How Do Finance Specialists Think?

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Outline

- The problem of finance
- The emergence of modern finance
- Finance in a world of market imperfections
- Conclusions

The problem of finance

- Historically, finance has emerged from thinking of economists who have studied the problem of *allocating resources over time*. Here the key questions are:

  1. How much should be saved and invested in projects that will pay off in the future?

  2. Among the alternatives available what is the best form for the investments to take?

- However, these questions inevitably hits up against the uncertainty of future events. The proper formulation of thinking about these problems awaited tools for the study of choice making under uncertainty.

Major developments in tools for the analysis of uncertainty

- Early building blocks were (a) *expected utility theory* by von Neumann, Morgenstern, and others and (b) the state contingent claims approach made the key insight that goods we consume can be distinguished by both the date when we consume and the *state of nature* prevailing when we consume them.

- Thus the scope of finance is to study the problem of *allocating resources over time and over states of nature.* This *risk shifting* is accomplished in financial contracts and securities.

- Arrow, Debreu and others showed that in *general equilibrium* with *complete market* securities markets will be efficient in the sense of Pareto.

- When some markets are missing Arrow also showed that if securities market allow all risks to be hedged, then markets will be *effectively complete.*

Early applications to finance

- Modern portfolio theory by Tobin and Markowitz in the early 1950's. Allocate wealth based on expected returns on projects and on the variances and covariances of all returns. This reaps the benefits of *diversification*.

- The Capital Asset Pricing Model (CAPM) was developed by Sharpe, Lintner, Mossin and Treyner about 1960. This was most prominent early example of general equilibrium analysis to be put to a very practical purpose. The CAPM is an example of a *linear pricing model*: the risk premium should be a linear function of that security's *systematic risk* (reflecting the correlation of that market with an index of returns for the market overall).

The emergence of modern finance

- *Diversification of risks*, *mean-variance analysis*, and *competitive equilibrium pricing* are part of the general intellectual bagage of all economists these days. What is special about finance?

- It think that the key developments that give a distinct finance approach to problems were contributions beteen the late 1950's and the late 1970's building on the economics of uncertainty which made us aware of the power of the logic of *arbitrage*.

- The time period here with reference to the publication of the paper by Modigliani and Miller in 1958 and to the publication in 1979 of the paper by Harrison and Kreps.

- Arbitrage is an old notion in economics and underlies the venerable *law of one price*. However, starting with Modigliani and Miller it was shown that often you can go very far in understanding a problem without postulating all the structure about preferences, production technology and so forth as typically required in a fully formulated economic model.

What is an arbitrage?

- *It is the purchase of one collection of goods or securities and the simultaneous sale of another collection of goods and securities which produces a gain in at least one state of nature without incurring a loss in any state of nature.*
- An arbitrage is a private money machine. As such, people can be relied upon to pursue arbitrage opportunities without limit.
- Therefore, prices will be forced to adjust to reflect *no arbitrage conditions*. Thus the logic of arbitrage leads us to rules for *relative pricing of securities*.

Modigliani-Miller

- In the absence of market frictions the value of the equity, debt and other liabilities of the firm must have a total value equal to the value of the assets of the firm.

- Merton Miller's pizza pie analogy: It does not matter whether you cut the pie into many slices or few, it is still has the same total number of calories.

- In symbols for a firm with only Debt and Equity,

$$A = D + E$$

- This has many implications. For example,

$$ROE - r = \frac{A}{E}(ROA - r)$$

  where r is return on debt. Higher returns to shareholders come from either increasing returns on assets or increasing leverage (and risk).

Modigliani-Miller with state contingent claims

- This leads us to an insight as to how values of securities can be related to their expected payoffs in the future.

- Suppose that there is a complete market for securities that payoff one unit of numraire in a given state of nature s and nothing otherwise, and let that security's price be given as π_s.

- What is the value of a security that pays off 1 unit in every state? This security is equivalent to a portfolio consisting of one unit of each state contingent claims. It is also equivalent to investing $1/(1+r)$ today at the risk-free rate r. So its no-arbitrage value is,

$$V_0 = \sum_s \pi_s = \frac{1}{(1+r)}$$

- Now let us use the prices π_s and the interest rate r to define a set of variables $\pi_s^* = \pi_s(1+r)$. These must be positive because otherwise there is an arbitrage. By construction they sum to unity $\Sigma\,\pi_s^* = 1$. So this set of modified prices defines a probability distribution over future states of the world. This distribution is called the "risk-neutral" probability distribution in finance.

- Apply this to a general security paying a variable amount y_s in each state s. Its no arbitrage value is is,

$$V = \sum_s \pi_s y_s = \sum_s \frac{\pi_s^*}{(1+r)} y_s = \frac{1}{(1+r)} E^* y_s$$

  That is, the value of any security can be expressed as the present discounted value of its expected payoff tomorrow where expectations are taken with respect to the *risk neutral probability distribution* and discounting is done at the risk-free rate.

- We stress that if the prices π_s are the market prices of the elementary state claims then this expression gives the fair value in the market of the security. Even though agents may be averse to risk, securities are priced in the market as though agents were risk-neutral but calculated expectations using the probability distribution π_s^*.

- Now the market value of the asset V computed in this way should be distinguished from the *actuarially fair value* of the asset, V^a which would be based on the *statistical* probability distribution over states, π_s^a,

$$V^a = \frac{1}{(1+r)} \sum_s \pi_s^a y_s = \frac{1}{(1+r)} E y_s$$

  Note that in Ey_s expectation is taken with respect to the statistical distribution of states.

- The difference $V^a - V$ reflects the discount that the market imposes in order the bear the risk involved in holding the payoffs $\{y_s\}$. Another way in finance the market value of the security is expressed is using the *risk adjusted* rate of return on the security r^* defined by, $V = \frac{1}{(1+r^*)} E y_s$.

Black-Scholes pricing

- The famous the Black-Scholes formula for the pricing of call options is an extension of these valuation formulae.
- Starting with Louis Bachelier (in 1900) and subsequently by others who found various expressions for the *actuarially fair value* which in our notation can be expressed as,

$$C^a = \frac{1}{(1+r)} EMax(S_T - X, 0)$$

  This has two problems: it is not the *market* value and it involves predicting equity prices, i.e, taking a view on the future course of the stock market.

- For example, Paul Samuelson solved for the *actuarial value* under the assumption that the stock price followed a geometric Brownian motion which can be denoted,

$$dS = \mu S dt + \sigma S dz \qquad (1)$$

where dS is the change of the stock price over a very small time interval dt and dz is a Brownian motion, i.e., random variable following a normal distribution over dt. But this solution involves the drift of the stock process μ.

- Black and Scholes found that they could construct a *dynamic sequence of arbitrage portfolios* involving the underlying stock and short term lending such that the portfolio was riskless over short time periods dt. Applying the logic of arbitrage they were able to derive a partial differential equation that must be obeyed by the price of the call option in the absence of arbitrage. They were able to solve that equation and found an expression which can be written in our notation as,

$$C = \frac{1}{(1+r)} E^* Max(S_T - X, 0)$$

where the expectation E^* is taken with respect to the risk-neutralized process,

$$dS = rSdt + \sigma Sdz \tag{2}$$

Let us write this solution as $C(S_t, X, r, T, \sigma)$. This is remarkable because it gives us an expression for the *fair market value* of the option and because its calculation does not require us to take a view on the direction of the stock market, i.e., the parameter μ.

Extending Black-Scholes theory

- This approach can be applied to *any* security dependent upon a risk following a geometric Brownian motion as in equation (1). If the payoff of the security at maturity satisfies a known function $y(S_T)$ at some future date T, then in the absence of arbitrage its market value today is $V^+ = \frac{1}{(1+r)} E^* y(S_T)$ in which expectations are taken with respect to the risk-neutral process (2).

- Furthermore, this probability distribution for pricing over all the possible realizations S_T can be written as $f(s)$ and can be inferred from the prices of a complete set of options on the stock using the equation,

  $$f(s) = \gamma C_X(S_t, s, r, T, \sigma)$$

  where C_X is the partial derivative with respect to the second argument, the exercise price X.

- As with finite states, the risk-neutral distribution will differ from the statistical distribution in a way that reflects the market's willing to bear risk.

- The way that the risk neutral probability distribution $f(s)$ will differ from the statistical distribution denoted $f^a(s)$, is given by an object called the *pricing kernal*, denoted $k(s)$, and defined implicitly by the valuation relations.

$$V^+ = \frac{1}{(1+r)} \int_{\underline{S}}^{\overline{S}} y(s) f(s) ds = \frac{1}{(1+r)} \int_{\underline{S}}^{\overline{S}} k(s) y(s) f^a(s) ds$$

- It was shown that in equilibrium the pricing kernal is related to agents' preferences by

$$k(s) = \frac{U'(C_T(s))}{U'(C_t)}$$

,

where $U'(C_T(s))$ is the marginal utility of consumption at time T in state s. That is, in equilibrium the pricing kernal is given by the marginal rate of substitution between state-time (s, T) and today, t.

- Finally, it was established in a variety of general settings, not just for geometric Brownian motions, that so long as the system of markets is effectively complete, the risk-neutral density and therefore the pricing kernal are unique.

No arbitrage theory in real world finance

- Is all this theory relevant to practical financial problems? Yes! Keynes' dictum that the practical business person of today is hostage to the thinking of some defunct academic of the recent past was never truer than in financial markets of today.
- Big banks seek consistency in pricing using common pricing kernals for segments of the market, e.g., from a single model of the short term interest rate.
- These pricing models used for hedging and risk management (e.g., calculation of Value at Risk by simulating underlying stochastic processes).
- The whole business of secruitisation, structured finance and all the other aspects of what has become known as the "slicing and dicing" of risks are nothing other than elaborate exercises in the application of the complete markets tools we have outlined here.

Finance in a world of market imperfections

- This theory of arbitrage pricing was essentially complete by the early 1980's. Since then academic finance has been busy studying what happens when real-world market imperfections are too big to be ignored.

- Unfortunately, introducing *market frictions* into the analysis destroys some of the precision of the theory of no-arbitrage in complete markets.

Transactions costs

- Loss of precision can be seen in in the example of trilateral currency arbitrage. In perfect markets, you can buy and sell at the same price. Suppose the US-UK exchange rate is \$2/£ and the US-EU exchange rate is \$1.5/€ . Then to prevent arbitrage the EU-UK exchange rate must be € 1.3333/ £ .

  When buying price differs from selling price, this precision is lost. Suppose you can buy sterling at the US-UK ask of \$2.01/£ and sell sterling at the bid of \$1.99/£ . You also face a similar spread of \$1.49/€ bid and \$1.51/€ ask. Then this implies a *wider* no-arbitrage bounds for EU-UK exchange of € 1.31$\leq$ $bid \leq ask \leq$€ 1.3490.

- *General point*: transactions costs can make complicated arbitrages uneconomic.

- This idea that a range of prices may be compatible with no-arbitrage carries over to arbitrages in general. When markets are incomplete there is a multiplicity of pricing kernals $k_t(s)$ consistent with the statistical distribution $f^a(s)$ of the underlying risk.

- How do finance specialists resolve this indeterminacy?

- *Financial economists* tend to rely on the idea of *equlibrium pricing*, i.e., that a security's price will reflect a balance of supply and demand.

  1. Some determine the pricing kernal $k_t(s) = \frac{U'(C_T(s))}{U'(C_t)}$ by reference to an equilibrium model where the preferences $U(.)$, technology etc are explicitly spelled out.
  2. Others take a more reduced-form approach and posit a convenient form for the pricing kernal as a function of conditioning variables, either observable (e.g., GDP, employment etc.) or unobservable (latent), and determine the estimate the kernal statistically.

- Mathematicians and statisticians working in finance tend select among alternative risk-neutral pricing distributions on the basis of additional properties (e.g., minimal entropy) which are thought to be plausible.

Corporate finance

- Modern corporate finance looks at various frictions that lead to the violation of the M-M result so that financial policy may have an impact on the value of the firm.
- Early on the study of *corporate taxes* and *bankruptcy costs* rise to "trade-off" theories which held that firms would choose leverage to balance the tax advantages of debt versus potential costs of financial distress.
- Starting with Jensen and Meckling analysts have studied the asymmetry between the position of corporate *insiders* such as senior managers or controlling share holders and *outsiders* such as small share holders or creditors. This tends to create a wedge between the *external cost of capital* and the *internal cost of capital.*
- In such a world, the way financial operations are organized can have considerable impact on the value of the firm, the efficiency, and indeed on the prosperity of the economy generally. Studies tend to take either of two distinct directions.

1. moral hazard (hidden actions)
2. adverse selection (hidden types).

- Some look for rules of (second-best) optimal financial structures and policies with an *exogenously given* array of possible financial instruments available, typically simple debt and equity.

- Later, analysts have considered how the nature of the financial contracts themselves are determined endogenously in the interaction of insiders and outsiders. This has led to the study of *security design* where analyst have tended to use the tools of incomplete contracts theory or mechanism design. The result has been rich in theory but poor in robust empirical predictions.

- More recently, interest has returned to relatively simple models taking into account basic frictions such as tax shields and bankruptcy costs, but in way that takes into account the inter-temporal nature of financing and investment decisions made by firms. Thus there has considerable recent interest in *dynamic trade-off* models of capital structure and financial policy.

Asset markets

- In asset market research, much of the work over the last two decades has emerged from empirical studies of the implications of the *efficient markets hypothesis*. For simplicity of notation assume the risk-rate is zero. Then the perfect markets theory above implies by property of iterated expectations,

$$E_t V_{t+1} = E_t E_{t+1} y(S_T) = E_t y(S_T) = V_t$$

.

  This says that the market value of an asset follows a martingale and asset prices are unpredictable at first order.

- This is a testable hypothesis. Initial studies of the random behavior of stock prices were generally supportive of this efficient markets hypothesis.

- However, further analysis uncovered a variety of ways these markets seem to violate the properties of efficient markets. Early examples of such pricing *anomalies* include the January effect, the small firm effect and the profitability of certain technical trading rules.

- Some of these apparent inefficiencies disappeared with closer scrutiny of the data or once transactions costs were taken into account. Others, such as the profitability of technical trading rules, could be explained by the fact that the predicted returns were not excessive once you took into account the greater riskiness of the the returns. Furthermore, some apparent predictability of risk-adjusted returns be accounted by time variations in the pricing kernal $k_t(s)$, or equivalently of the marginal rate of substitution function.

- This pattern of empirical work uncovering apparent pricing anomalies and theorists coming up with more general theoretical explanations to account for them has continued unabated to the present. Roughly there are two lines of work:

  1. In the last fifteen years or so there has been great interest in *behavioral* explanations. Investor behavior which might be irrational in the sense that they are do not maximize a well-defined utility function or they do not process information in a correct manner (e.g., by doing Bayesian updating). This has borrowed insights of psychology where

concepts such as *over-confidence*, *envy* and *biased perceptions*.

2. The other line assumes agents are rational, and instead looks for explanations of pricing anomalies in more general representations of *agents' objectives* or in the *institutional environment*. Examples of the former line of research are those models that posit preferences exhibiting *habit formation* or *ambiguity aversion*. Examples of the latter are models that take into account *agency problems* that can emerge for example in *delegated investment management* or through *imperfect incentive schemes* for financial analysts.

Final comments

- Thus there is no general, settled theory of financial market imperfections. Nevertheless, I would argue that there is an implicit common thrust in most of current finance research.

- This is the shared goal to achieve a coherent body of theory as tight and consistent as the theory of no-arbitrage in complete markets that is also consistent with data.

- The paradigm of self-interested agents maximizing some objective subject to constraints imposed by the institutional environment has proved so rich and malleable that virtually no financial economist of my acquaintance makes any pretense of offering a revolutionary idea that would sweep this framework away.