

Risk & Stochastics and Financial Mathematics Joint Seminar in 2016

Seminars are listed in reverse chronological order, most recent first.

24 November - Aditi Dandapani (ETH Zurich)

Strict Local Martingales and Initial Expansions of Filtrations

Beginning with a non negative model following a stochastic differential equation with stochastic volatility, we show how a strict local martingale might arise from a true martingale as a result of an enlargement of the underlying filtration. More precisely, we implement a particular type of enlargement, an "initial expansion" of the filtration, for various kinds of stochastic differential equation models, and we provide sufficient conditions such that this expansion can turn a martingale into a strict local martingale. Applications of our work include the modeling and detection of financial bubbles. For example, one might postulate that a bubble arises as a result of the arrival of new information, which we can model via an enlargement of the filtration.

17 November - Martin Larsson (ETH Zurich)

Conditional infimum and recovery of monotone processes

Monotone processes, just like martingales, can often be reconstructed from their final values. Examples include the running maximum of supermartingales, of fractional Brownian motion, and more generally, running maxima and local times of sticky processes. An interesting corollary is that any positive local martingale can be reconstructed from its final value and its global maximum. These results are derived from a simple no-arbitrage principle for monotone processes on certain complete lattices, analogous to the fundamental theorem of asset pricing in mathematical finance. The framework of complete lattices is sufficiently general to handle also more exotic examples, such as the process of convex hulls of multidimensional diffusions, and the process of sites visited by a random walk. The notion of conditional infimum is at the center of all of these results.

10 November - Paolo Guasoni (Dublin City)

Leveraged Funds: Robust Replication and Performance Evaluation

Leveraged and inverse exchange-traded funds and certificates seek daily returns equal to a multiple of an index' return. The trading costs implied by the frequent portfolio adjustments required create a tension between tracking error, which reflects short-term correlation with the index, and excess return, the long-term deviation from the leveraged index' performance. With proportional trading costs, the optimal replication policy is robust to the index' dynamics. Overall fund

performance depends on the implied spread, the product of tracking error and excess return, rescaled for leverage and volatility. The implied spread is insensitive to the risk premia and allows to compare funds tracking different factors of the same index.

27 October - Christoph Frei (Alberta)

Systemic Influences on Optimal Investment in Stocks and Credit Default Swaps

Recent events have shown that the dependence structure of financial markets is more complex than what is captured by classical models. For example, during the 2008 financial crisis, the financial instability of some companies spread out to affect other companies. The goal of this talk is to analyze how such systemic influences are reflected in optimal investment decisions. To this end, we introduce a model with dependence structure between market risk and default risk of the companies. An investor can use stocks and credit default swaps (CDSs) to participate in the market. We derive an explicit expression for the optimal investment strategy in stocks and CDSs. This allows us to analyze the mechanisms driving the optimal investment decisions. We then develop a novel calibration procedure so that we can fit the model to historical time series of stock and CDS data. An empirical analysis reveals the critical role of systemic risk in portfolio monitoring. This talk is based on joint work with Agostino Capponi (Columbia).

13 October - Frank Seifried (Trier)

Epstein-Zin Stochastic Differential Utility: Foundations, Properties, and Portfolio Optimization

This talk presents some recent contributions to the theory and applications of Epstein-Zin (EZ) stochastic differential utility.

First, we provide novel results on existence, uniqueness and concavity as well as a utility gradient inequality for EZ utility in a general semimartingale setting. In the second part, I would like to address consumption-portfolio choice with EZ utility. We develop a new approach to solve such problems in a large class of incomplete market models, based on fixed point arguments and the associated FBSDE system. Finally, using an asymptotic analysis we show how small proportional transaction costs influence optimal consumption and investment decisions of an agent with EZ utility.

30 June - Johannes Muhle-Karbe (Michigan)

Information and Inventories in High-Frequency Trading

We study how short-term informational advantages can be monetized in a high-frequency setting, when large inventories are explicitly penalized. We find that if most of the additional information is revealed regardless of the high-frequency

traders' actions, then optimized inventory management allows to minimize positions with only second-order losses to expected returns. (Joint work with Kevin Webster)

12 May - Remi Peyre (École des Mines de Nancy)

Fractional Brownian motion, financial mathematics and stopping times

Fractional Brownian motion (fBm) is a Gaussian continuous stochastic process with stationary increments, which generalises ordinary Brownian motion by allowing for any Hölder regularity between 0 and 1. Despite being a very natural model for stochastic evolutions, fBm a priori irrelevant in financial mathematics, as it is not a semimartingale. However if we restrict the authorised trading strategies, or if we introduce transaction costs, it becomes a nontrivial natural question whether the price of a financial asset may be modelled by a (geometric) fBm while satisfying the non-arbitrage condition.

This kind of questions lead to look at what may or may not happen to the trajectory of a fBm just after an arbitrarily chosen stopping time. Obviously the behaviour of the trajectory may be very different from what it is typically, but to which extent? For example, can one find a stopping time after which the fBm would have nonzero probability to go on upwards or downwards? If not, fBm would be said to have the two-way crossing property, which has been beautifully shown to imply non-arbitrage results.

In this talk I will present a work of mine in which I proved the two-way crossing property for fBm, by studying sharp properties of its behaviour after a stopping time, in particular at the order of the local law of the iterated logarithm. Emphasis will be set of the ideas and techniques involved, which seem to me interesting as such.

5 May - Andreea Minca (Cornell)

Dynamics and Stability of Debt Capacity

We propose a dynamic model that explains the build-up of short term debt when the creditors are strategic and have different beliefs about the prospects of the borrowers' fundamentals. We define a dynamic game among creditors, whose outcome is the short term debt. As common in the literature, this game features multiple Nash equilibria. We give a refinement of the Nash equilibrium concept that leads to a unique equilibrium.

For the resulting debt-to-asset process of the borrower we define a notion of stability and find the debt ceiling which marks the point when the borrower becomes illiquid.

We show existence of early warning signals of bank runs: a bank run begins when the debt-to-asset process leaves the stability region and becomes a mean-fleeing sub-martingale with tendency to reach the debt ceiling.

Our results are robust across a wide variety of specifications for the distribution of the capital across creditors' beliefs. This is joint work Johannes Wissel.

24 March - Teemu Pennanen (King's College London)

Convex duality in optimal investment and contingent claim valuation in illiquid markets

Abstract: This paper studies convex duality in optimal investment and contingent claim valuation in markets where traded assets may be subject to nonlinear trading costs and portfolio constraints. Under fairly general conditions, the dual expressions decompose into three terms, corresponding to the agent's risk preferences, trading costs and portfolio constraints, respectively. The dual representations are shown to be valid when the market model satisfies an appropriate generalization of the no-arbitrage condition and the agent's utility function satisfies an appropriate generalization of asymptotic elasticity conditions. When applied to classical liquid market models or models with bid-ask spreads, we recover well-known pricing formulas in terms of martingale measures and consistent price systems. Building on the general theory of convex stochastic optimization, we also derive optimality conditions in terms of an extended notion of a shadow price.

17 March - Jérôme Renault (University Toulouse 1 Capitole)

Optimal Dynamic Information Provision

We study a dynamic model of information provision. A state of nature evolves according to a Markov chain. An advisor with commitment power decides how much information to provide to an uninformed decision maker, so as to influence his short-term decisions. We deal with a stylized class of situations, in which the decision maker has a risky action and a safe action, and the payoff to the advisor only depends on the action chosen by the decision maker. The greedy disclosure policy is the policy which, at each round, minimizes the amount of information being disclosed in that round, under the constraint that it maximizes the current payoff of the advisor. We prove that the greedy policy is optimal (whatever the discount factor) in many cases - but not always.

J. Renault (TSE) , E. Solan (U Tel-Aviv) and N. Vieille (HEC Paris)

10 March - Zhenjie Ren (CMAP, École Polytechnique)

Viscosity Solution of Path-dependent PDEs

This talk will give an overview of the recent study on the path-dependent PDEs. We will show the general motivations of this new topic, and explain why the viscosity-type solutions are interesting. By comparing with the classical notion of viscosity solutions to PDEs, we try to understand the advantage of the new definition of viscosity solutions in the path-dependent context. Also, in this talk we will see what important role the comparison principle plays in the theory of viscosity solutions, and will show the recent progress in proving the comparison result. At the end of the talk, we will see a collection of references on the theory of path-dependent PDEs including both the theoretical improvements and the applications.

25 February - Christa Cuchiero (Vienna)

Polynomial Processes in Stochastic Portfolio Theory

Inspired by volatility stabilized market models introduced by Robert Fernholz and Ioannis Karatzas [2], we introduce a class of processes which are polynomial in the sense of [1] to model both, asset prices (or market capitalizations of companies in an equity market) and their corresponding market weights. More precisely, we characterize the class of polynomial diffusion models for the asset price process whose market weights process is again a polynomial diffusion process on the unit simplex. Explicit parameter conditions assuring the existence of relative arbitrages with respect to the market portfolio are given and the connection to non-attainment of the boundary is discussed. We also consider extensions to models with jumps and the computation of optimal relative arbitrage strategies.

References

- [1] C. Cuchiero, M. Keller-Ressel, and J. Teichmann. Polynomial processes and their applications to mathematical finance. *Finance and Stochastics*, 16(4):711–740, 2012.
- [2] R. Fernholz and I. Karatzas. Relative arbitrage in volatility-stabilized markets. *Annals of Finance*, 1(2):149–177, 2005.

11 February - Xiaolu Tan (Paris-Dauphine)

Tightness and duality of martingale transport on the Skorokhod space

The martingale optimal transport aims to optimally transfer a probability measure to another along the class of martingales. This problem is mainly motivated by the robust superhedging of exotic derivatives in financial mathematics, which turns out to be the corresponding Kantorovich dual. In this paper we consider the continuous-time martingale transport on the Skorokhod space of cadlag paths. Similar to the classical setting of optimal transport, we introduce different dual problems and establish the corresponding dualities by a crucial use of the S-topology and the dynamic programming principle.

28 January - Alexandru Hening (Oxford)

Killed Brownian motion with a prescribed lifetime distribution and models of default

Abstract: In finance, the default time of a counterparty is sometimes modeled as the first passage time of a credit index process below a barrier. It is therefore relevant to consider the following question: If we know the distribution of the default time, can we find a unique barrier which gives this distribution? This is known as the Inverse First Passage Time (IFPT) problem in the literature. We consider a more general 'smoothed' version of the inverse first passage time problem in which the first passage time is replaced by the first instant that the time spent below the barrier exceeds an independent exponential random variable. We show that any smooth distribution results from some unique continuously differentiable barrier. In current work with B. Ettinger and T. K. Wong, we use PDE methods to show the uniqueness and existence of solutions to a discontinuous version of the IFPT problem.

25 January - Julio Bakhoff (Technical University of Vienna)

On optimal transport under the causality constraint

In this talk we shall examine causal transports and the associated optimal transportation problem under the causality constraint (P_c) introduced by Rémi Lasalle. Loosely speaking, causal transports are a relaxation of adapted processes in the same sense as Kantorovich transport plans are the extension of Monge-type transport maps. We will establish a simple primal-dual picture of both (P_c) and the so-called bicausal transportation problem (whereby causality runs in both directions) in euclidean space or equiv. for discrete-time processes, and discuss the limit to continuous-time.

14 January - Blanka Horvath (ETH)

Robust methods for the SABR model

During its decade and a half of existence, the SABR model has changed the daily routine of interest rate modelling. The tractability of the SABR formula—derived by Hagan et al— has led to the formula rather than the model itself becoming an industry standard. While this formula is prone to yield inconsistent prices around zero interest rates, thereby inducing distortions and arbitrage into modelling, the model itself yields more consistent prices. In the historically low interest-rate environments of the past years this has become increasingly relevant and spurred research both among practitioners and academics. From an academic perspective, the concrete but delicate example of SABR-type models exhibit several advantageous but also certain challenging properties. In this talk we review some approaches from this perspective and propose new effective solutions to the arbitrage problem.