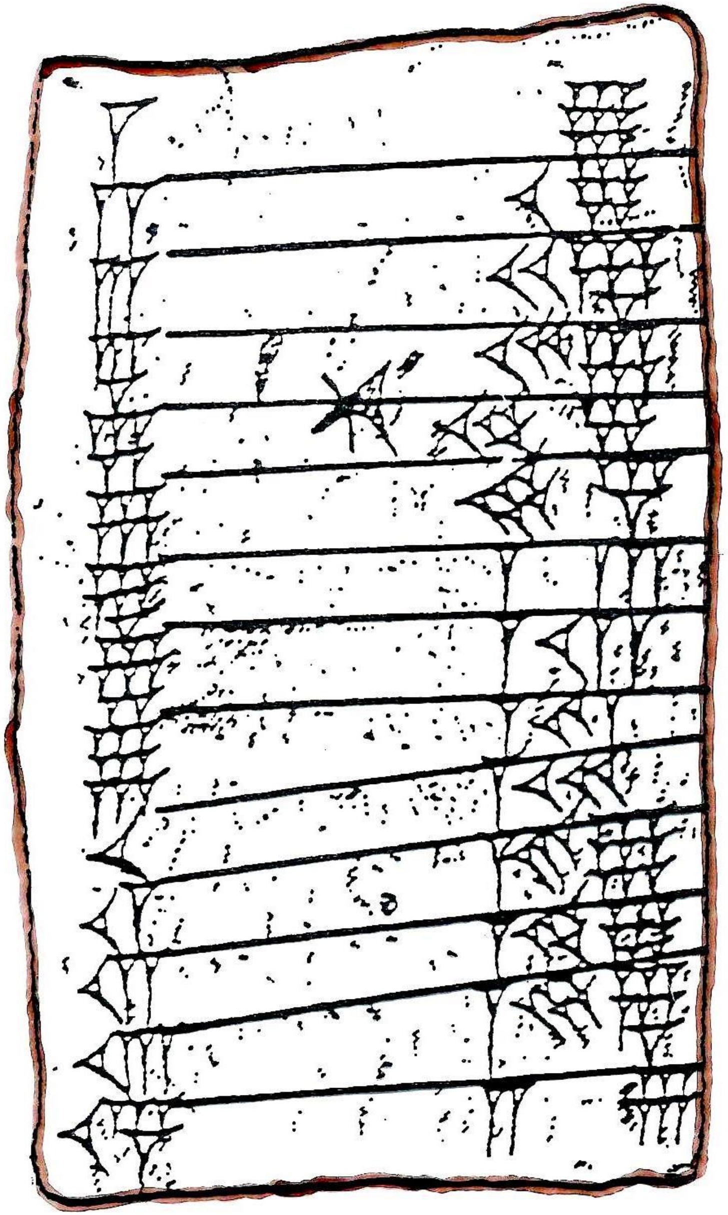
Cuneiform writing: place-value system (based on 60)

symbols: Y (or I) and <

= (41 x 60) + 40,
or $41^{40}/_{60}$, or . . .

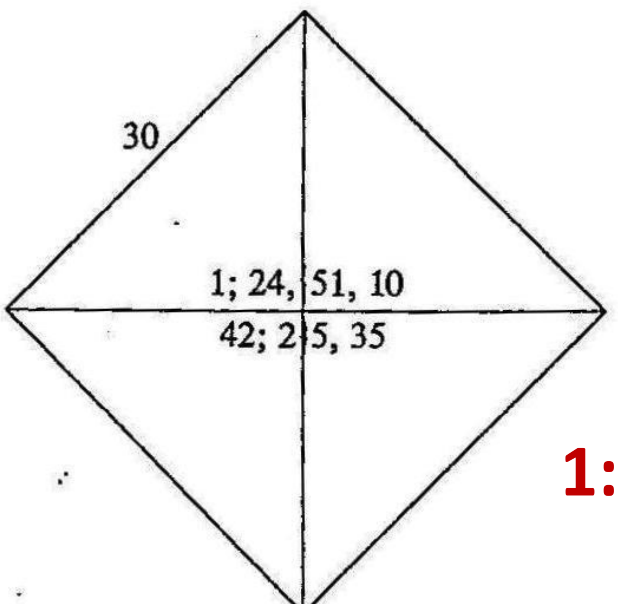


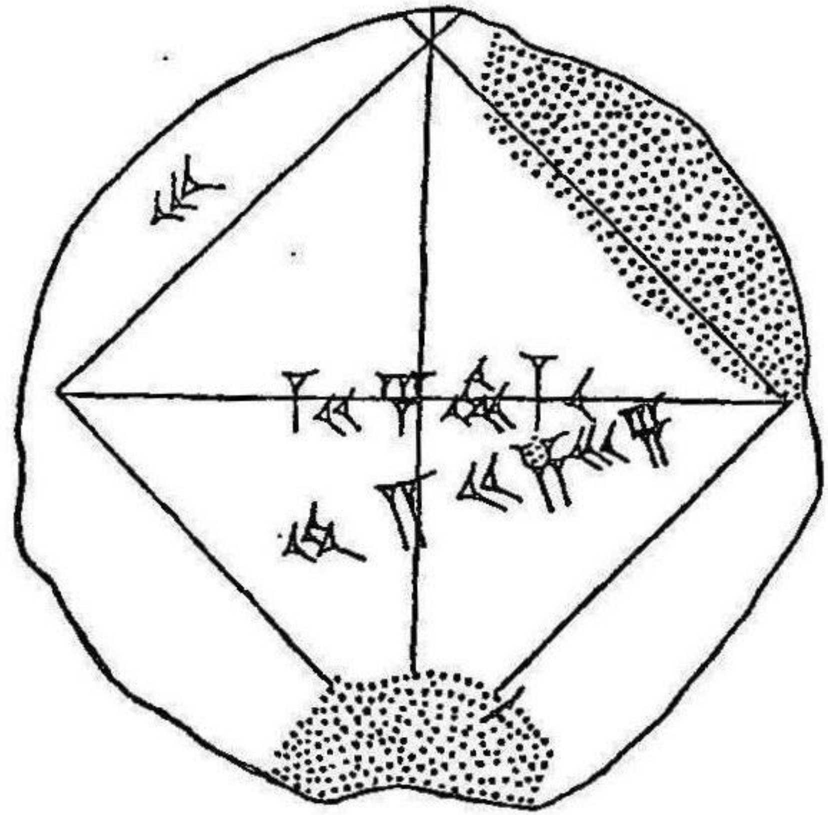
9-times table



1	9
2	18
3	27
4	36
5	45
6	54
7	63
8	72
9	81
10	90
11	99
12	108
13	117
14	126

The square root of 2





$$1: 24,51,10 = 1 + {}^{24}/_{60} + {}^{51}/_{3600} + {}^{10}/_{216000}$$

= 1.4142128… (in decimals)

A Problem Tablet – Weighing a Stone

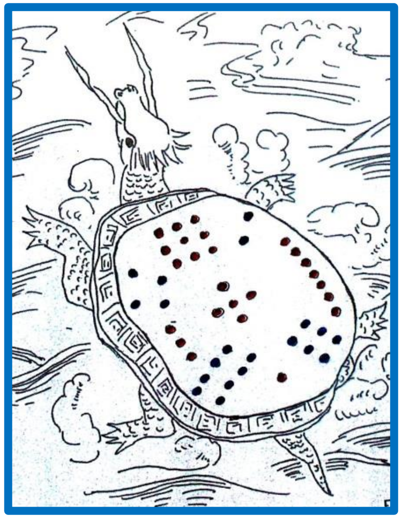
I found a stone, but did not weigh it;
After I weighed out 6 times its weight,
added 2 gin,
and added one-third of one-seventh multiplied by 24,
I weighed it: 1 ma-na.
What was the original weight of the stone?

Solution:

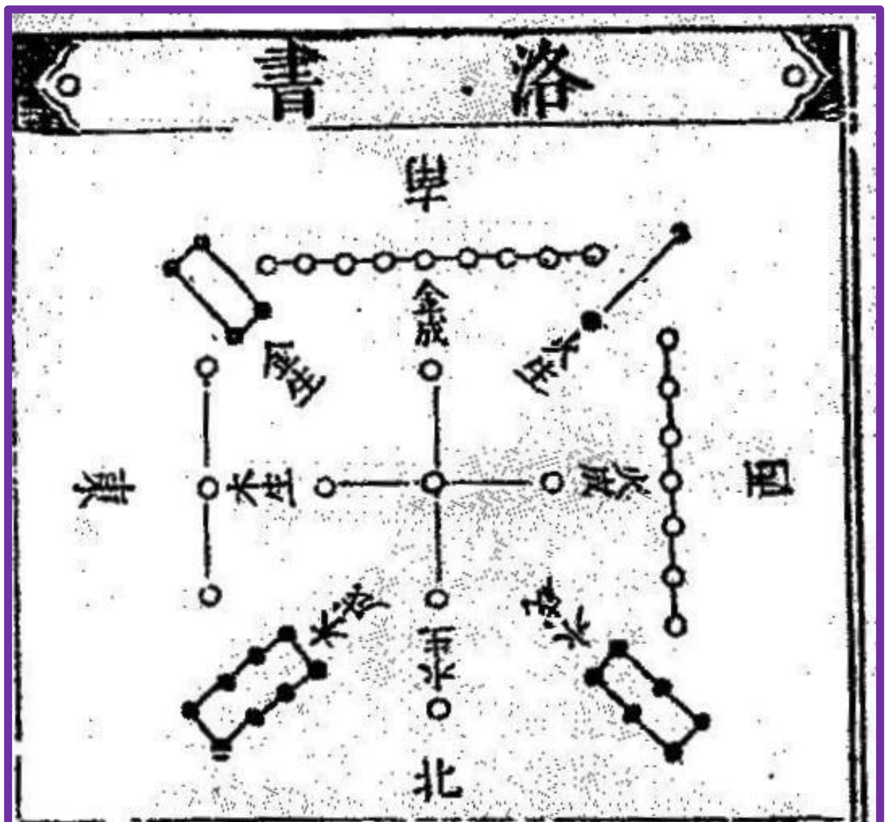
$$(6x + 2) + \frac{1}{3} \cdot \frac{1}{7} \cdot 24\,(6x+2) = 60 \text{ gin}$$

so $x = 4\,{}^{1}\!/_{3}$ gin.

Chinese magic squares

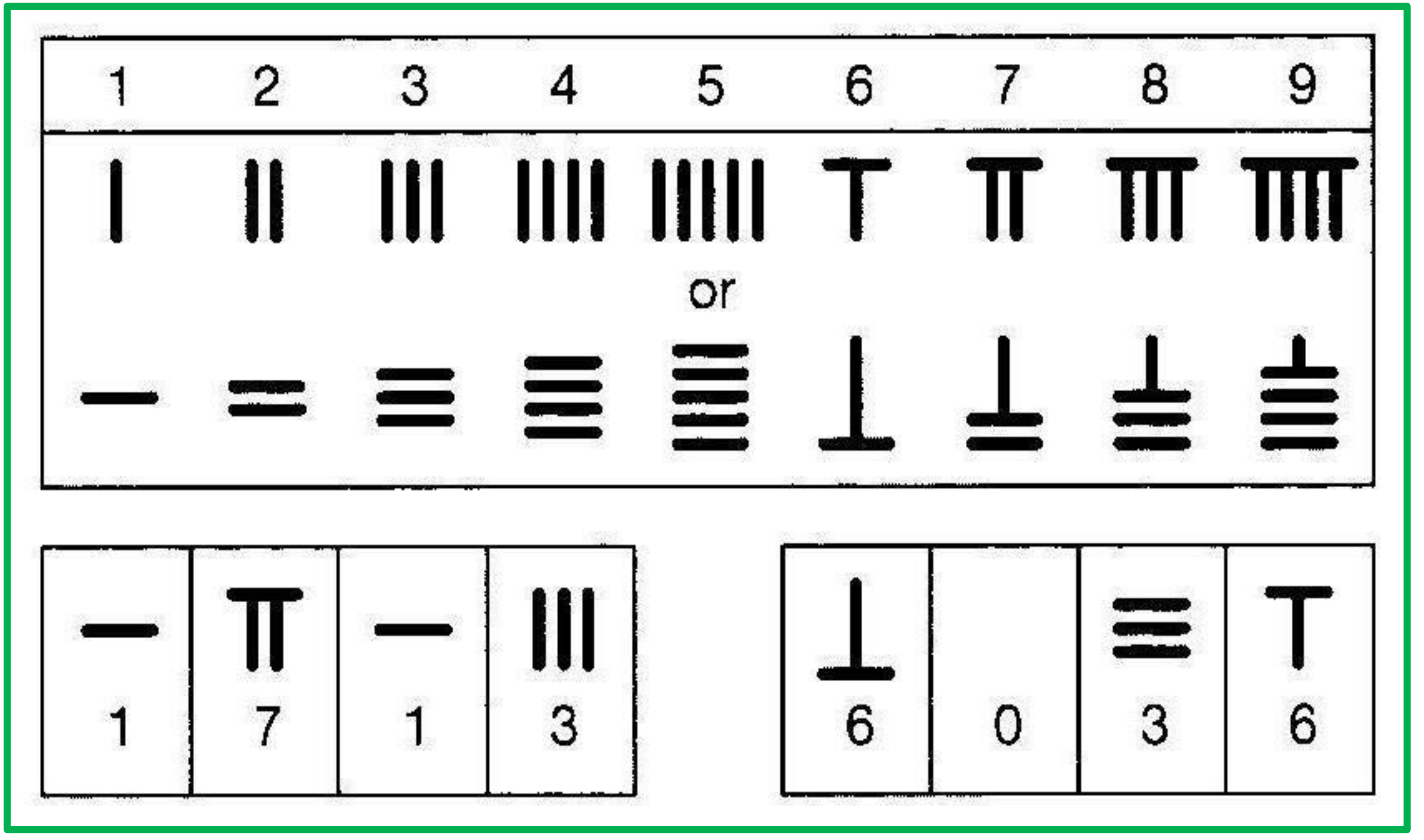


4	9	2
3	5	7
8	1	6



31	76	13	36	81	18	29	74	11
22	40	58	27	45	63	20	38	56
67	4	49	72	9	54	65	2	47
30	75	12	32	77	14	34	79	16
21	39	57	23	41	59	25	43	61
66	3	48	68	5	50	70	7	52
35	80	17	28	73	10	33	78	15
26	44	62	19	37	55	24	42	60
71	8	53	64	1	46	69	6	51

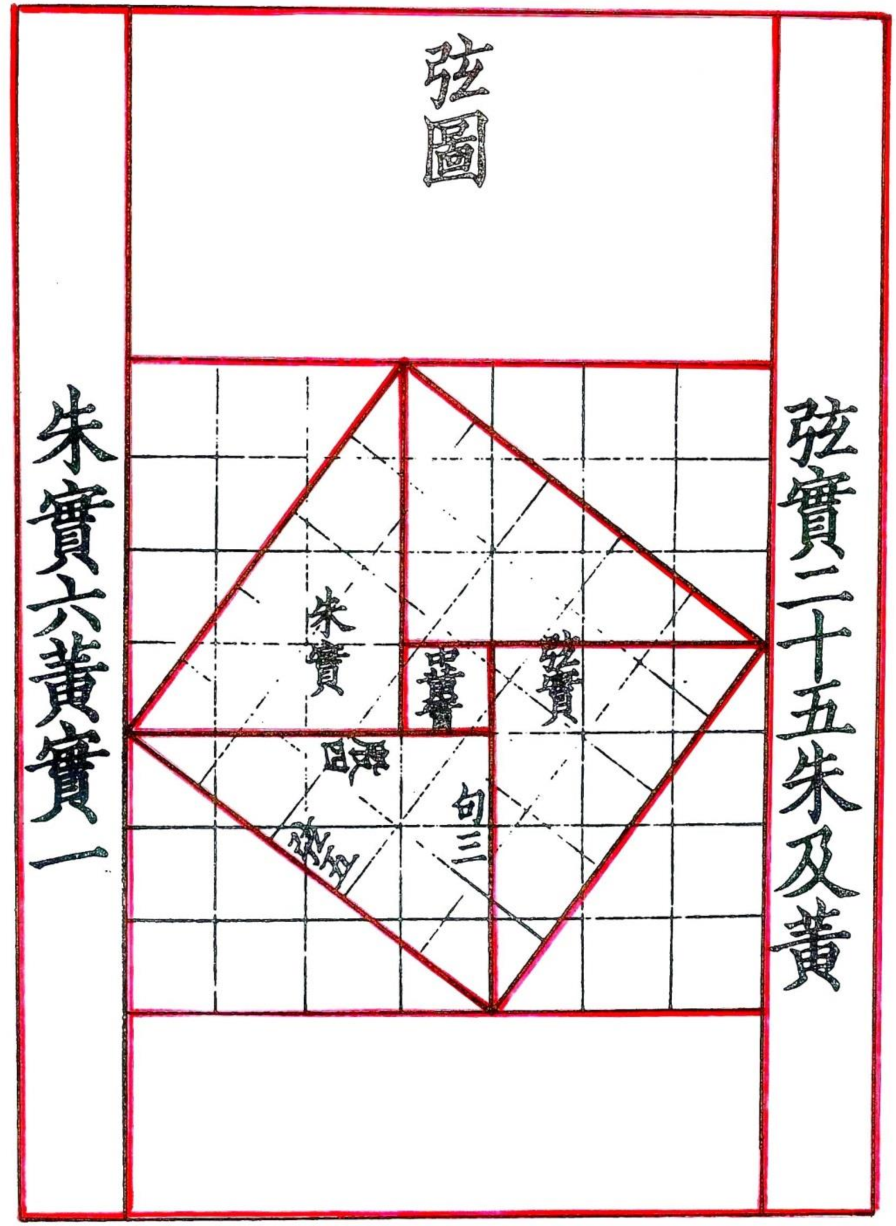
Chinese decimal counting boards

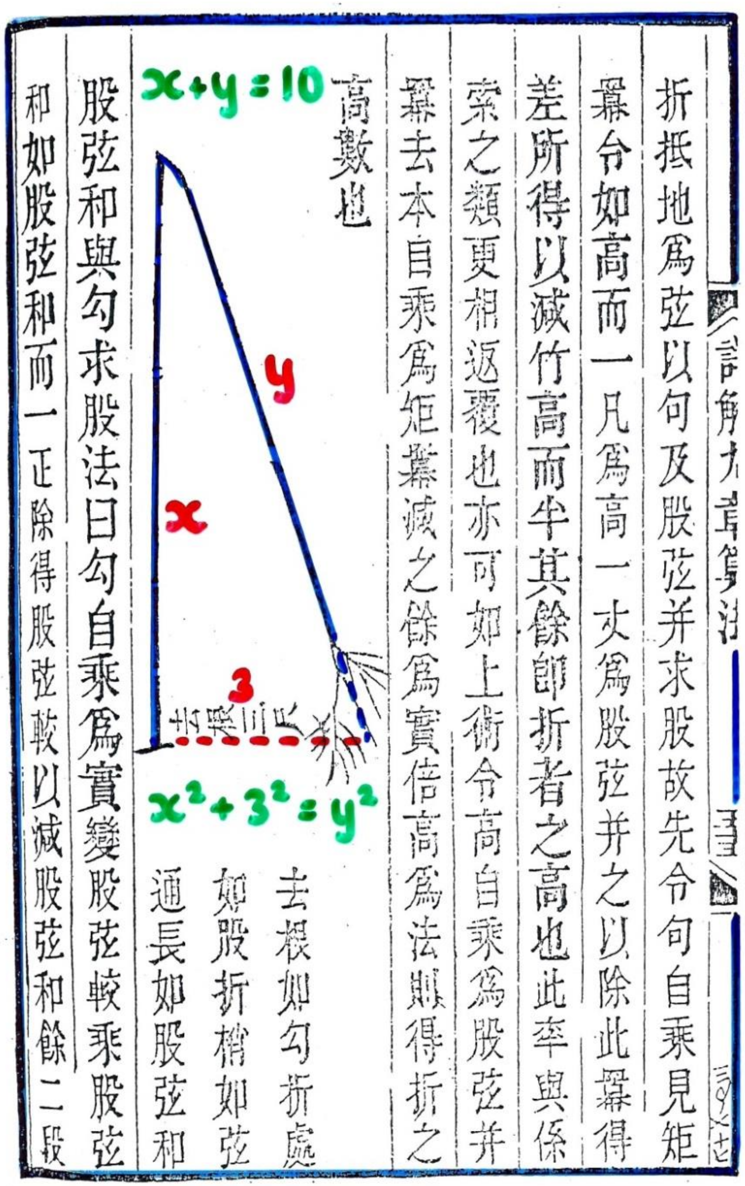


Zhou-bei suanjing

(*The arithmetical classic of the gnomon . . .*)

Dissection proof of the *gou-gu* (Pythagorean theorem)





折抵地爲弦以句及股弦并求股故先令句自乘見矩
冪令如高而一凡爲高一丈爲股弦并之以除此冪得
差所得以減竹高而半其餘即折者之高也此率與係
索之類更相返覆也亦可如上術令高自乘爲股弦并
冪去本自乘爲矩冪減之餘爲實倍高爲法則得折之
高數也

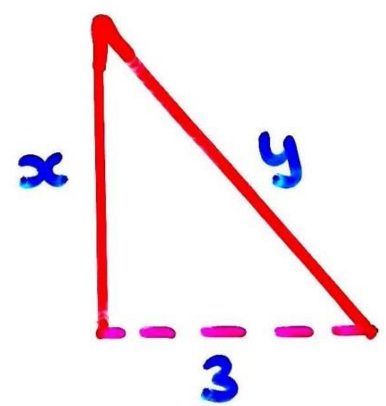
去根如勾折處
如股折梢如弦
通長如股弦和

股弦和與勾求股法曰勾自乘爲實變股弦較乘股弦
和如股弦和而一正除得股弦較以減股弦和餘二股

詳解九章算法

The broken bamboo problem

A bamboo 10 chi high is broken, and the upper end reaches the ground 3 chi from the stem. Find the height of the break.



$$x + y = 10$$

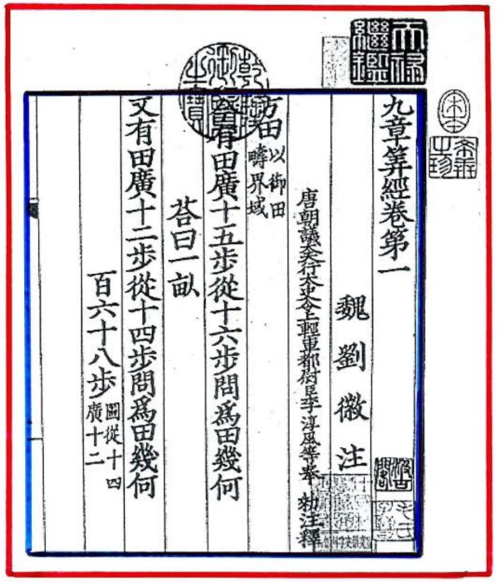
$$x^2 + 3^2 = y^2$$

The Chinese remainder theorem

Sun Zi (AD 250) in *Sunzi suanjing*
(Master Sun's mathematical manual)

We have things of which
we do not know the number.
If we count them by 3s the remainder is 2
If we count them by 5s the remainder is 3
If we count them by 7s the remainder is 2
How many things are there?

Jiuzhang suanshu
(200 BC?)
Nine Chapters on the Mathematical Art



Agriculture, business, surveying, etc.

- **calculation of areas and volumes**
- **calculation of square and cube roots**
- **study of right-angled triangles**
- **simultaneous equations**

Chinese simultaneous equations

Given 3 bundles of top grade paddy, 2 bundles of medium grade paddy, and 1 bundle of low grade paddy, yield 39 *dou* of grain.

2 bundles of top grade paddy, 3 bundles of medium grade paddy, and 1 bundle of low grade paddy, yield 34 *dou*.

1 bundle of top grade paddy, 2 bundles of medium grade paddy, and 3 bundles of low grade paddy, yield 26 *dou*.

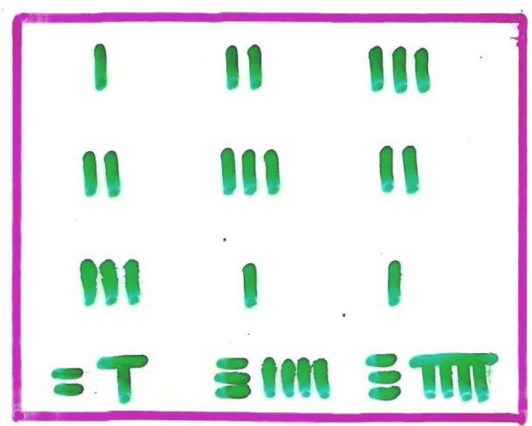
Tell: how much paddy does one bundle of each grade yield?

***Answer*: Top grade paddy yields $9^1/_4$ *dou per* bundle; medium grade paddy $4^1/_4$ *dou*; and low grade paddy $2^3/_4$ *dou*.**

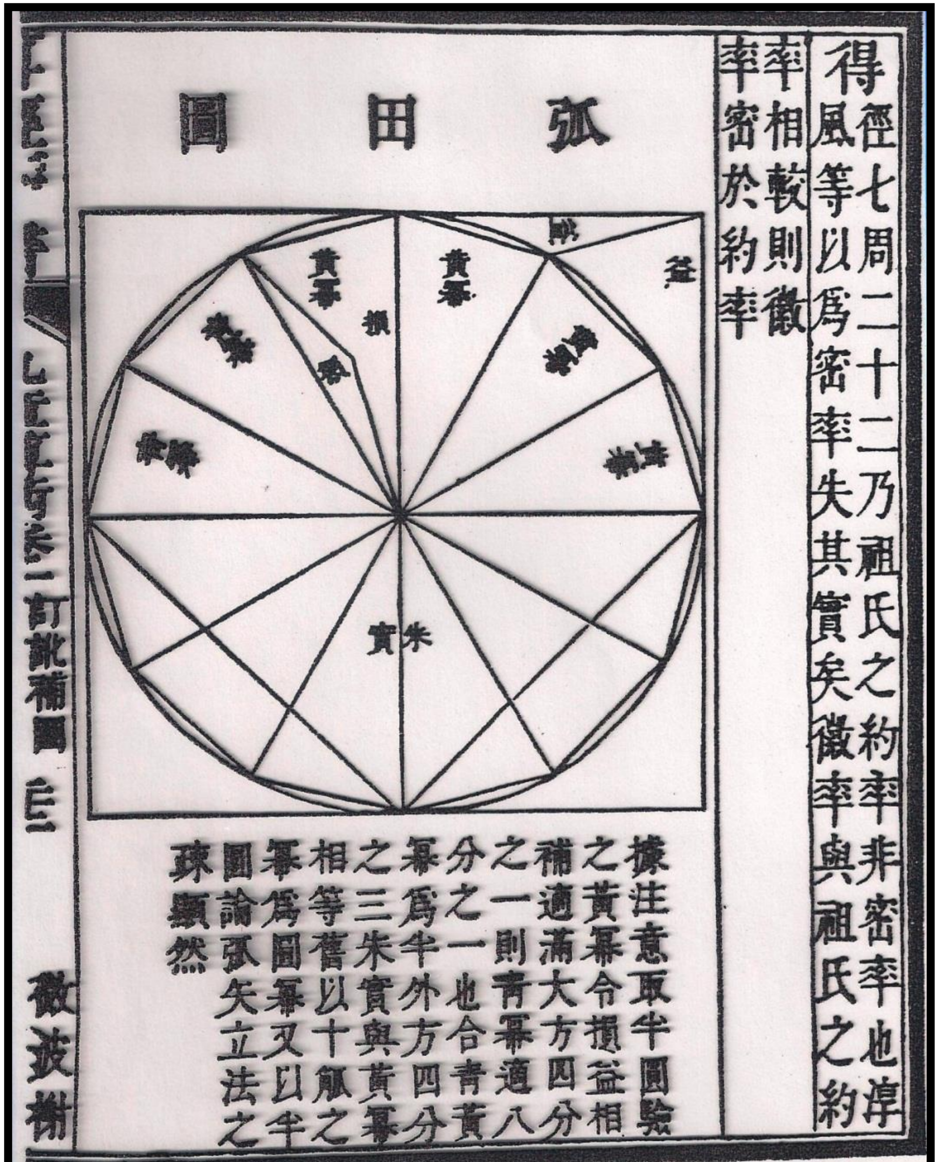
$$3A + 2B + 1C = 39$$

$$2A + 3B + 1C = 34$$

$$1A + 2B + 3C = 26$$



Chinese values for π



弧田圖

得徑七周二十二乃祖氏之約率非密率也焞風等以爲密率失其實矣徽率與祖氏之約率相較則徽率密於約率

據注意取半圓驗之黃冪令損益相補適滿大方四分之一則青冪適八分之一也合青黃冪爲半外方四分之三朱實與黃冪相等舊以十觚之冪爲圓冪又以半圓論弧矢立法之疎顯然

Zhang Heng (AD 100)

$\pi = \sqrt{10}$

Liu Hui (AD 263)

$\pi = 3.14159$

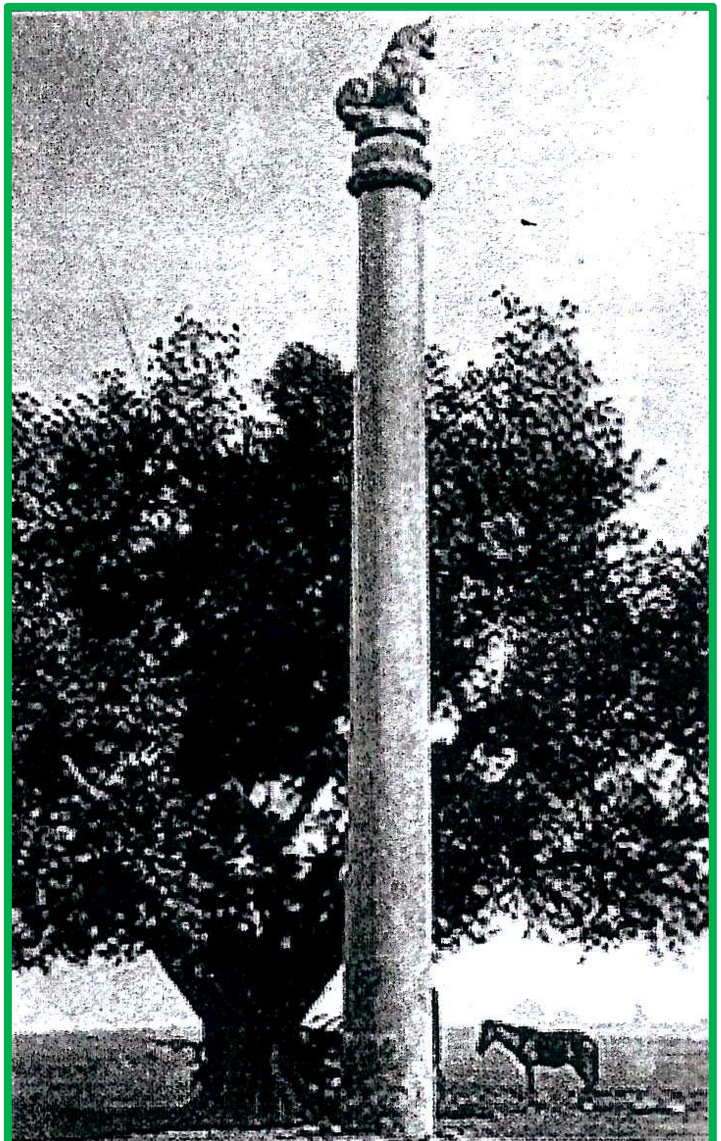
(3072 sides)

Zu Chongzhi (AD 500)

$\pi = 3.1415926$

(24576 sides)

and $\pi = {}^{355}/_{113}$

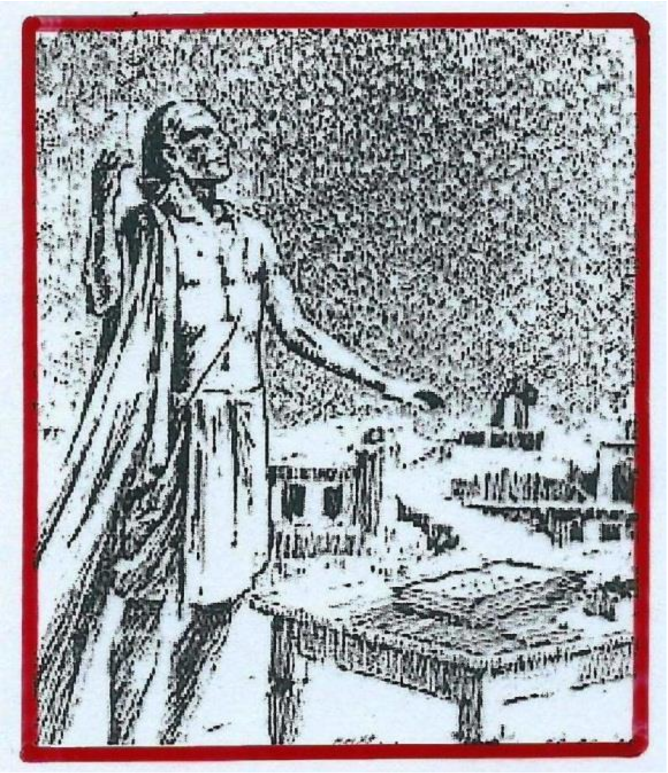


Indian counting

King Asoka (c. 250 BC), the first Buddhist monarch: numbers were inscribed on pillars around the kingdom

They used a place-value system based on 10
– with only
1, 2, 3, . . . , 9
– and eventually also 0

Aryabhata (AD 500)



Sum of an arithmetic progression:

$$6 + 9 + 12 + 15 + 18 + 21 = ?$$

The desired number of terms, minus one,
halved, multiplied by the common
difference between the terms,
plus the first term, is the middle term.
This, multiplied by the number
of terms desired, is the sum
of the desired number of terms.

OR

The sum of the first and last terms
is multiplied by half the number of terms.

$$\text{sum} = n\left\{\left(\frac{n-1}{2}\right)d + a\right\}$$

$$= \frac{n}{2}\left\{a + (a + (n-1)d)\right\}.$$

Brahmagupta (c. AD 600)

Calculating with zero and negative numbers.

The sum of cipher and negative
is negative;
Of positive and nought, positive;
Of two ciphers, cipher.

Negative taken from cipher
becomes positive,
and positive from cipher is negative;
Cipher taken from cipher is nought.

The product of cipher and positive,
or of cipher and negative, is nought;
Of two ciphers, it is cipher.

Cipher divided by cipher is nought.

Brahmagupta: ‘Pell’s equation’

Tell me, O mathematician,
what is that square which multiplied by 8
becomes – together with unity – a square?

$8x^2 + 1 = y^2$: so $x = 1, y = 3$ or $x = 6, y = 17$, or . . .

In general, given C,
solve $Cx^2 + 1 = y^2$

$C = 67$:

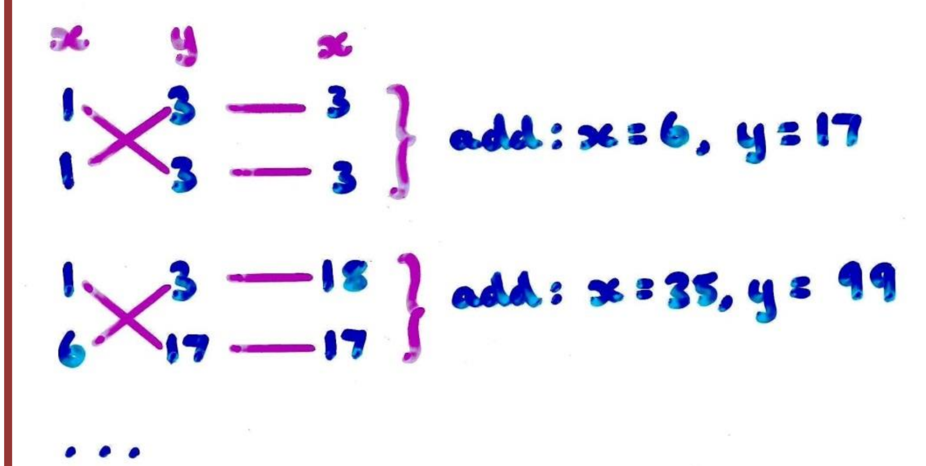
$67x^2 + 1 = y^2$

Solution:

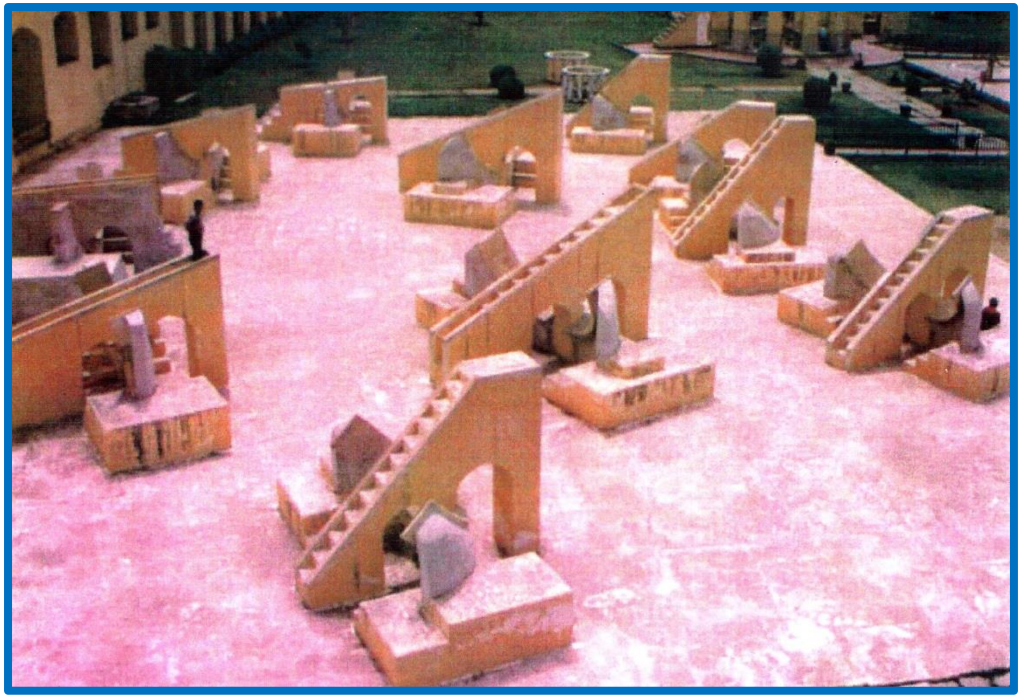
$x = 5967$

$y = 48,842$

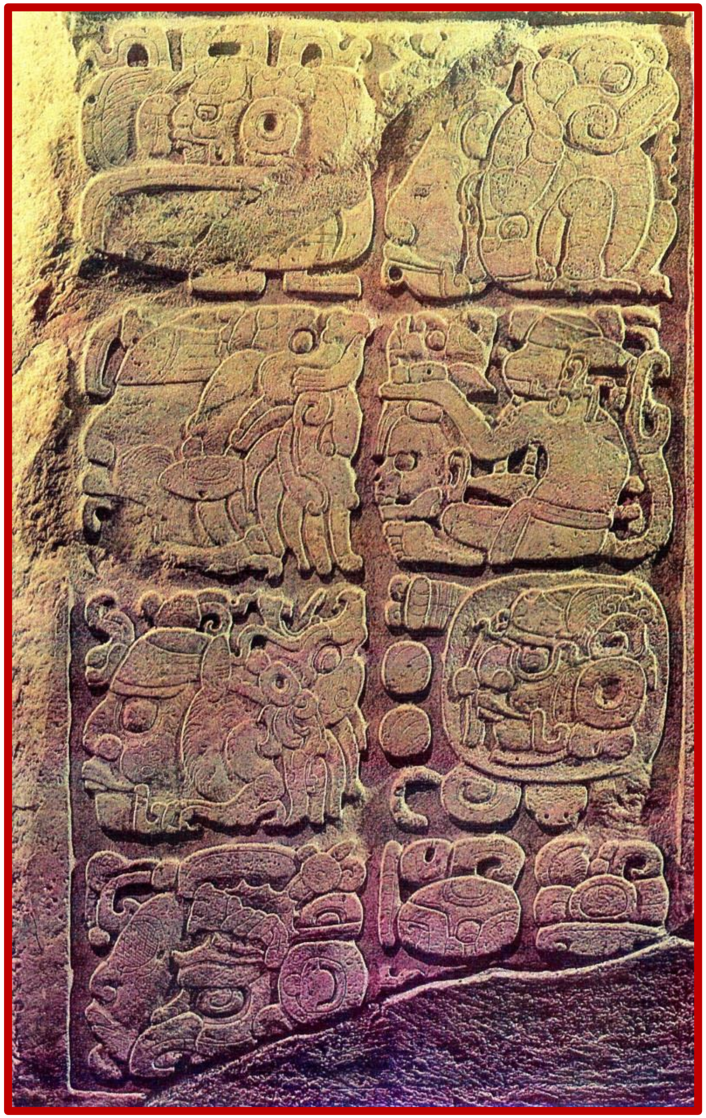
To find solutions:



Jantar Mantar (Jaipur & Delhi)

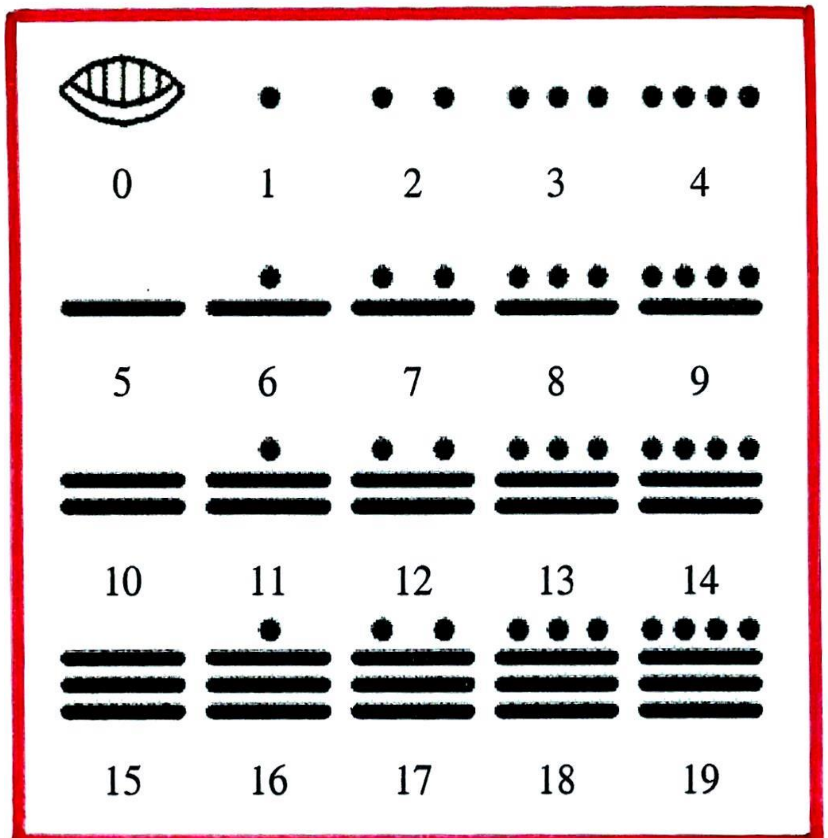


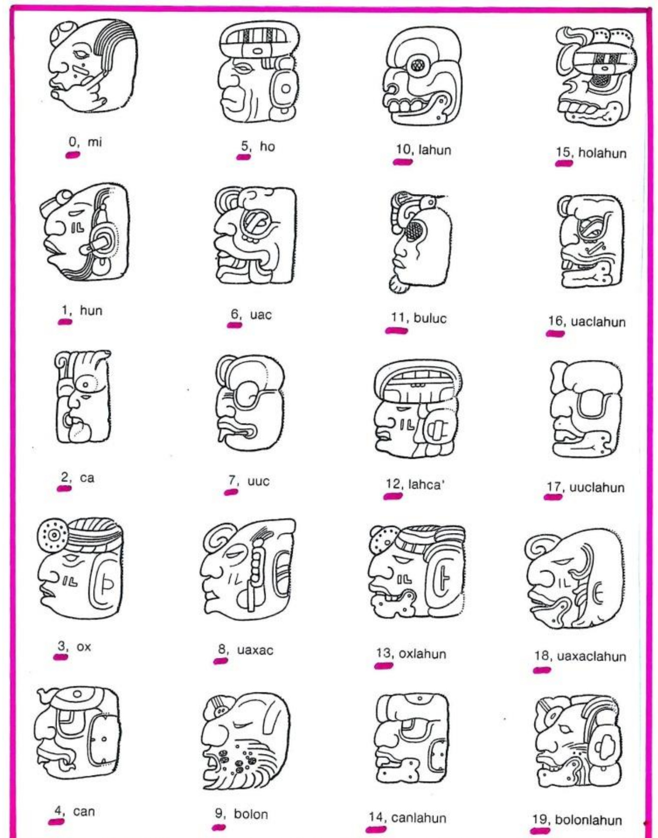
The Mayans of Central America



A Mayan codex

Mayan counting



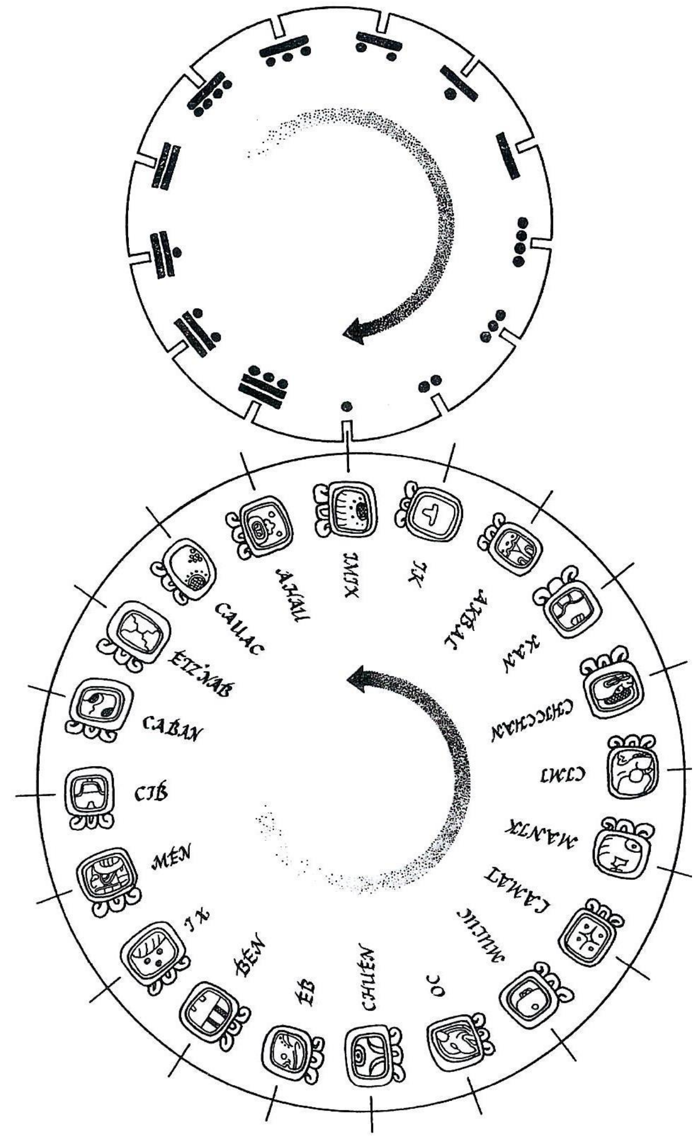


The Mayan calendar

Two forms:

260 days: 13 months of 20 days
365 days: 18 months of 20 days
(+ 5 'evil' days)

These combine to give a 'calendar round' of 18980 days (= 52 years), and these rounds are then combined into longer periods



Mayan timekeeping

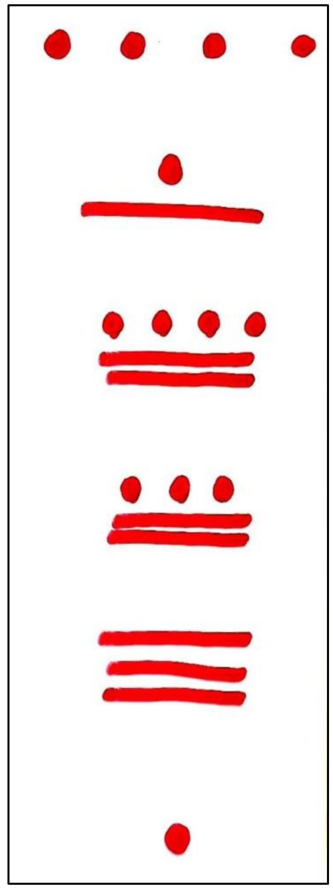
1 kin = 1 day

20 kins = 1 uinal
= 20 days

18 uinals = 1 tun
= 360 days

20 tuns = 1 katun
= 720 days

20 katuns
= 1 baktun
= 144000 days



= 4 × 2880000 = 11520000 days

= 6 × 144000 = 864000 days

= 14 × 7200 = 100800 days

= 13 × 360 = 4680 days

= 15 × 20 = 300 days

= 1 × 1 = 1 day

Total: 12,489,781 days

Dating a calendar stone

This limestone calendar stone from Yaxchilan notes a particular date.

The Mayan calendar started in 3114 BC, and the numbers on this stone date it as 11 February 526 AD