2023 LSE-Warwick Workshop on Search Games and Patrolling

Monday 3rd July

10:30-11:45 Tutorial: *An introduction to network search games.*
Thomas Lidbetter, Rutgers Business School

12:00-12:30 *Search games with predictions*
Spyros Angelopoulos, CNRS and Sorbonne University

12:30-14:00 Lunch (provided by LSE)

14:00-14:30 *A stochastic game framework for patrolling a border*
Matthew Darlington, Lancaster University

14:45-15:15 *Recent Advances in Bamboo Garden Trimming Problem*
Leszek Gasieniec, University of Liverpool

15:45-16:15 *General Lotto Games with Scouts: Information versus Strength*
Jan-Tino Brethouwer, Delft University of Technology

16:30-17:00 *Exploring Multi-Agent Behaviour in Social Distancing and Search Games on Simple Networks*
Li Zeng, University of Warwick

Tuesday 4th July

10:30-11:00 *A Game Theoretic Approach To Search For A Moving Target*
Thuy (Christy) Bui, Rutgers Business School

11:15-11:45 *A recursive search game with a decremental budget: a step to bridge foraging ecology and search games*
Paul Clemencon, Université de Tours

12:00-12:30 *On a Mate Choice Problem with Scramble Competition, but without Discounting*
David Ramsey, Wroclaw University of Science and Technology

12:30-14:00 Lunch

14:00-14:30 *Some versions of Wythoff Nim, solved automatically*
Robbert Fokkink, Delft University of Technology

14:40-15:10 *The ‘three means' problem in winner-take-all games*
John Howard, LSE

15:20-15:50 *A comparative analysis of maze searching algorithms*
Kieran Drury, University of Warwick

Abstracts

Monday 3rd July

Speaker: **Thomas Lidbetter**, Rutgers Business School
Title: *An introduction to network search games.*
Abstract: Search games are typically played between a Searcher and a Hider. The Hider wishes to remain hidden and the Searcher wishes to find him. We discuss the classic search game on a network introduced by S. Gal in the 1970's, and present the solution to the game on so-called weakly Eulerian networks (networks consisting of a number of Eulerian cycles that, when reduced to a single point, leave a tree). We then discuss more recent variations to the classic model, including variable speed networks, where the time to traverse an arc depends on the direction of travel; and expanding search, where the Searcher can instantaneously return to any point of the network already visited.

Speaker: **Spyros Angelopoulos**, CNRS and Sorbonne University
Title: *Search games with predictions*
Abstract: Motivated by recent advances on algorithms with machine-learned predictions, we study search games in which the searcher enhances its strategy with some (unreliable) hint about the hider's position in the environment. The objective is to quantify the tradeoffs between the performance of the strategy in settings in which the hint is trusted, adversarially obtained or, more generally, has unknown, but bounded error. I will discuss applications of this framework to line and star search under pure strategies, but also some ongoing work on mixed strategies for problems such as searching on boxes and expanding search.

Speaker: **Matthew Darlington**, Lancaster University
Title: *A stochastic game framework for patrolling a border*
Abstract: We consider a stochastic game for modelling the interactions between smugglers and a patroller. A group of cooperating smugglers make regular attempts at bringing small amounts of illicit goods across a border. A single patroller has the goal of preventing the smugglers from doing so, but must pay a cost to travel from location to location. Our framework extends the literature by allowing the smugglers to choose a continuous quantity of contraband. We prove properties of the Nash equilibria in the game, and explore how parameters such as the penalties applied to the smugglers by the patroller and topology of the border can affect the patroller's strategy.

Speaker: **Leszek Gasieniec**, University of Liverpool
Title: *Recent Advances in Bamboo Garden Trimming Problem*
Abstract: A garden is populated by $n > 1$ bamboos $b_1, b_2, \ldots, b_n$ with the respective daily growth rates $h_1, h_2, \ldots h_n$. It is assumed that the initial heights of bamboos are zero. The gardener (or a robot) maintaining the bamboo garden is attending bamboos and trimming them, one after another, to height zero according to some predefined schedule. The Bamboo Garden Trimming Problem, or simply BGT, is to design a perpetual schedule of cuts to maintain the elevation (defined by the highest ever observed bamboo) of the garden as low as possible. We are considering two variants of BGT. In discrete BGT the gardener is allowed to trim only one bamboo at the end of each day. In continuous BGT the bamboos can be cut at any time, however, the gardener needs time to move from one

bamboo to the next one which is proportional to the distance (in some metric space) between the two bamboos. The problem is known to be NP-hard and several constant approximation algorithms for discrete BGT have been designed. In contrast, for continuous BGT only O(log n) approximation is currently known. This talk will summarize the newest developments in this area.

Speaker: **Jan-Tino Brethouwer**, Delft University of Technology
Title: *General Lotto Games with Scouts: Information versus Strength*
Abstract: We introduce General Lotto games with Scouts: a two-stage General Lotto game with asymmetric information. There are two players, Red and Blue, who both deploy troops to a field. With some probability, Blue gets information on Red's troops. We find optimal strategies for the case of a single field. Then we provide upper and lower bounds of the game value in a multi-stage case with multiple battlefields. These bounds are tight in some settings. We devise several ways to characterize the value of information versus strength. We conclude by drawing qualitative insights from these characterizations and the game values, and draw parallels with military practice.

Speaker: **Li Zeng**, University of Warwick
Title: *Exploring Multi-Agent Behaviour in Social Distancing and Search Games on Simple Networks*
Abstract: During the COVID-19 pandemic, social distancing has played a crucial role in preventing the spread of the virus. In this study, we investigate the impact of social distancing policy on agents' behaviour in spatial dispersion games. Specifically, we analyse the social distancing game where agents follow a common lazy random walk, remaining in their current location with probability p and randomly moving to adjacent nodes with probability 1-p. Our goal is to determine an optimal value of p that minimizes the expected time for all agents to achieve a socially distanced state. Furthermore, we extend search game to a multi-agent context on simple networks. Two searchers aim to locate a hider within a simple network, and they can choose to collaborate or compete with each other during the search. Interestingly, we find that the optimal search strategy for the searchers remains consistent regardless of teamwork, while the hider's behaviour is influenced by the teamwork of searchers, resulting in more frequent movement.

Tuesday 4th July

Speaker: **Thuy (Christy) Bui**, Rutgers Business School
Title: *A Game Theoretic Approach To Search For A Moving Target*
Abstract: We present a zero-sum game between a Hider and a Searcher. The Hider places a target in one of n boxes. The target then moves among boxes according to a time-homogeneous Markov chain. Each box i has a detection probability q_i. That is, given the target is in box i, a search of that box will find the target independently with probability q_i. The Searcher picks a search sequence of length m corresponding to the order in which the boxes are searched. The payoff is the probability of finding the target in m searches, which the Searcher wishes to maximize, and the Hider wishes to minimize. We prove the game has a value and each Player has an optimal strategy. We also present the solution of the game for some classes of Markov chains.

Speaker: **Paul Clemencon**, Université de Tours
Title: *A recursive search game with a decremental budget: a step to bridge foraging ecology and search games*
Abstract: Combining the search and pursuit aspects of predator-prey interactions into a single game, where the payoff to the Searcher (predator) is the probability of finding and capturing the Hider (prey) within a fixed number of searches was proposed by Gal and Casas (2014). Subsequent models allowed the predator to continue his search (in another 'round') if the prey was found but escaped the chase. However, it is unrealistic to allow this pattern of prey relocation to go on forever. Here we introduce a limit of the total number of searches, in all 'rounds', that the predator can carry out. We show how habitat structural complexity affects the mean time until capture: the quality of the location with the lowest capture probability matters more than the number of hiding locations. Moreover, we show that the parameter space defined by the capture probabilities in each location and the budget of the predator can be divided into distinct domains, defining whether the prey ought to play with pure or mixed hiding strategies. We discuss these findings in the light of foraging ecology (e.g. patch giving-up time rules), highlighting the bidirectional information flow between behavioural ecology and operational research.

Speaker: **David Ramsey**, Wroclaw University of Science and Technology
Title: *On a Mate Choice Problem with Scramble Competition, but without Discounting*
Abstract: This talk presents a game-theoretical model of mate choice. At the beginning of the breeding season each member of a large population searches for a mate of the opposite sex. Mutual acceptance is required for a partnership to be formed. On forming a partnership, a male and a female leave the pool of searchers. The length of time allowed for search is assumed to be finite. Given that an individual finds a partner, then his/her reward from search is assumed to be the value of his/her partner. The sex ratio is equal to one (i.e. there are the same number of males and females) and the distribution of the value of an individual is assumed to be discrete and independent of sex. The possible values of mates are $v_1, v_2, \dots, v_n$, where $v_1 > v_2 > \cdots > v_n > 0$. An individual of value v_i is said to be of type *i*. If an individual does not find a partner, then his/her reward from search is assumed to be zero. It follows that the game is symmetric with respect to sex. The rate at which prospective partners are found is assumed to be non-decreasing in the proportion of individuals still searching for a partner, i.e. as time progresses the rate at which prospective partners are

found is non-increasing. At one extreme of such a spectrum, the rate at which prospective partners are found is assumed to be proportional to the fraction of individuals still searching for a mate (the random mixing model). At the other end of this spectrum, the rate at which prospective partners are found is assumed to be constant (the singles bar model). When a searcher meets a prospective partner, then the value of this partner is taken from the distribution of the value of searchers in the current mating pool. Hence, the pressure on an individual to accept a partner results from the risk of not finding a partner (which increases as time progresses) and the fact that the pool of potential partners tends to become less attractive on average as times passes. We look for a symmetric equilibrium (according to sex) at which the strategy used by an individual only depends on his/her value (i.e. type). In the case where the reward of an individual is discounted according to the time spent searching, it has been shown that multiple equilibria can exist. This talk shows that when the reward of searchers is not discounted, then there exists a unique symmetric equilibrium. Given the expected reward of a type 1 searcher at this equilibrium, then the equilibrium can be derived by an inductive procedure. Based on this, a value induction procedure is defined to approximate the equilibrium.

Speaker: Robbert Fokkink, Delft University of Technology
Title: *Some versions of Wythoff Nim, solved automatically*
Abstract: To solve a take away game, you first try to find a pattern in the P-positions. This can be hard if not impossible, as illustrated by many examples in Winning Ways. Once a pattern is found, you have to prove that it holds. This can also be hard. However, I have some good news for you! There exists a machine that can do the job for you provided you can express your theorem in first order logic and a suitable number system. The machine is called Walnut, is freely available and easy to program, find it right here https://cs.uwaterloo.ca/~shallit/walnut.html. In this talk I will explain how Walnut solved all my problems in practically no time at all. More info through https://arxiv.org/abs/2210.03996 and https://arxiv.org/abs/2204.11805

Speaker: John Howard, LSE
Title: *The ‘three means' problem in winner-take-all games*
Abstract: A group of searchers compete to be first to find their (or a common) target. Each searcher has a given set of distributions for their search time. These distributions can be imagined as resulting from different choices of strategies for searching a forest (say) for a treasure trove. They submit their choices to a referee who samples from each distribution to obtain a discovery time for that player, and then the player with the shortest time wins. A general theory of such ``winner-take-all'' games was developed by Alpern and Howard (Operations Research 65, 2017). Their paper showed how the theory could be used to give an interesting alternative approach to solving the silent duel (and since then it has also been applied to the all-pay auction). Symmetric many-player games and asymmetric two-player games were analysed in the paper, but the many-player asymmetric case proved more difficult to solve. Here, I will show how to find solutions to asymmetric three player winner-take-all games with constraints on the means.

Speaker: **Kieran Drury**, University of Warwick
Title: *A comparative analysis of maze searching algorithms*
Abstract: This research analyses the performance of certain algorithms that search a maze for an exit or hider with the aim to minimise the expected time to finish the search. We consider a maze mathematically as an undirected graph G with vertices representing junctions and dead ends in the maze and edges representing passages between these vertices. We have a searcher who starts at a given location s in the graph G and we provide certain assumptions that we use to build and analyse algorithms that determine how the searcher traverses the maze. In particular, we assume that the searcher has no prior information about the maze - the searcher only has information about the parts of the maze they have traversed, but not of parts of the maze they have not traversed. This distinguishes this problem from typical shortest path problems. We focus on four main algorithms - a simple symmetric random walk, the Random Limited Backtracking Algorithm, the Randomised Tarry Algorithm and the Earlier In Later Out Algorithm – named the 'normal strategy' by Edward Anderson in his 1981 paper. For algorithms that guarantee a search within a finite number of steps, we present a method for generating all search paths the searcher can take, as well as the probability of each search path being taken in one search of the maze for the given algorithm. Therefore, we can calculate an exact expected search time. However, the combinatorial complexity of this method deems it unsuitable for analysis of even medium-sized graphs, and we rely on a simulation-based approach to draw conclusions by simulating each algorithm on a range of graphs. We determine which algorithm produces the lowest expected search time for hider who is distributed uniformly at random over the maze and which algorithm provides the lowest cover time of the maze, providing the worst-case scenario for the searcher who finds the hider located at the last point of the search. We find that there are significant differences in the performances of these algorithms, especially as the maze being searched becomes more complex.