Seminar on Discrete and Applicable Mathematics in 2011

Seminars are listed in reverse chronological order, most recent first.

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8 December - Imre Bárány (UCL)
Tensors, colours, and octahedral

Several classical results in convexity, like the theorems of Caratheodory, Helly, and Tverberg, have colourful versions. In this talk I plan to explain how two methods, the octahedral construction and Sarkaria's tensor trick, can be used to prove further extensions and generalizations of such colourful theorems.

24 November - Ron Peretz (Tel Aviv)
The Entropy Method and Repeated Games with Bounded Complexity

In the recent years there has been a significant progress in the study of repeated games with bounded complexity due to an information theoretic lemma by Neyman and Okada (2005). One of the main challenges in the study of repeated games with bounded complexity is to characterize the set of equilibrium payoffs for n-player games (n>2).

The difficulty comes from the fact that given the information of one player the actions of the other players need not be independent (as predicted by Kuhn's Theorem for perfect recall games); therefore the min-max value in the repeated game may be lower than that of the one-stage game.

We apply Neyman-Okada's Lemma to obtain new results on the extent of correlation that can be achieved by two bounded memory strategies. Our results have implications to the min-max value in 3-player bounded memory games.

17 November - Stephen J. Young (California)
Braess's Paradox in Expanders

Expander graphs are known to facilitate effective routing and most real-world networks have expansion properties. At the other extreme, it has been shown that in some special graphs, removing certain edges can lead to more efficient routing. This phenomenon is known as Braess's paradox and is usually regarded as a rare event. In contrast to what one might expect, we show that Braess's paradox is ubiquitous in expander graphs. Specifically, we prove that Braess's paradox occurs in a large class of expander graphs with continuous convex latency functions. Our results extend previous work which held only when the graph was both denser and

random and for random linear latency functions. We identify deterministic sufficient conditions for a graph with as few as a linear number of edges, such that Braess's Paradox almost always occurs, with respect to a general family of random latency functions.
Joint work with Fan Chung and Wenbo Zhao.

10 November - Bjarne Toft (Southern Denmark)
Graph colouring problems revisited

In 1995 the book Graph Colouring Problems (by Jensen and Toft) appeared. It contains a detailed description of 211 (then) unsolved problems. Since then there has been much development, and the talk will try to update some areas, like the four colour theorem, edge-colourings, Hadwiger's conjecture and/or colour-critical graphs.

3 November - Vassili Kolokoltsov (Warwick)
Game-theoretic analysis of rainbow options in incomplete markets

We develop a pure game-theoretic approach to option pricing in a multi-dimensional market (rainbow options), where risk-neutral probabilities emerge automatically from the robust control evaluation. The process of investment is considered as a zero-sum game of an investor with the Nature. Our method is robust enough to be able to accommodate various markets rules and settings including path dependent payoffs, American options, real options, stochastic volatilties and transaction costs.On the other hand, it leads to rather simple numerical algorithms. The continuous-time limit is described by nonlinear and/or fractional (in time) Black-Scholes type equations.

27 October - Amol Sasane (LSE)
A new v-metric in Control Theory

The need for measuring the distance between control systems is basic in control theory. For example, in robust control theory, one knows that the unstable control system P to be stabilized is just an approximation of reality, and so one would really like the stabilizing controller to not only stabilize the nominal control system P, but also all sufficiently close systems P' to P. The question of what one means by "closeness"of control systems thus arises naturally. So one needs a metric on the set of unstable systems which is amenable to computation and it has good properties in the robust stabilization problem.Such a desirable metric was introduced by Glenn Vinnicombe in 1993, and is called the v-metric. There essentially the ring R of transfer functions of stable control systems was taken to be the set of the rational functions without poles in the closed unit disk.The problem of what happens when R is some other ring of stable transfer functions

(for example the Hardy algebra) was left open.In this talk, we address this issue, and give an extension of the v-metric.

Friday 21 October - Endre Boros (Rutgers)
Every Stochastic Game with Perfect Information Admits a Canonical Form

Stochastic games, introduced by Lloyd Shapley in the early 1950s, generalize Markov decision processes, the so called repeated games, as well as some classical combinatorial optimization problems, like the minimum/maximum mean cycle problem (Karp, 1978). Certain special cases of stochastic games are known as simple stochastic games, parity games or mean-payoff games.

A central computational problem is to compute the value of such games, when it exists. This computational problem was considered for many special variants of these games, and generally considered to be a very hard problem, even though it is known to belong to both NP and co-NP. For the simplest case of mean- payoff games (or BW games, in short) pseudo-polynomial algorithms are known (Gurvich, Karzanov, Khachiyan, 1988; Zwick and Paterson, 1996; Pisaruk 1999) and also there are some subexponential algorithms (Bjorklund and Vorobyov, 2005, 2007). This was extended to the family of simple stochastic games by (Halman, 2007). On the other hand for most types of algorithms for these games there are known negative results, such as the cycling of many variants (Condon, 1992), the exponential lower bounds for strategy improvement approach for parity games (Friedmann, 2009), or the inadequacy of discounted approximations of BWR games even with a single random node (Boros, Elbassioni, Gurvich and Makino, 2010). Furthemore, BW games were shown to be equivalent with a linear program having exponentially large coefficients (Chatterjee, Majumdar and Henzinger, 2008; see also Akian, Gaubert and Guterman, 2009; Sergeev 2010; etc).

In this talk we consider the BWR model that is a common generalization of all the mentioned special stochastic game classes proposed in a different from by (Gillette, 1957), and show that such a game can always be transformed into an equivalent game in which locally optimal strategies are also globally optimal. The existence of such canonical transformations also give rise to a new type algorithm, which directly aims at finding such a transformation. With the aim of this algorithm we could derive polynomial time approximation for BW games, and for BWR games with a constant number of random nodes. Let us finally note that the existence of a pseudo- polynomial algorithm for general BWR games would already imply the existence of a polynomial time algorithm for the above mentioned special cases, including simple stochastic games.

Co-Authors: Khaled Elbassioni (MPI, Saarbrucken, Germany), Vladimir Gurvich (RUTCOR, Rutgers University, New Jersey, USA) and Kazuhisa Makino (Tokyo University, Tokyo, Japan)

20 October - Diana Piquet (Birmingham)
Embedding Trees in Sparse Graphs

No abstract available

13 October - Prof. W. Zelazko (Polish Academy of Sciences)
Around the Grauert-Remmert Theorem

The GR Theorem asserts that a commutative Noetherian Banach algebra is finitely dimensional. In more general situations finite-dimensionality may be replaced by the property that all ideals in the algebra are closed. I will discuss generalizations of the theorem and associated open problems.

Friday 7 October - Michael Stiebitz (TU Ilmenau)
Edge Colouring of Multigraphs: Tashkinov Trees and Goldberg's Conjecture

There are two trivial lower bounds for the **chromatic index** $\chi'(G)$ of a (multi)graph G, namely the **maximum degree** $\Delta(G)$ of G and the **density** $w(G)$ of G; the last graph parameter is defined by

$$w(G) = \max_{H \subseteq G, |V(H)| \geq 2} \left\lceil \frac{|E(H)|}{\lfloor \frac{1}{2}|V(H)| \rfloor} \right\rceil .$$

A famous conjecture made, independently, by Anderson, Goldberg, Gupta and Seymour in the 1970s says that every graph G satisfies

$$\chi'(G) \leq \max\{\Delta(G) + 1, w(G)\}.$$

In 1990 Nishizeki and Kashiwagi proved that $\chi'(G) \leq \max\{(11\Delta(G) + 8)/10, w(G)\}$ for every graph G. The proof was based on the so-called critical chain method. A shorter proof of this result was given by Tashkinov in 2000. The main tool in Tashkinov's proof are Tashkinov trees, a common generalization of both Vizing fans and Kierstaed paths. Based on Tashkinov's method Favrholdt, Stiebitz and Toft proved in 2006 that $\chi'(G) \leq \max\{(13\Delta(G) + 10)/12, w(G)\}$. In 2007 Scheide extended this result to $\chi'(G) \leq \max\{(15\Delta(G) + 12)/14, w(G)\}$. Furthermore, he proved that every graph G satisfy $\chi'(G) \leq \max\{\Delta(G) + \sqrt{\Delta(G)/2}, w(G)\}$ as well as $\chi'(G) \leq \chi'_f(G) + \sqrt{\chi'_f(G)/2}$, where $\chi'_f(G)$ denotes the fractional chromatic index of G. The last result extends a result of Kahn from 1996 as well as a result of Sanders and Steurer from 2005. The proofs of all these results are constructive and based on an extension of Tashkinov's method. In particular, the proof of the inequality $\chi'(G) \leq \max\{\Delta(G) + \sqrt{\Delta(G)/2}, w(G)\} =: \tau(G)$ yields an algorithm that computes, for every graph $G = (V, E)$, an edge colouring of G using at most $\tau(G)$ colours, where the algorithm has time complexity bounded from above by a polynomial in $|V|$ and $|E|$ (and also in Δ).

6 October - Dario Bauso (Palermo)
Attainability in Repeated Games with Vector Payoffs

We study two-player repeated games with vector payoffs in continuous time and introduce the concept of attainable sets of payoff vectors. A set A of payoff vectors is called strongly attainable by player 1 if player 1 has a strategy such that the distance between A and the total payoff up to time T goes to 0 as T goes to infinity. The set A is called attainable by player 1 if for every epsilon > 0, player 1 has a strategy that ensures that this distance is asymptotically at most epsilon. This concept is motivated by applications in multi-inventory control. We provide geometric characterization for the attainability of a specific vector x, as well as to the attainability of all vectors.

Co-authors: Ehud Lehrer and Eilon Solan (Tel Aviv University)

30 June - **Matêj Stehlík (Grenoble)**
Simultaneous colouring of plane graphs

In a simultaneous colouring of a plane graph G = (V,E,F), the elements of a subset of {V,E,F} (of cardinality at least 2) are coloured so that no two adjacent or incident elements receive the same colour. This concept was introduced in 1965 by Ringel, who considered the problem of simultaneously colouring the vertices and faces. The general problem is to determine the minimum number of colours which admit such a colouring. I will give an overview of known results and open problems related to simultaneous colouring, and present two results on edge-face colouring which I recently obtained with Ross Kang and Jean-Sébastien Sereni.

26 May - Martin Gairing (Liverpool)
Computing Stable Outcomes in Hedonic Games

We study the computational complexity of finding stable outcomes in hedonic games, which are a class of coalition formation games. We restrict our attention to a nontrivial subclass of such games, which are guaranteed to possess stable outcomes, i.e., the set of symmetric additively-separable hedonic games. These games are specified by an undirected edge-weighted graph: nodes are players, an outcome of the game is a partition of the nodes into coalitions, and the utility of a node is the sum of incident edge weights in the same coalition. We consider several stability requirements defined in the literature. These are based on restricting feasible player deviations, for example, by giving existing coalition members veto power. We extend these restrictions by considering more general forms of preference aggregation for coalition members. In particular, we consider voting schemes to decide if coalition members will allow a player to enter or leave their coalition. For all of the stability requirements we consider, the existence of a stable outcome is guaranteed by a potential function argument, and local improvements

will converge to a stable outcome. We provide an almost complete characterization of these games in terms of the tractability of computing such stable outcomes. Our findings comprise positive results in the form of polynomial-time algorithms, and negative (PLS-completeness) results. The negative results extend to more general hedonic games.

Joint work with Rahul Savani.

20 May - Guillermo Owen (Naval Postgraduate School)
A Game-theoretic Approach to Networks

We consider teams whose members (a manager, workers, robots) are represented as nodes of a graph. By cooperation, the several members can accomplish much more than by acting individually; we represent this by a super-additive game in characteristic function form. This collaboration is however only possible if the several members can communicate, and this requires links in the graph. The links can be more or less effective; more effective links are generally more costly.We use the Myerson approach to games on graphs to represent the amount of work that can be done, and the method of multilinear extensions to represent more or less efficient links. This allows us to look for optimal links. An example is worked out in some detail.

24 March - Selin Damla Ahipasaoglu (LSE)
Analytical Results on the PAUSE Auction Procedure

In this talk, we focus on the analytical properties of a decentralized auction, namely the PAUSE Auction Procedure. We prove that the revenue of the auctioneer from PAUSE is greater than or equal to the profit from the well-known VCG auction when there are only two bidders and provide lower bounds on the profit for arbitrary number of bidders. Based on these bounds and observations from auctions with few items, we propose a modification of the procedure that increases the profit. We believe that this study, which is still in progress, will be a milestone in designing better decentralized auctions since it is the first analytical study on such auctions with promising results.

10 March - Bernhard von Stengel (LSE)
Nash Codes for Noisy Channels

We consider a coordination game between a sender and a receiver who communicate over a noisy channel.

The sender wants to inform the receiver about the state by transmitting a message over the channel. Both receive positive payoff only if the receiver decodes the received signal as the correct state. The sender uses a known "codebook" to map

states to messages. When does this codebook define a Nash equilibrium?

The receiver's best response is to decode the received signal as the most likely message that has been sent.Given this decoding, an equilibrium or "Nash code" results if the sender encodes every state as prescribed by the codebook, which is not always the case. We show two theorems that give sufficient conditions for Nash codes.First, the "best" codebook for the receiver (which gives maximum expected receiver payoff) defines a Nash code.

A second, more surprising observation holds for communication over a binary channel which is used independently a number of times, a basic model of information theory: Given a consistent tie-breaking decoding rule which holds generically, ANY codebook of binary codewords defines a Nash code. This holds irrespective of the quality of the code and also for nonsymmetric errors of the binary channel.

Joint work with P. Hernandez.

3 March - **Jozsef Solymosi (British Columbia)**
On point-line incidences

In this talk we review some old and new problems on geometric incidences. We present various bounds on point-line incidences. For a given arrangement of lines L and a set of points P the number of incidences is the number of point line pairs, {p.l}, (p from P and l from L) so that p is a point of l. We are interested about upper bounds on incidences over the Euclidean plane, 3-space, projective complex plane/space, and finite fields. The tools we are using for bounding the number of incidences are from algebra, discrete geometry, and algebraic geometry.

17 February - Rann Smorodinsky (Technion University)
Approximately Optimal Mechanism Design via Differential Privacy

In this paper we study the implementation challenge in an abstract interdependent values model and an arbitrary objective function. We design a mechanism that allows for approximate optimal implementation of insensitive objective functions in ex-post Nash equilibrium. If, furthermore, values are private then the same mechanism is strategy proof. We cast our results onto two specific models: pricing and facility location. The mechanism we design is optimal up to an additive factor of the order of magnitude of the square root of the number of agents and involves no utility transfers.

Underlying our mechanism is a lottery between two auxiliary mechanisms - with high probability we actuate a mechanism that reduces players influence on the choice of the social alternative, while choosing the optimal outcome with high

probability. This is where the recent notion of `differential privacy' is employed. With the complementary probability we actuate a mechanism that is typically far from optimal but is incentive compatible. The joint mechanism inherits the desired properties from both.

This is joint work with Kobbi Nissim and Moshe Tennenholtz.

3 February - Colin Cooper (King's College London)
Component structure of the vacant set induced by a random walk on a random graph

We consider random walks on two classes of random graphs and explore the likely structure of the set of unvisited vertices or vacant set. In both cases, the size of the vacant set $N(t)$ can be obtained explicitly as a function of t.

Let $\Gamma(t)$ be the subgraph induced by the vacant set. We show that for random graphs $G_{n,p}$ above the connectivity threshold, and for random regular graphs G_r, for constant $r\geq 3$, there is a phase transition in the sense of the well-known Erd\H{os}-Renyi phase transition. Thus for $t\leq (1-\epsilon)t^*$ we have a unique giant plus components of size $O(\log n)$ and for $t\geq (1+\epsilon)t^*$ we have only components of size $O(\log n)$.

In the case of G_r we describe the likely degree sequence, size of the giant component and structure of the small ($O(\log n)$) size components.

20 January - Mark Walters (Queen Mary)
Euclidean Ramsey Theory

A finite set X in some Euclidean space R^n is called Ramsey if for any k there is a d such that whenever R^d is k-coloured it contains a monochromatic set congruent to X. A long standing open problem is to characterise the Ramsey sets.

In this talk I will discuss the background to this problem, a new conjecture, and some group theoretic questions this new conjecture raises.

13 January - **Akrira Okada (Hitotsubashi)**
Dynamic Group Formation in the Repeated Prisoner's Dilemma

We consider dynamic group formation in repeated n-person prisoner's dilemma. Agreements in coalitional bargaining are self-binding in that they are supported as subgame perfect equilibria of repeated games. Individuals are allowed to renegotiate the cooperating group agreement through a process of voluntary participation. We prove that a cooperating group forms as an absorbing state of a Markov perfect equilibrium after a finite number of renegotiations if and only if the

group is Pareto efficient, provided that individuals are patient. The cooperating group can only expand.

Joint work with Toshimasa Maruta.