

# Operations Research Seminar Series in 2015

Seminars are listed in reverse chronological order, most recent first.

---

**8 December at 14.00 - Nisheeth Vishnoi (EPFL)**

## **Natural Algorithms for Flows and Linear Programming**

In the last few years, there has been a significant interest in the computational abilities of *Physarum Polycephalum* (a slime mold). This drew from a remarkable experiment which showed that this organism can compute shortest paths in a maze. Subsequently, the workings of *Physarum* were mathematically modeled as a dynamical system and algorithms inspired by this model were proposed to solve several basic optimization problems such as flows and linear programs. These dynamics are arrived at by a local and mechanistic interpretation of the inner workings of the slime mold and a global optimization perspective has been lacking even in the simplest of instances. If these algorithms could be shown to work, it would raise the tantalizing possibility that nature, via evolution, has developed algorithms that could efficiently solve some of the most complex optimization problems.

In this talk I will focus on the *Physarum* dynamics for flows and linear programming and prove convergence and complexity bounds for them. I will also shed some light on the question of how one could arrive at these "local" dynamics from an optimization point of view.

This is based on joint works with Damian Straszak.

**13 November - R. Ravi (CMU)**

## **Improved Approximations for Graph-TSP in Regular Graphs**

A tour in a graph is a connected walk that visits every vertex at least once, and returns to the starting vertex. We describe improved approximation results for a tour with the minimum number of edges in regular graphs. En route we illustrate the main ideas used recently in designing improved approximation algorithms for graph TSP.

**28 October - Xue Lu (LSE)**

## **A Generalized Dantzig-Wolfe Decomposition Algorithm for Mixed Integer Programming Problems**

We propose a generalized Dantzig-Wolfe decomposition algorithm for mixed integer programming. By generating copy variables, we can reformulate the original problem to have a diagonal structure which is amendable to the Dantzig-Wolfe

decomposition. We apply the proposed algorithm to multi-level capacitated lot sizing problem and production routing problem. Rigorous computational results show that our algorithm provides a tighter bound of the optimal solution than all the existing methods.

This is joint work with Zeger Degraeve.

**21 October - Eleni Pratsini (IBM Research Ireland)**  
**Big and Fast Data in Cities: Trends and Challenges**

It is expected that by 2050, seventy percent of the world's population will live in cities. This causes great challenges but also opportunities as cities look at how information can be used to manage services and communities, and assess the impact and benefits to citizens. The Smarter Cities Technology Centre at IBM Research - Ireland, houses a cross-disciplinary research and development team helping cities around the world to better understand, interconnect and manage their core operational systems such as transport, communication, water and energy and also to improve citizen services and care. In this presentation, we will first explain what defines a "smart city", and through the use of examples, present how the intersection of mathematics, information technology, economics and engineering is used to address some of the challenges in the area of citizen care.

**16 September - Jugal Garg (Max Planck Institute for Informatics, Saarbrücken)**  
**Polynomial-Time Complementary Pivot Algorithms for Market Equilibria**

We consider the problem of computing equilibria in the Fisher market for utility functions such as linear, spending constraint and perfect price-discrimination. In each case we start with a convex program that captures market equilibria, and in a systematic way, convert it into a linear complementary problem (LCP) formulation. To obtain a polynomial-time algorithm, we depart from previous approaches of pivoting on a single polyhedron associated with the LCP. Instead, we carefully construct a polynomial-length sequence of polyhedra, one containing the other, such that starting from an optimal solution to one allows us to obtain an optimal solution to the next in the sequence in a polynomial number of complementary pivot steps.

This is a joint work with Ruta Mehta, Milind Sohoni and Nisheeth Vishnoi.

**2 September - Zeev Nutov (The Open University of Israel)**  
**On LP-Relaxations for the Tree Augmentation Problem**

In the Tree Augmentation Problem (TAP) we are given a tree  $T$  (a laminar family) on node set  $V$  and an edge set  $E$  on  $V$ . The goal is to augment  $T$  by a min-size edge-set  $F \subseteq E$  such that  $T+F$  is 2-edge-connected. In the weighted version of the

problem (Weighted TAP),  $F$  should be of minimum weight.

Weighted TAP is currently a "threshold" problem in connectivity network design. It is the simplest problem for which ratio better than 2 is not known, but is suspected to exist – since TAP admits ratio 1.5.

In this talk I will give a new LP-relaxation for (Weighted) TAP, and describe a relatively simple dual-fitting 1.75-approximation algorithm for TAP that is based on this LP-relaxation.

**10 June - Ola Svensson (École polytechnique fédérale de Lausanne)  
Approximating ATSP by Relaxing Connectivity**

The standard LP relaxation of the asymmetric traveling salesman problem has been conjectured to have a constant integrality gap in the metric case. We prove this conjecture when restricted to shortest path metrics of node-weighted digraphs. Our arguments are constructive and give a constant factor approximation algorithm for these metrics. We remark that the considered case is more general than the directed analog of the special case of the symmetric traveling salesman problem for which there were recent improvements on Christofides' algorithm.

The main idea of our approach is to first consider an easier problem obtained by relaxing the general connectivity requirements into local connectivity conditions. For this relaxed problem, it is quite easy to give an algorithm with a guarantee of 3 on node-weighted shortest path metrics. More surprisingly, we then show that any algorithm (irrespective of the metric) for the relaxed problem can be turned into an algorithm for the asymmetric traveling salesman problem by only losing a small constant factor in the performance guarantee. This leaves open the intriguing task of designing a "good" algorithm for the relaxed problem on general metrics.

**27 May - Sergei Kulivets (Trapeznikov Institute of Control Sciences of Russian Academy of Sciences)  
Game Theoretic Model on a Cognitive Map**

Some problems of complex control in fields connected with public life (i.e., social-economic, political and other fields) include a semi-structured control object. A situation appears semi-structured if the basic parameters have a qualitative (not quantitative) nature, and their values are subjective expert evaluations. Cognitive maps serve to solve control problems for semi-structured situations. The concept of a cognitive map was proposed by R. Axelrod as a digraph-based mathematical model of a decision maker belief system about some limited domain, such as a policy problem. Cognitive map nodes correspond to situation concepts. Edges are interpreted as direct causal links from one concept to another. We consider a game of players with cognitive maps (that represent players' belief systems). There are

concepts that values could be controlled by some player and concepts that values have desired values for this player. The equilibrium of the game is calculated according to players' beliefs that are represented as the hierarchy of cognitive maps. We consider the rule of changes of concept values during discrete time. Since the input data for the model is expert evaluation prone to subjectivity, it is necessary to estimate the tolerance of model results to errors in input data. For example the experts evaluate the weight of edges in the cognitive map of the player. The sign of the edge weight indicates the type of the causal link and the value of edge weight indicates the strength of the causal link. We consider the problem of the model's tolerance to errors in input data. We also consider the problem of information control possibility when one player can manipulate the decision making process of other players through a message of false information. We illustrate all theoretical results with examples.

### **6 May - Mark Walters (Queen Mary)**

#### **Transitive Avoidance Games**

There are many games where two players compete to be the first to make something – one well known example is noughts and crosses: each player is competing to be the first to get three in a row. It is well known that all such games are either a draw or a first player win.

In this talk we look, instead, at games where the first player to make something loses. One natural example is the avoidance version of noughts and crosses, where the first to get three in a row loses.

It seems natural to expect the second player to have an advantage – at least when the game is reasonably symmetrical – but this turns out to be false. In this talk we discuss when the first player might have a win, and whether any general result can be recovered.

### **24 April - Paul Slovic (Oregon)**

#### **When (In)Actions Speak Louder than Words: Confronting the Collapse of Humanitarian Values in Foreign Policy Decisions**

Decisions to save civilian lives by intervening in foreign countries are some of the most difficult and controversial choices facing national decision makers. Although each situation is unique, decisions involving trade-offs that pit the value of human lives against other important objectives are quite common. Furthermore, there is often a striking disconnect between the high value placed on saving human lives expressed by top government officials and the apparent low value revealed by government decisions not to intervene. Specifically, when multiple objectives are in play, highly regarded humanitarian values appear to collapse in the competition with national security and economic security objectives.

On the basis of theoretical models of judgment and choice, research in social cognition, and careful reading of official pronouncements, we have developed a hypothesis to explain this collapse. Underlying our hypothesis is the “prominence effect” (Tversky, Sattath, and Slovic, 1988), which asserts that choice is inherently more lexicographic than expressed judgments of value. That is, the more prominent attributes of a proposed action will be weighted more heavily in choice than in judgments and decisions based on expressed preferences or values.

We argue that the prominence effect may underlie the disconnect between expressed and revealed values regarding whether or not to act to save large numbers of civilian lives under attack in foreign countries. Specifically, we hypothesize that national security is the prominent dimension in the context we are studying here. Chosen actions need to be justified, and deciding in favour of security likely makes a stronger argument than deciding in favour of protecting foreign lives, no matter how many thousands or millions of lives are at stake. We shall discuss the moral, ethical, and strategic implications of this bias that devalues efforts to intervene in massive humanitarian crises.

**11 March - Kevin Glazebrook (Lancaster)**

**A graph patrol problem with random attack times**

A patroller traverses a graph to detect and then thwart potential attacks at nodes. In deciding how to patrol, the patroller needs to take account of many things: the structure of the graph, the possibly different attack time distributions at distinct nodes, the different costs which may be incurred depending on where an attack takes place. Simple, natural ways of patrolling (like repeatedly walking up and down a line graph) may perform poorly. Both random and strategic attackers are considered in the work. We use Lagrangian relaxation to develop index-based heuristics which are easy to compute and which typically achieve within 1% of (cost) optimality.

Based on joint work with Kyle Y Lin, Michael Atkinson and Timothy Chung.

**25 February - Daniel Lokshtanov (Bergen)**

**Fixed-parameter tractable canonization and isomorphism test for graphs of bounded treewidth**

We give a fixed-parameter tractable algorithm that, given a parameter  $k$  and two graphs  $G_1, G_2$ , either concludes one of these graphs has treewidth at least  $k$ , or determines whether  $G_1$  and  $G_2$  are isomorphic. The running time of the algorithm on an  $n$ -vertex graph is  $2^{O(k^5 \log k)} \cdot n^5$ , and this is the first fixed-parameter algorithm for Graph Isomorphism parameterized by treewidth.

Our algorithm in fact solves the more general canonization problem. We namely design a procedure working in  $2^{O(k^5 \log k)} n^5$ , time that, for a given graph  $G$  on  $n$  vertices, either concludes that the treewidth of  $G$  is at least  $k$ , or finds an isomorphism-invariant construction term – an algebraic expression that encodes  $G$  together with a tree decomposition of  $G$  of width  $O(k^4)$ . Hence, a canonical graph isomorphic to  $G$  can be constructed by simply evaluating the obtained construction term, while the isomorphism test reduces to verifying whether the computed construction terms for  $G_1$  and  $G_2$  are equal.

Based on joint work with Marcin Pilipczuk, Michal Pilipczuk and Saket Saurabh.

**11 February - Antoine Deza (McMaster)**

**On the polynomial Hirsch conjecture and its continuous analogue**

The simplex and primal-dual interior point methods are the most computationally successful algorithms for linear optimization. While simplex methods follow an edge path, interior point methods follow the central path. Within this framework, the curvature of a polytope, defined as the largest possible total curvature of the associated central path, can be regarded as the continuous analogue of its diameter. In this talk, we highlight links between the edge and central paths, and between the diameter and the curvature of a polytope. We recall continuous results of Dedieu, Malajovich, and Shub, and discrete results of Holt and Klee and of Klee and Walkup, as well as related conjectures such as the Hirsch conjecture that was disproved by Santos. We also present analogous results dealing with average and worst-case behaviour of the curvature and diameter of polytopes, including a result of Allamigeon, Benchimol, Gaubert, and Joswig who constructed a counterexample to the continuous analogue of the polynomial Hirsch conjecture. Based on joint work with Tamás Terlaky (Lehigh), Feng Xie (Microsoft), and Yuriy Zinchenko (Calgary).

**4 February - Colin Cooper (King's College London)**

**Coalescing Random Walks and Models of Voting on Graphs**

A simple model of voting on connected graphs is as follows:

The voting proceeds in rounds. Initially each vertex has a distinct opinion. In each round each vertex contacts a randomly chosen neighbour and adopts their opinion. We call this model single-sample voting. The quantities of interest are the time for voting to complete, and the probability a particular opinion wins. Examples include: Single-sample voting. The relationship between single-sample voting and coalescing random walks. These processes are dual and we can use random walks to analyze the performance of voting. Using this, we give an upper bound for the expected time for voting to complete on general connected graphs.

Two-party voting. In two-party voting, each vertex initially holds one of two opinions (e.g. 0 or 1). We discuss the speed-up in two-party voting when the opinion of one or more neighbour is sampled at each step. We give results for two-party voting on regular expanders when we sample two or more opinions. Compared to single-sample voting, this process is more consistent (democratic) and finishes much quicker.

## **28 January - John Fearnley (Liverpool)**

### **The complexity of the Simplex Method**

The simplex method is a well-studied and widely-used pivoting method for solving linear programs. When Dantzig originally formulated the simplex method, he gave a natural pivot rule that pivots into the basis a variable with the most violated reduced cost. In their seminal work, Klee and Minty showed that this pivot rule takes exponential time in the worst case. We prove two main results on the simplex method. Firstly, we show that it is PSPACE-complete to find the solution that is computed by the simplex method using Dantzig's pivot rule. Secondly, we prove that deciding whether Dantzig's rule ever chooses a specific variable to enter the basis is PSPACE-complete. We use the known connection between Markov decision processes (MDPs) and linear programming, and an equivalence between Dantzig's pivot rule and a natural variant of policy iteration for average-reward MDPs. We construct MDPs and show PSPACE-completeness results for single-switch policy iteration, which in turn imply our main results for the simplex method.

## **21 January - Endre Csóka (Warwick)**

### **Efficient teamwork**

In real-life multi-agent projects, agents often choose actions that are highly inefficient for the project or damaging for other agents because they care only about their own contracts and interests. We show that this can be avoided by the right project management. We model agents with private workflows including hidden actions and chance events, which can influence each other through publicly observable actions and events. We design an efficient mechanism for this model which is prior-free, incentive-compatible, collusion-resistant, individually rational and avoids free-riders.