PhD Seminar on Discrete and Applicable Mathematics in 2017

Seminars are listed in reverse chronological order, most recent first.

Friday 8 December - Benny Sudakov (ETH)
Unavoidable patterns in words

A word w is said to contain the pattern P if there is a way to substitute a nonempty word for each letter in P so that the resulting word is a subword of w. Bean, Ehrenfeucht and McNulty and, independently, Zimin characterised the patterns P which are unavoidable, in the sense that any sufficiently long word over a fixed alphabet contains P. Zimin's characterisation says that a pattern is unavoidable if and only if it is contained in a Zimin word, where the Zimin words are defined by $Z_1 = x_1$ and $Z_n=Z_{n-1} x_n Z_{n-1}$.

We study the quantitative aspects of this theorem, showing that there are extremely long words avoiding Z_n. Our results are asymptotically tight.

Joint with David Conlon and Jacob Fox

Friday 1 December - Will Perkins (Birmingham)
Bethe states of random factor graphs

We present some general results on how arbitrary probability distributions on discrete cubes can be approximated by mixtures of product measures. These results are related to and inspired by the decomposition results of Szemeredi and Frieze/Kannan from graph theory. We then apply these results to prove a conjecture of Mezard and Montanari: that Gibbs measures on random factor graphs can be represented as a convex combination of a moderate number of "Bethe states", probability measures which have a simple local and global structure. Based on joint work with Amin Coja-Oghlan.

Friday 24 November - Jan Corsten (LSE)
Cycle Partitioning in Hypergraphs

Erdős, Gyárfás and Pyber proved in 1991 that for every r-edge-coloured complete graph (on any number of vertices) the vertices can be partitioned into $O(r^2 \log r)$ monochromatic cycles and conjectured that r cycles should be enough. In case $r=2$, this conjecture was proved for very large n by Łuczak, Rödl and Szemerédi, for large n by Allen and finally for all n by Bessy and Thomassé. For $r>2$, the conjecture was disproved by Pokrovskiy who proposed an alternative conjecture which is still open. The same problem for loose cycles in hypergraphs was studied by Gyárfás and Sárközy. For tight cycles however, not much is known and we will investigate them in this talk.

Friday 17 November - Aistis Atminas (LSE)
Classes of graphs without star forests and related graphs

This work provides structural characterization of hereditary graph classes that do not contain a star forest, several related graphs obtained from star forests by subset complementation, a union of cliques and a complement of union of cliques as induced subgraphs. This provides, for instance, structural results for graph classes not containing a matching and several complements of a matching. In terms of the speed of hereditary graph classes, our results imply that all such classes have at most factorial speed of growth.

Friday 10 November - Edin Husic (LSE)
Independent set is FPT for even hole-free graphs

An independent set in a graph is a subset of pairwise non-adjacent vertices. The well-studied maximum independent set problem is to find an independent set of the maximum possible size. The problem is known to be NP-hard and there is no fixed parameter tractable algorithm (FPT) for the problem, unless W[1]=FPT. For the class of even hole-free graphs the complexity of the problem is still open, but using the augmentation technique we show that the problem is FPT (parameterized by the size of the solution k). Augmenting graphs represent a generalization of augmenting paths. The algorithm relies on the fact that every minimal even hole-free augmenting graph is a tree on at most 2k+1 vertices.

Friday 20 October - Attila Dankovics (LSE)
Powers of Hamiltonian cycle in Kneser graphs

The seminar will be about the extremal question: Given l and k what is the smallest value n_0 that for all $n\ge n_0$ the Kneser graph $K_{n,k}$ contains the l-th power of a Hamiltonian cycle. I show two different approaches that lead to an upper bound around k^2l.

Friday 6 October - Pablo Moscato (University of Newcastle, Australia)
"We have it all wrong"… so what are you doing to change practice?

Along with many other researchers, I share the view that a systematically coherent research program, in both theory and applications of algorithms, is definitely needed to accelerate innovation in computing. We routinely design computational approaches and engage in healthy competitions where the performance of our methods is tested… but what if "We have it all wrong"? What if we need a paradigmatic change in our practice for the development and design of computational methods? We may need to enrich our practice with a new approach.

In fact, John N. Hooker already alerted the computing and mathematical community more than 20 years ago [Hooker, 1995; Journal of Heuristics]: “Competitive testing tells us which algorithm is faster but not why.” Hooker argued for a more scientific approach and he proposed the use of ‘controlled experimentation’. This is common in empirical sciences. “Based on one’s insights into an algorithm”, he said, “one may expect good performance to depend on a certain problem characteristic”. Then “design a controlled experiment that checks how the presence or absence of this characteristic affects performance” and, finally, “build an exploratory mathematical model that captures the insight [...] and deduce from its precise consequences that can be put to the test”. In this talk, I will address how a new thinking is needed for the development of our field. I will have an with emphasis in our success on both speeding up solutions for the traveling salesman problem as well as our success to create very hard instances for the world’s fastest solver.

Friday 29 September - Jens Vygen (University of Bonn)
Approaching 3/2 for the s-t-path TSP

The s-t-path TSP is a variant of the traveling salesman problem in which the endpoints of the tour are given and distinct. The integrality ratio of the natural linear programming relaxation is believed to be 3/2, but all approximation algorithms known so far have worse performance ratio. We show that there is a polynomial-time algorithm with approximation guarantee 3/2+epsilon, for any fixed \epsilon>0.

It is well known that Wolsey’s analysis of Christofides' algorithm also works for the s-t-path TSP except for the narrow cuts (in which the LP solution has value less than two). A fixed optimum tour has either a single edge in a narrow cut (then call the edge and the cut lonely) or at least three (then call the cut busy). Our algorithm "guesses" (by dynamic programming) lonely cuts and edges. Then we partition the instance into smaller instances and strengthen the LP, requiring value at least three for busy cuts. By setting up a k-stage recursive dynamic program, we can compute a spanning tree (V,S) and an LP solution y such that 1/2+O(2^{-k}))y is in the T-join polyhedron, where T is the set of vertices whose degree in S has the wrong parity. This is joint work with Vera Traub.

Friday 9 June - Tuğkan Batu (LSE)
Generalised Uniformity Testing

In this work, we revisit the problem of uniformity testing of probability distributions. A fundamental problem in distribution testing, testing uniformity over a known domain has been addressed over a significant line of works, and is by now fully understood.

The complexity of deciding whether an unknown distribution is uniform over its unknown (and arbitrary) support, however, is much less clear.

Yet, this task arises as soon as no prior knowledge on the domain is available, or whenever the samples originate from an unknown and unstructured universe. In this work, we introduce and study this generalized uniformity testing question, and establish nearly tight upper and lower bound showing that -- quite surprisingly -- its sample complexity significantly differs from the known-domain case. Moreover, our algorithm is intrinsically adaptive, in contrast to the overwhelming majority of known distribution testing algorithms.

Friday 26 May - Sebastián Bustamante (Universidad de Chile)
Monochromatic cycle partitions

Given an edge-colouring of a graph or hypergraph K, the problem of partitioning the vertices of K into the smallest number of monochromatic cycles has received much attention. Central to this area has been an old conjecture of Lehel (1967) stating that two monochromatic disjoint cycles in different colours are sufficient to cover the vertex set of any 2-edge-colouring of the complete graph. This conjecture was confirmed 31 years later by Bessy and Thomassé. In the hypergraph setting, we show that at most three monochromatic loose cycles are sufficient to partitition the vertex set of any 2-edge-colouring of the complete k-uniform hypergraph, two of them in different colours and covering all but at most 2k-2 vertices. We also discuss several generalisations and open problems. This is joint work Maya Stein.

Friday 5 May - Misha Rudnev (Bristol)
On the sum-product problem and variations

The sum-product problem is due to Erdös and Szemerédi, who suggested that the cardinality of the set of all pairwise sums or the set of all pairwise products of a finite integer set A should always be almost |A|^2. It was arguably considered second to the famous Erdös distinct distance conjecture. However, the latter got resolved by Guth and Katz in 2010, while the sum-product type questions, now mostly asked over fields, are wide open.

The work of Guth and Katz, which was concerned with reals, has changed our vision of discrete geometry questions, but had arguably no impact on those arising from the real sum-product problem. However, it was used by myself to prove a theorem on point-plane incidences in 3D over any field; this theorem is generally sharp and for applications just slightly weaker than what had been known over the reals, where the main tools are order and the Szemerédi-Trotter theorem. Yet its applications to sum-product questions are almost as strong, and now we have a reasonably uniform state of the art over both reals and, say prime residue fields, where the earlier generation of estimates that stemmed from the famous paper of Bourgain , Katz and Tao was just qualitatively, rather than quantitatively, stronger than trivial.

I will review, as much as time allows, today's "halfway" state of the art of sum-product type questions over both the real and general fields, with an emphasis on key concepts, tools and tricks.

Friday 28 April - Nóra Frankl (LSE)
Embedding graphs in Euclidean space

The dimension of a graph is the smallest d for which it can be embedded in R^d as a unit distance graph. Answering a question of Erdős and Simonovits, we show that any graph with less than binom{d+2}{2} edges has dimension at most d. Improving their result, we also show that the dimension of a graph with maximum degree d is at most d.

This is a joint work with Andrey Kupavskii and Konrad Swanepoel.

Friday 10 March - Janusz Brzdek (Cracow)
Fixed points in function spaces and Ulam stability

Stan Ulam asked a series of questions concerning stability of various mathematical relations. The issue of stability of an equation (e.g., difference, differential, functional, integral) can be very roughly expressed in the following way: When must a function satisfying an equation approximately (in some sense) be close to an exact solution to the equation?

Numerous results concerning such stability can be re-stated in the form of fixed point theorems in certain function spaces (for various operators, including also nonlinear ones) and, conversely, fixed point theorems can be applied to prove the stability of equations of various kinds. Moreover, some fixed point theorems have been established in recent years specifically in connection with investigations of Ulam-type stability. We discuss these connections and provide several recent particular examples. We also present some basic definitions and examples of simple classical results.

Friday 3 March - Olaf Parczyk (Frankfurt)
Explicit construction of universal hypergraphs

A hypergraph H is called universal for a family $\mathbb{F}$ of hypergraphs, if it contains every hypergraph $F \in \mathbb{F}$ as a copy. For the family of r-

uniform hypergraphs with maximum vertex degree bounded by Δ and at most n vertices any universal hypergraph has to contain $\Omega(n^{r-r/\Delta})$ many edges. We exploit constructions of Alon and Capalbo to obtain universal r-uniform hypergraphs with the optimal number of edges $O(n^{r-r/\Delta})$ when r is even, $r \mid \Delta$ or $\Delta=2$.
This is joint work with Samuel Hetterich and Yury Person.

Friday 24 February - Eng Keat Hng (LSE)
Erdos-Hajnal conjecture

No abstract available

Friday 27 January - Ewan Davies (LSE)
A probabilistic approach to bounding graph polynomials

No abstract available

Friday 20 January - Jan Corsten (LSE)
Grid Ramsey Problem

The *Grid Graph* $\Gamma_{m,n}$ is the graph product of K_n and K_m, i.e. the graph with vertices $[m] \times [n]$ and $\{(i,j),(i',j')\}$ being an edge if either $i = j$ or $i' = j'$. For a positive integer r, Shelah's number $G(r)$ is the smallest number n such that every r-colouring of $E(\Gamma_{n,n})$ induces an alternating rectangle, i.e. a rectangle whose parallel edges receive the same colour.

These numbers first appeared in Shelah's proof of the Hales-Jewett theorem in 1988, where he used the trivial bound $G(r) \leq r^{\binom{r+1}{2}} + 1$. The so far only improvement to the upper bound was made by Gyárfás in 1994, who showed $G(r) \leq r^{\binom{r+1}{2}} - r^{\binom{r-1}{2}+1} + 1$ for all $r \geq 3$. Conlon, Fox, Lee and Sudakov recently showed that $G(r)$ grows super-polynomially in r destroying the hope for a significant improvement of the Hales-Jewett number via the Grid Ramsey Problem.

In this talk we try to explain why the problem of determining $G(r)$ is difficult and sketch a proof of the slightly better upper bound $G(r) \leq r^{\binom{r+1}{2}} - r^{\binom{r}{2}} + 1$ for all $r \geq 3$.

Friday 13 January - Julian Sahasrabudhe (Memphis)
A Problem of Littlewood : Counting Zeros of Cosine Polynomials

For a finite set A of non-negative integers, we define the "Fourier transform" of this set to be the function on $[0, 2\pi]$ defined by

$$f_A(\theta) = \sum_{a \in A} \cos(a\theta).$$

In 1968, J.E. Littlewood asked for a lower bound on the minimum number of zeros attained by a function of the form f_A, where A is of a fixed size. In this talk, we show that f_A must have at least $(\log\log\log|A|)^{1/2-\epsilon}$ roots. This gives the first unconditional lowerbound on the problem of Littlewood, solves a conjecture of Borwein, Erdélyi, Ferguson & Lockhart and improves results of Borwein & Erdélyi. Along the way, we also mention a new result that is perhaps of independent interest: Every exponential polynomial f which "correlates" with a low-degree exponential polynomial P and has rational coefficients from a "small" set, must have a very particular structure.