Improved Approximation Algorithms for Weighted k-Set Packing

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6 March 2024

Abstract

The weighted k-Set Packing problem constitutes a fundamental problem in combinatorial optimization. It is defined as follows: As input, we are given a collection $\mathcal{S}$ of sets, each of cardinality at most k, and positive weights $w : \mathcal{S} \to \mathbb{R}_{>0}$. The task is to compute a sub-collection $A \subseteq \mathcal{S}$ such that the sets in A are pairwise disjoint, and the total weight of A is maximum. For $k \geq 3$, already the special case of unit weights, the *unweighted k-Set Packing problem*, is NP-hard.

The technique that has proven most successful in designing approximation algorithms for both the unweighted and the weighted k-Set Packing problem is *local search*. Prior to our work, the best that was known for general weights was Berman's $\frac{k+1}{2} + \epsilon$-approximation algorithm SquareImp. It uses local improvements of constant size with a special structure.

In this talk, we show that by considering a certain class of local improvements of up to logarithmic size, we can, in polynomial time, obtain improved approximation guarantees of $\frac{k+\epsilon_k}{2}$, where $\epsilon_k \in (0, 1)$ for all $k \geq 3$ and $\lim_{k\to\infty} \epsilon_k = 0$. We further prove that one cannot achieve a better guarantee than $\frac{k}{2}$ if one only takes local improvements of logarithmically bounded size into account.

At a first glance, this result seems to conclude the story of local improvement algorithms for the weighted k-Set Packing problem. However, we show how to design a polynomial time $(\frac{k}{2} - \Omega(k))$-approximation algorithm for weighted k-Set Packing that is still based on the local search paradigm.

This talk is based on the papers "The Limits of Local Search for Weighted k-Set Packing" (IPCO 2022, Mathematical Programming) and "Passing the Limits of Pure Local Search for Weighted k-Set Packing" (SODA 2023).