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The endowment effect and environmental discounting

Simon Dietz[1,2] and Frank Venmans[3,1]

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Abstract

There is a considerable body of evidence from behavioural economics and contingent valuation showing that our preferences exhibit both reference dependence and loss aversion, a.k.a. the endowment effect. In this paper we consider the implications of the endowment effect for discounting future improvements in the environment. We show that the endowment effect modifies the discount rate via (i) an instantaneous endowment effect and (ii) a reference-level effect. Moreover we show that these two effects often combine to dampen the preference to smooth consumption over time. What this implies for discounting future environmental benefits may then depend critically on whether environmental quality is merely a factor of production of material consumption, or whether it is an amenity. On an increasing path of material consumption, dampened consumption smoothing implies a lower discount rate. But on a declining path of environmental quality and where we derive utility directly from environmental quality, it implies a higher discount rate. On non-monotonic paths, the endowment effect can give rise to substantial discontinuities in the discount rate.

Keywords: discounting, endowment effect, environmental discount rate, loss aversion, reference dependence, relative prices

JEL codes: D03, D61, H43, Q51

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"There is substantial evidence that initial entitlements do matter and that the rate of exchange between goods can be quite different depending on which is acquired and which is given up" (Tversky and Kahneman, 1991, p1039)

1 Introduction

The discounting debate is of enduring importance to environmental economics, because many investments in improving environmental quality provide pay-offs far into the future. Numerous aspects of environmental discounting have been discussed in the literature over the years (e.g. Lind et al., 1982; Portney and Weyant, 1999; Gollier, 2012; Arrow et al., 2013). However, one that has been missing is the implications of the endowment effect.

In one of their classic experiments, Kahneman et al. (1990) endowed half of their subjects with a coffee mug and asked them for the lowest price at which they would sell it. By contrast, the other half were asked how much they would pay for the same mug. Conventional consumer theory would predict no difference between the selling and buying prices. However, subjects endowed with the mug – those who would stand to lose it – were prepared to sell for more than twice as much as the remaining subjects – those who would stand to gain it – were willing to pay (also see Knetsch 1989; 1992).[1] Kahneman et al. therefore showed the initial endowment creates a reference point that matters, and in particular losses are ascribed more value than equivalent gains, which has been termed the 'endowment effect' (Thaler, 1980). As well as experiments, the endowment effect is consistent with a ubiquitous feature of contingent valuation studies into non-market goods, whereby there is a spread between stated willingness to accept compensation and willingness to pay (Horowitz and McConnell, 2002). It is also consistent with studies of various sorts into status quo bias (e.g. Samuelson and Zeckhauser, 1988; Knetsch, 1989),[2] and has been demonstrated in field studies (e.g. Genesove and Mayer, 2001).

The grounds for suspecting the endowment effect could modify the discount rate are fundamental. The discount factor is the marginal rate of substitution between consumption today and in the future. The endowment effect changes how we conceive of the relationship between changes in consumption, utility and welfare at different times (Tversky and Kahneman, 1991).

One aspect of the debate about environmental discounting, for which the endowment effect may be particularly germane, is the relative scarcity of environmental quality (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Sterner and Persson, 2008; Traeger, 2011). In a model where utility is obtained from the consumption of at least two goods, there is a discount rate for each of these goods. Furthermore each discount rate depends not only on consumption of the good in question, it also depends on the consumption of other goods, i.e. on relative scarcity. Hoel and Sterner (2007) and Traeger (2011) explore the effects of discounting and relative scarcity combined. Suppose material consumption

[1] To allay concerns that the disparity could have been due to differences in wealth between subjects, Kahneman et al. conducted a further experiment, in which, rather than being asked how much they would be willing to pay to buy the mug, subjects were given the option of being gifted the mug or a sum of money, and asked at what value they would choose money over the mug. Those endowed with the mug were still prepared to sell for over twice as much as the valuation put on the mug by those invited to choose.

[2] Status quo bias can, of course, be explained in other ways, such as the existence of search and transaction costs.

grows faster than environmental quality. The wedge between the discount rate on material consumption and that on environmental quality is an increasing function of the difference between the two goods' growth rates, and it is higher, the more limited is the degree of substitutability between the two goods.[3] Moreover, the larger is this wedge between the discount rates, the greater is the relative weight given to environmental quality.

So what is the connection between the role of relative price changes in discounting on the one hand, and the endowment effect on the other? The connection is that the scenario most often motivating interest in relative scarcity is one in which material consumption is increasing, while environmental quality is *decreasing*. Not only does environmental quality become relatively more scarce in this situation, gains in material consumption are being weighed against losses in environmental quality, which is the most interesting feature of the problem as far as the endowment effect is concerned.

The overall purpose of this paper is to integrate the endowment effect in models of environmental discounting. In doing so, it follows in the tradition of previous studies, which have examined the effects of other behavioural anomalies on discounting, most notably studies into so-called hyperbolic discounting.[4] We consider both a single-good setting, in which we invest in the environment in order to obtain extra material consumption in the future, and the aforementioned two-good setting, in which we invest to obtain future environmental amenities.

We show that the endowment effect modifies the discount rate via (i) an instantaneous endowment effect and (ii) a reference-level effect, and that the overall effect on the discount rate depends on the combination of these. However, we show that the two effects often combine to dampen the standard preference to smooth consumption over time. What this means for discounting can be fundamentally different depending on whether we are in a single-good or two-good world. In the former, if consumption is increasing as it usually does, the implication of dampened consumption smoothing is that the endowment effect reduces the discount rate. Conversely in the latter setting, assuming environmental quality is in decline, the implication is that the discount rate is increased. We complete our analysis by examining non-monotonic growth paths. These raise distinct issues, in particular loss aversion becomes an important factor. We show that on non-monotonic paths the endowment effect can give rise to substantial discontinuities in the discount rate.

We begin in Section 2 by characterising the endowment effect in a single-good setting, before repeating the analysis in Section 3 in a two-good setting. At the end of these two sections we are able to condense the instantaneous endowment effect and the reference-level effect into a single 'endowment factor', the sign and size of which Section 4 attempts to establish analytically on stylised growth paths, and Section 5 evaluates numerically. Section 6 provides a discussion.

[3] Traeger's Proposition 1 (p218) is a particularly clear expression of this.

[4] See Laibson (1997) and O'Donoghue and Rabin (1999), as well as Hepburn et al. (2010) for an application of hyperbolic discounting to natural-resource management. Karp and Traeger (2009) provide a general framework for analysing how the intertemporal dependence of preferences affects discounting. Our paper is in some respects a special case of their framework, but also extends the analysis to include multiple goods and loss aversion.

2 The single-good setting

This paper is about discounting the future benefits and costs of projects to improve the environment. These projects are assumed to be marginal.[5] The first setting we consider is one in which it is appropriate to discount environmental benefits in the future at the rate pertaining to consumption of material goods, denoted $C \in [0, \infty)$. When would this be appropriate? The answer is when the environment does not directly affect utility – when it has no amenity value, so to speak. Rather, improving the environment enhances our ability to produce material goods.[6]

2.1 Preferences

We begin by characterising welfare in the usual, discounted-utilitarian way as

$$J = \int_0^\infty U_t e^{-\delta t} dt, \tag{1}$$

where instantaneous utility is discounted at the constant rate $\delta > 0$.

Instantaneous utility depends on C. But it does not just depend on the level of C, it also depends on the difference between the level of consumption and a reference level. Our instantaneous utility function $U : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$ is then

$$U(C_t, \underline{C}_t) = v\left(C_t\right) + g\left(C_t - \underline{C}_t\right), \tag{2}$$

where $\underline{C}$ is the reference level (we will elaborate on the formation of $\underline{C}$ in a moment). Instantaneous utility therefore represents a mixed objective. The function $v(.)$ corresponds with the standard theory of preferences, in that individual utility remains directly responsive to the absolute level of consumption. Hence we shall refer to this element of the instantaneous utility function as *consumption-level utility*. We assume $v(.)$ is continuous, twice continuously differentiable, and that $v'(C_t) > 0$ and $v''(C_t) < 0$.

By contrast, the *gain-loss* function $g(.)$ captures the endowment effect. It is assumed to be continuous and twice continuously differentiable except when $C_t - \underline{C}_t = 0$. We impose three further behavioural restrictions on $g(.)$, as a formal representation of the famous value function in Kahneman and Tversky (1979).[7] Let $x \equiv C_t - \underline{C}_t$:

Assumption 1. *[Bigger gains and smaller losses are weakly preferred]* $g'(x) \geq 0$.

Assumption 1 just ensures the gain-loss function is weakly increasing over its entire domain.

Assumption 2. *[Loss aversion] If* $x > 0$, *then* $g'(-x) > g'(x)$.

Assumption 2 represents loss aversion with respect to both large changes in consumption and small ones. The former property is of course shared with strictly concave

[5] See Dietz and Hepburn (2013) on discounting non-marginal environmental improvements.

[6] This is the conventional way in which improving the environment is conceptualised in economic models of climate change, for example. In these models, carbon dioxide emissions cause temperatures to rise and rising temperatures reduce the aggregate output obtainable with given factor inputs (e.g. Nordhaus, 2013).

[7] Building on Bowman et al. (1999) and later Kőszegi and Rabin (2006).

consumption-level utility functions, but the latter property – specifically $\lim_{x\to 0} g'(-x)/g'(x) > 1$ – is a distinctive feature of the gain-loss function, which was made famous by Kahneman and Tversky (1979).

Assumption 3. *[Non-increasing sensitivity]* $g''(x) \leq 0$ *for all* $x > 0$*, and* $g''(x) \geq 0$ *for all* $x < 0$*.*

Assumption 3 ensures the gain-loss function is weakly convex over the domain of losses and weakly concave over the domain of gains. Diminishing sensitivity is required to represent a preference such as: "the difference between a yearly salary of $60,000 and a yearly salary of $70,000 has a bigger impact when current salary is $50,000 than when it is $40,000" (Tversky and Kahneman, 1991, p1048). Constant sensitivity implies the impact of the difference in salary does not depend on current salary.

The reference level $\underline{C}$ depends on the history of consumption as in Ryder and Heal (1973):

$$\underline{C}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} C_\tau d\tau, \tag{3}$$

where the parameter $\alpha \in [0,1]$ represents the responsiveness of the reference level to changes over time in the level of consumption, i.e. it captures an individual's memory for past consumption. The smaller is α, the longer that memory is. If $\alpha = 1$, then the current reference level is just consumption at the last instant. At the other extreme, if $\alpha = 0$ then the current reference level is the same as consumption far in the past.[8] It is worth noting that there is empirical support for the idea that a long history of consumption levels determines the reference level (Strahilevitz and Loewenstein, 1998). Combining (2) and (3) means preferences are inter-temporally dependent.

2.2 Discounting with the endowment effect

For an individual with preferences given by Equations (1)-(3), Appendix 1 shows that the marginal rate of substitution between material consumption at date 0 and date t, the discount factor, is

$$\begin{aligned} D^C(t,0) &\equiv J_{C_t}/J_{C_0} \\ &= e^{-\delta t} \frac{v'(C_t) + \overbrace{g'\left(C_t - \underline{C}_t\right)}^{\substack{\text{Instantaneous} \\ \text{endowment} \\ \text{effect}}} \overbrace{- \alpha \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g'\left(C_\tau - \underline{C}_\tau\right) d\tau}^{\substack{\text{Reference-level} \\ \text{effect}}}}{v'(C_0) + g'\left(C_0 - \underline{C}_0\right) - \alpha \int_{\tau=0}^{\infty} e^{-(\alpha+\delta)\tau} g'\left(C_\tau - \underline{C}_\tau\right) d\tau}, \end{aligned} \tag{4}$$

where the important feature is that J_{C_t} is a functional derivative.

It turns out that simply inspecting this expression for the discount factor helps a great deal in comprehending the mechanisms driving our main results later in the paper. As well as providing consumption-level utility v', a unit of consumption at time t provides a

[8]We shall make this notion precise later in the paper.

contemporaneous gain, which we will refer to as the *instantaneous endowment effect*. In addition, a unit of consumption at time t affects the reference level from which gains are evaluated after time t. This we will describe as the *reference-level effect*. In evaluating investment projects, forward-looking individuals will anticipate the effect that changes in consumption have on reference levels thereafter. The reference-level effect is negative, because an increase in consumption today raises future reference levels, and thereby reduces future gains in consumption, or increases future losses. By how much an increase in consumption today raises future reference levels depends on the memory parameter α, and what effect this in turn has on welfare depends on the pure time discount rate δ. Another way to think of the reference-level effect is in relation to the literature on habit formation (e.g. Constantinides, 1990; Campbell and Cochrane, 1999): a unit of consumption at time t contributes to our becoming habituated to higher consumption, which in turn reduces the marginal contribution to welfare of future increments in consumption.

It is important to note the role of loss aversion is implicit so far. That is, (4) could just as well describe a model with reference dependence, but without loss aversion (i.e. without Assumption 2). But loss aversion implies both the instantaneous endowment effect and the reference-level effect will be higher on decreasing consumption paths, because g' will be higher.

Appendix 1 goes on to show that the discount rate in the presence of the endowment effect can be expressed as

$$\begin{aligned} r^C &\equiv -\frac{d}{dt}\ln D^C(t,0), \\ &= \delta - \frac{v''\dot{C} + g''(\dot{C} - \alpha C + \alpha \underline{C}) + \alpha g' - \alpha(\alpha+\delta)\int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}{v' + g' - \alpha \int_t^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}, \end{aligned} \tag{5}$$

where we drop the time subscripts for convenience's sake. Since this is still a rather complex expression, it is helpful to clean it up. To do so, we define the shadow price of reference consumption as

$$\mu^{\underline{C}} = \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau. \tag{6}$$

This is the marginal effect on welfare at time t of reducing the reference level, without changing the consumption path. Substituting $\mu^{\underline{C}}$ into (5), we obtain

$$r^C = \delta - \frac{\dot{v'} + \dot{g'} - \alpha \dot{\mu}^{\underline{C}}}{v' + g' - \alpha \mu^{\underline{C}}}. \tag{7}$$

This characterises the discount rate on an arbitrary consumption path. Appendix 2 shows that it also characterises the discount rate on an optimal path, where $\mu^{\underline{C}}$ is the negative of the costate variable on reference consumption.

If we define the absolute value of the elasticity of consumption-level marginal utility as

$$\eta^{CC} \equiv \frac{-v''C}{v'},$$

then we finally obtain a more convenient and recognisable expression for r^C:

Definition 1. In the presence of the endowment effect as characterised by Eq. (2), the material discount rate is

$$r^C = \delta + \theta^C \eta^{CC} \frac{\dot{C}}{C}, \tag{8}$$

where the 'material endowment factor' is

$$\theta^C = \frac{1 + \frac{\dot{g}'}{v'} + \frac{\alpha \dot{\mu}^C}{v'}}{1 + \frac{g'}{v'} + \frac{\alpha \mu^C}{v'}}. \tag{9}$$

Equation (8) shows that the endowment effect modifies the consumption discount rate through the factor θ^C. In the absence of the endowment effect, $\theta^C = 1$. We would like to know the sign and size of θ^C. If consumption is increasing, as we would normally suppose, and $\theta^C < 1$, the endowment effect decreases the discount rate and this increases our willingness to pay to provide future environmental benefits. Conversely if $\theta^C > 1$, the endowment effect increases the discount rate and our willingness to pay is lower.

3 The two-good setting

Before we embark on the exercise of establishing the sign and size of θ^C, it is important to show that (8) is not in fact the appropriate discount rate for future environmental benefits in a two-good setting, unless it is adjusted for changes in relative prices (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Sterner and Persson, 2008; Traeger, 2011). In this section, we therefore suppose that, in addition to the composite produced good C, instantaneous utility also depends on the quality of the natural environment $E \in \left[0, \overline{E}\right]$.[9]

3.1 Preferences

Preferences are a minimal extension of the single-good setting. The welfare functional remains discounted utilitarian as in (1). The instantaneous utility function $U : \mathcal{R}^2 \times \mathcal{R}^2_+ \rightarrow \mathcal{R}$ in period t is now

$$U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) = v\left(C_t, E_t\right) + g^C\left(C_t - \underline{C}_t\right) + g^E\left(E_t - \underline{E}_t\right), \tag{10}$$

where $\underline{E}$ is the reference level of environmental quality. We assume $\underline{E}$ evolves in just the same way as the reference level of material consumption, i.e.

$$\underline{E}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} E_\tau d\tau. \tag{11}$$

We could of course easily assign different memory parameters to each of the goods, however it is an unnecessary complication. We make the same assumptions about $v(.)$ as above, and add that $v_E > 0$ and $v_{EE} < 0$. No restriction is placed on v_{CE}.

We add the gain-loss function $g^E(.)$ to capture the endowment effect with respect to environmental quality. It is conditioned by the same assumptions as what we now call

[9] It would be obvious to interpret $\overline{E}$ as a pristine natural environment, for instance a primary rainforest, or the pre-industrial concentration of greenhouse gases in the atmosphere. But it can in principle stand for a human-modified natural environment instead, for instance an ancient agricultural landscape.

$g^C(.)$. By virtue of the additive way in which reference dependence enters the utility function, $g^{C\prime}$ is assumed independent of the level or change in E, and $g^{E\prime}$ is likewise assumed independent of the level or change in C.

3.2 Discounting with the endowment effect

An environmental project paid for in units of material goods at date 0, which increases environmental quality at a future date t, is welfare-preserving if and only if $J_{C_0}dC_0 = -J_{E_t}dE_t$. Define the accounting price of environmental quality as $p_t \equiv J_{E_t}/J_{C_t}$. Then the project is welfare-preserving if and only if

$$J_{C_0}dC_0 = -J_{E_t}\frac{dC_t}{p_t}.$$

The appropriate discount factor in a trade-off between consumption of the produced good at date 0 and environmental quality at date t depends on how environmental quality is priced. If environmental quality at date t is converted into units of the produced good using the relative price p_t, the appropriate comparison is between consumption of the produced good today and in the future using the discount factor D^C from (4). If, by contrast, environmental quality is converted into units of the produced good at p_0, the appropriate comparison is between environmental quality today and in the future using an environmental discount factor D^E. This is

$$D^E(t,0) = \frac{J_{E_t}}{J_{E_0}} = \frac{J_{C_t}}{J_{C_0}}\frac{p_t}{p_0},$$

which leads to the following equivalence between environmental and consumption discount rates (Weikard and Zhu, 2005; Hoel and Sterner, 2007):

$$r = r^E = r^C - \frac{\dot{p}}{p}, \tag{12}$$

where r stands for the internal rate of return (IRR) of the project and $\dot{p}/p$ is the relative price change . Expanding the term $\dot{p}/p$ we get

$$\begin{aligned}
\frac{\dot{p}}{p} &\equiv \frac{\frac{d}{dt}(J_E/J_C)}{J_E/J_C} \\
&= \frac{v_{EE}\dot{E} + v_{CE}\dot{C} + \dot{g^{E\prime}} + \alpha g^{E\prime} - \alpha(\alpha+\delta)\int_t^\infty e^{-(\alpha+\delta)(\tau-t)}g^{E\prime}d\tau}{v_E + g^{E\prime} - \alpha\int_t^\infty e^{-(\alpha+\delta)(\tau-t)}g^{E\prime}d\tau} \\
&\quad - \frac{v_{CC}\dot{C} + v_{CE}\dot{E} + \dot{g^{C\prime}} + \alpha g^{C\prime} - \alpha(\alpha+\delta)\int_t^\infty e^{-(\alpha+\delta)(\tau-t)}g^{C\prime}d\tau}{v_C + g^{C\prime} - \alpha\int_t^\infty e^{-(\alpha+\delta)(\tau-t)}g^{C\prime}d\tau} \\
&= \theta^E\left(\eta^{EC}\frac{\dot{C}}{C} - \eta^{EE}\frac{\dot{E}}{E}\right) - \theta^C\left(\eta^{CE}\frac{\dot{E}}{E} - \eta^{CC}\frac{\dot{C}}{C}\right),
\end{aligned} \tag{13}$$

where η^{EC} is the elasticity of consumption-level marginal utility of environmental quality with respect to consumption of material goods,

$$\eta^{EC} \equiv \frac{v_{CE}C}{v_E},$$

and η^{EE} is the elasticity of consumption-level marginal utility of environmental quality with respect to environmental quality,

$$\eta^{EE} \equiv \frac{-v_{EE}E}{v_E}.$$

Substituting (7) and (13) into (12), and defining the shadow price of reference environmental quality as

$$\mu^{\underline{E}} = \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g^{E\prime} d\tau, \tag{14}$$

we obtain the IRR of the project or equivalently the discount rate on environmental quality:

Definition 2. In the presence of the endowment effect as characterised by Eq. (2), the environmental discount rate is

$$r^E = \delta + \theta^E \left(\eta^{EE} \frac{\dot{E}}{E} - \eta^{EC} \frac{\dot{C}}{C} \right) \tag{15}$$

where the 'environmental endowment factor' is

$$\theta^E = \frac{1 + \frac{\dot{g^{E\prime}}}{\dot{v_E}} + \frac{\alpha \dot{\mu^{\underline{E}}}}{\dot{v_E}}}{1 + \frac{g^{E\prime}}{v_E} + \frac{\alpha \mu^{\underline{E}}}{v_E}}. \tag{16}$$

Appendix 2 shows how the environmental discount rate r^E can be derived from an optimal control problem, in which environmental degradation has either a flow or stock character.

Equation (15) shows the distinct roles played by the endowment effect and changes in the relative scarcity of environmental quality in modifying the material discount rate for the purposes of evaluating an environmental project that has amenity value. If we are in a setting where environmental quality is declining while consumption of material goods is increasing, $\eta^{EE} \cdot \dot{E}/E < 0$, which will reduce the discount rate. The sign of $\eta^{EC} \cdot \dot{C}/C$ is ambiguous *a priori*, but ordinarily we would expect $\eta^{EC} > 0$ so that, as environmental quality becomes relatively more scarce, the environmental discount rate is lower still (Hoel and Sterner, 2007; Traeger, 2011).

Parallel to the previous analysis, the endowment effect enters via the environmental endowment factor θ^E. While the implications of θ^E for r^E are obvious from Definition 2, the implications of θ^E for willingness to pay to provide future environmental benefits are less obvious in the two-good setting: not only does the endowment effect impact upon the discount rate, it is clear that since the initial accounting price $p_0 \equiv J_{E_0}/J_{C_0}$, this will also be modified by the endowment effect. For long-run investments in the environment, the effect of θ^E on r^E will dominate, and *vice versa* for short-run investments.

4 The endowment factor on stylised growth paths

Given Definitions 1 and 2, the crux of the paper is the question: what is the sign and size of the endowment factor θ^i, $i \in \{C, E\}$?

4.1 Linear paths

First, note that in the particular case of constant sensitivity, i.e. $g^{i\prime\prime}(x) = 0$ for all $x \neq 0$, the endowment factor on a strictly increasing/decreasing consumption path simplifies to

$$\theta^i = \frac{1}{\left[1 + \left(\frac{\delta}{\alpha + \delta}\right) \frac{g^{i\prime}}{v_i}\right]}. \tag{17}$$

Thus under constant sensitivity $0 < \theta^i < 1$ if and only if $\delta > 0$. We assume this is so. Positive pure time preference, however small, is an uncontroversial assumption. Even if the view is taken that the discount rate is derived from a social welfare functional and it should be impartial to the date at which utility is enjoyed, (very) small positive utility discounting still follows from taking into account the probability of extinction of society (e.g. Stern, 2007; Llavador et al., 2015).

But that is not all. Equality (17) will also hold even if preferences obey diminishing sensitivity – i.e. $g^{i\prime\prime}(x) < (>)0$ for all $x > (<)0$ – as long as the consumption path is linear or in other words arithmetically increasing/decreasing. Let us prove this, and capture both of these results, in the following Proposition:

Proposition 1. *[The endowment effect dampens consumption smoothing on a linear path] On a linear increasing or decreasing consumption path, or on any strictly increasing/decreasing consumption path with constant sensitivity,* $0 < \theta^i < 1$ *if and only if* $\delta > 0$.

Proof. See Appendix 3. □

Standard consumption-level utility makes us want to smooth consumption between dates at which the level of consumption is different. When consumption is increasing, marginal consumption-level utility falls. When consumption is decreasing, marginal consumption-level utility rises. Therefore in a growing economy we want to consume earlier by discounting the future at a higher rate. Conversely when consumption is falling we want to postpone it to the future by discounting at a lower rate. All of this is in the economist's DNA.

Yet the endowment effect interferes with these preferences. Let us attempt an intuitive explanation of what happens. On a linear path, marginal gain-loss utility is clearly constant. Moreover when $\delta > 0$ the instantaneous endowment effect is larger than the reference-level effect on a linear path, so marginal gain-loss utility is constant and positive. Since overall marginal utility is the sum of marginal consumption-level utility and marginal gain-loss utility as in (4), the endowment effect causes overall marginal utility to decrease at a slower rate on an increasing path, and increase at a slower rate on a decreasing path.

Most of the time consumption of material goods is increasing. Therefore the implication of Proposition 1 is that the endowment effect reduces the material discount rate r^C. This increases our willingness to pay to provide future environmental improvements, which increase material consumption but do not otherwise contribute to utility. By contrast, oftentimes we seek to improve environmental quality, because it is on a decreasing path and it matters directly for our utility. In this case, Proposition 1 implies that the endowment effect *increases* the environmental discount rate r^E and this may reduce our willingness to pay to provide future environmental improvements.

4.2 Non-linear paths

Proposition 1 considers paths along which marginal gain-loss utility is constant. Either preferences are characterised by constant sensitivity, or consumption follows a linear increasing/decreasing path. One way of expressing this is to say that

$$g_t^{i\prime} = g_0^{i\prime} e^{kt} \tag{18}$$

and the constant rate of change of marginal gain-loss utility $k \equiv \dot{g^{i\prime}}/g^{i\prime} = 0$. We can then see that, under diminishing sensitivity, $k < 0$ corresponds with a consumption path that is either convex increasing or concave decreasing, while $k > 0$ corresponds with a consumption path that is either concave increasing or convex decreasing. Remember, diminishing sensitivity implies marginal gain-loss utility falls, the larger is the gain/loss. Convex increasing and concave decreasing consumption paths imply ever larger gains and losses respectively, while concave increasing and convex decreasing consumption paths imply ever smaller gains and losses respectively.

What kinds of path could be represented by (18), where k is constant? Appendix 4 shows that, if the gain-loss function has a constant elasticity η^g over the appropriate domain of gains or losses, then paths with constant k correspond with paths with a constant rate of consumption growth/decline, which we call h. Specifically $k = -\eta^g h$ if and only if $h > -\alpha$. Convex consumption paths can be represented by $C_t = C_{t_0} e^{h(t-t_0)}$, where $h > 0$ gives rise to convex increasing paths, while $h < 0$ gives rise to convex decreasing paths. Concave consumption paths can be represented by $C_t = \Upsilon - \Upsilon e^{h(t-T)}$. When $h > 0$, consumption is concave decreasing from a horizontal asymptote at $C_{-\infty} = \Upsilon$ to $C_T = 0$. On the whole path, $t-T < 0$. When $h < 0$, consumption is concave increasing from $C_T = 0$ and converges asymptotically to $C_\infty = \Upsilon$. On the whole path, $t - T > 0$.

All of this opens up further insights into the sign and size of the endowment factor on non-linear consumption paths, because we can combine (9) and (16) with (18) to describe a functional relationship between θ^i and k:

$$\theta^i = \frac{1 + \frac{\dot{g^{i\prime}}}{\dot{v_i}} \left(\frac{\delta - k}{\alpha + \delta - k} \right)}{1 + \frac{g^{i\prime}}{v_i} \left(\frac{\delta - k}{\alpha + \delta - k} \right)}. \tag{19}$$

This functional relationship can be plotted. However, before we do so, note that if we also rewrite the discount rate as a function of k,

$$r^i \quad = \delta - \frac{\dot{v_i} + \dot{g^{i\prime}} - \left(\frac{\alpha}{\alpha + \delta - k} \right) \dot{g^{i\prime}}}{v_i + g^{i\prime} - \left(\frac{\alpha}{\alpha + \delta - k} \right) g^{i\prime}}, \tag{20}$$

we obtain a result that will be helpful in interpreting the analysis that follows.

Lemma 1. *The reference-level effect is a fixed proportion of the instantaneous endowment effect if k is constant.*

Keeping this in mind, we turn to Figure 1, which plots θ^i as a function of k on a strictly increasing consumption path. We always assume $\delta > 0$. Looking first at convex increasing paths, the following Proposition is established:

Proposition 2. *[The endowment factor on convex increasing paths] When consumption is strictly increasing and $k < 0$,*

$$\begin{cases} 0 < \theta^i < 1 & \iff k > \frac{\dot{v}_i}{v_i} \\ \theta^i > 1 & \iff k < \frac{\dot{v}_i}{v_i} \end{cases}.$$

On a convex increasing path, gains grow over time, hence $g^{i\prime}$ falls over time and so do both the instantaneous endowment effect and the reference-level effect. We know from Lemma 1 that the latter is a fixed fraction of the former. Moreover along a convex increasing consumption path $k < 0$, so the reference-level effect is always smaller than the instantaneous endowment effect. This means overall marginal gain utility is positive and falls along the path.

What then becomes crucial is the relationship between the rate of decrease of marginal gain utility, k, and the rate of decrease of marginal consumption-level utility, $\dot{v}_i/v_i$. When $k > \dot{v}_i/v_i$, marginal gain utility falls more slowly than marginal consumption-level utility, hence overall marginal utility does not fall as quickly as it would otherwise do and $0 < \theta^i < 1$, like on linear paths. The discount rate is consequently lower. By contrast, when $k < \dot{v}_i/v_i$, marginal gain utility falls faster than marginal consumption-level utility, overall marginal utility falls more quickly than it would otherwise do and our preference to bring consumption forward is amplified, increasing the discount rate ($\theta^i > 1$).

On a concave increasing path, the sign of θ^i is ambiguous:

Proposition 3. *[The endowment factor on concave increasing paths] When consumption is strictly increasing and $k > 0$,*

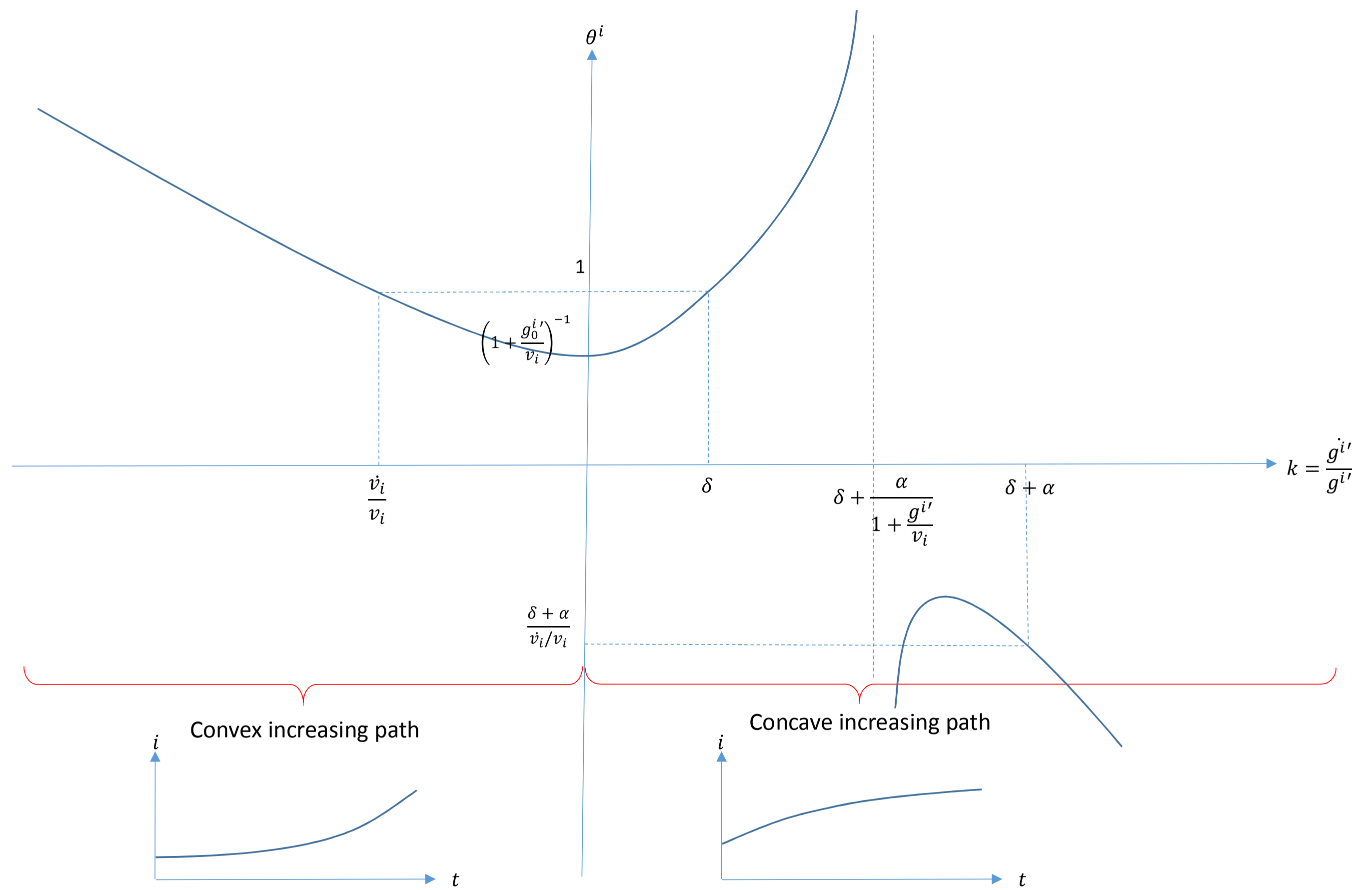
$$\begin{cases} 0 < \theta^i < 1 & \iff k < \delta \\ \theta^i > 1 & \iff \delta < k < \delta + \alpha \\ \theta^i < 0 & \iff \delta + \alpha < k \end{cases}.$$

The reference-level effect is increasing in k. If $k < \delta$, the instantaneous endowment effect is larger than the reference-level effect according to (20), which means that marginal gain utility is positive overall and this time it is increasing. This gives us another case in which $0 < \theta^i < 1$. If $k > \delta$, the reference-level effect is larger than the instantaneous endowment effect, which means that marginal gain utility is negative and increasing. This increases the discount rate, because the rate at which overall marginal utility falls is amplified ($\theta^i > 1$). However, in the limit as $k \to \delta + \alpha / \left(1 + g^{i\prime}/v_i\right)$, θ^i becomes unbounded. To the right of the asymptote, $\theta^i < 0$.

Figures 2 and 3 plot θ^i as a function of k on strictly decreasing consumption paths. The rate of decrease of consumption turns out to matter here. Therefore Figure 2 depicts a setting of rapidly decreasing consumption, defined as $\dot{v}_i/v_i > \delta + \alpha / \left(1 + g^{i\prime}/v_i\right)$. By contrast, Figure 3 depicts the opposite setting of slowly decreasing consumption, where $\dot{v}_i/v_i < \delta + \alpha / \left(1 + g^{i\prime}/v_i\right)$.[10]

[10]Note the exact placement of $\dot{v}_i/v_i$ on the Figures is arbitrary; we only know where it lies in relation to $\delta + \alpha / \left(1 + g^{i\prime}/v_i\right)$. This affects the strength of the conclusions we can draw.

Figure 1: The endowment factor as a function of k on strictly increasing consumption paths.



Looking first at concave decreasing paths, the following Proposition is plain to see:

Proposition 4. *[The endowment factor is less than unity on concave decreasing consumption paths]* $\theta^i < 1$ *if consumption is strictly decreasing and* $k < 0$.

The preference to smooth consumption, in this case by postponing it, is also dampened on concave decreasing consumption paths. Indeed, if marginal loss utility falls quickly enough, the endowment effect can actually reverse the preference to postpone consumption (although whether the discount rate changes sign depends on δ). Concave decreasing consumption paths might be of particular relevance to environmental discounting at rate r^E. For instance, in their exploration of the concept of the 'Anthropocene', Steffen et al. (2011) plotted the evolution of 12 global environmental indicators, ranging from the atmospheric stock of greenhouse gases and ozone, to the depletion of fisheries, forests and biological diversity. They showed that in all of the aforementioned cases environmental quality has been on a concave decreasing path since the beginning of the industrial revolution, branding the last 70 years in particular the 'Great Acceleration'.[11] Proposition 4 implies that, when the endowment effect is taken into account, r^E is higher and our willingness to pay to provide future environmental improvements may be reduced.

An intuitive explanation for Proposition 4 draws once again on Lemma 1, which says that the reference-level effect falls as a fixed proportion of the instantaneous endowment effect. The case of concave decreasing consumption is also one where the reference-level effect is the smaller of the two effects ($k < 0$). On a concave decreasing consumption path, marginal consumption-level utility is increasing. On the other hand, marginal loss utility is positive and decreasing, so overall marginal utility increases more slowly. The effect could be sufficiently large that overall marginal utility itself is decreasing overall.

On a convex decreasing path, diminishing sensitivity instead results in an increase in marginal loss utility over time. The instantaneous endowment and reference-level effects grow in step with each other as will be familiar by now, but of course when $k > 0$ it is no longer assured that the reference-level effect is smaller than the instantaneous endowment effect. It will be the case if $k < \delta$, a situation in which consumption decreases relatively slowly. If, in addition to this, $k < \dot{v}_i/v_i$, $0 < \theta^i < 1$ and the discount rate increases:

Proposition 5. *[On convex decreasing paths, the endowment effect dampens consumption smoothing if marginal loss utility increases at a slower rate than pure time preference or marginal consumption-level utility, whichever is smaller] When consumption is strictly decreasing and* $k > 0$, $0 < \theta^i < 1$ *if* $k < \delta < \dot{v}_i/v_i$ *or if* $k < \dot{v}_i/v_i < \delta$.

If $k > \delta$ – consumption is decreasing more rapidly – the reference-level effect is larger than the instantaneous endowment effect. As the Figures show, this can result in $\theta^i > 1$, but the picture is complicated. There is the asymptote of θ^i, reached as $k \to \delta + \alpha/\left(1 + g^{i\prime}/v_i\right)$, with the limit behaviour of θ^i in the region of the vertical asymptote varying between the two panels (see Appendix 5). In this situation, the reference-level

[11] Where environmental quality is the inverse of the stock of pollution (carbon dioxide and ozone), the stock of pollution has increased exponentially. The percentage of global fisheries fully exploited, and the percentage of global forest cover destroyed since 1700, have both increased exponentially. The rate of species extinctions has increased exponentially, with approximately no species additions. See Steffen et al. (2011), figure 1.

Figure 2: The endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v}_i/v_i > \delta + \alpha/\left(1 + g^{i\prime}/v_i\right)$.

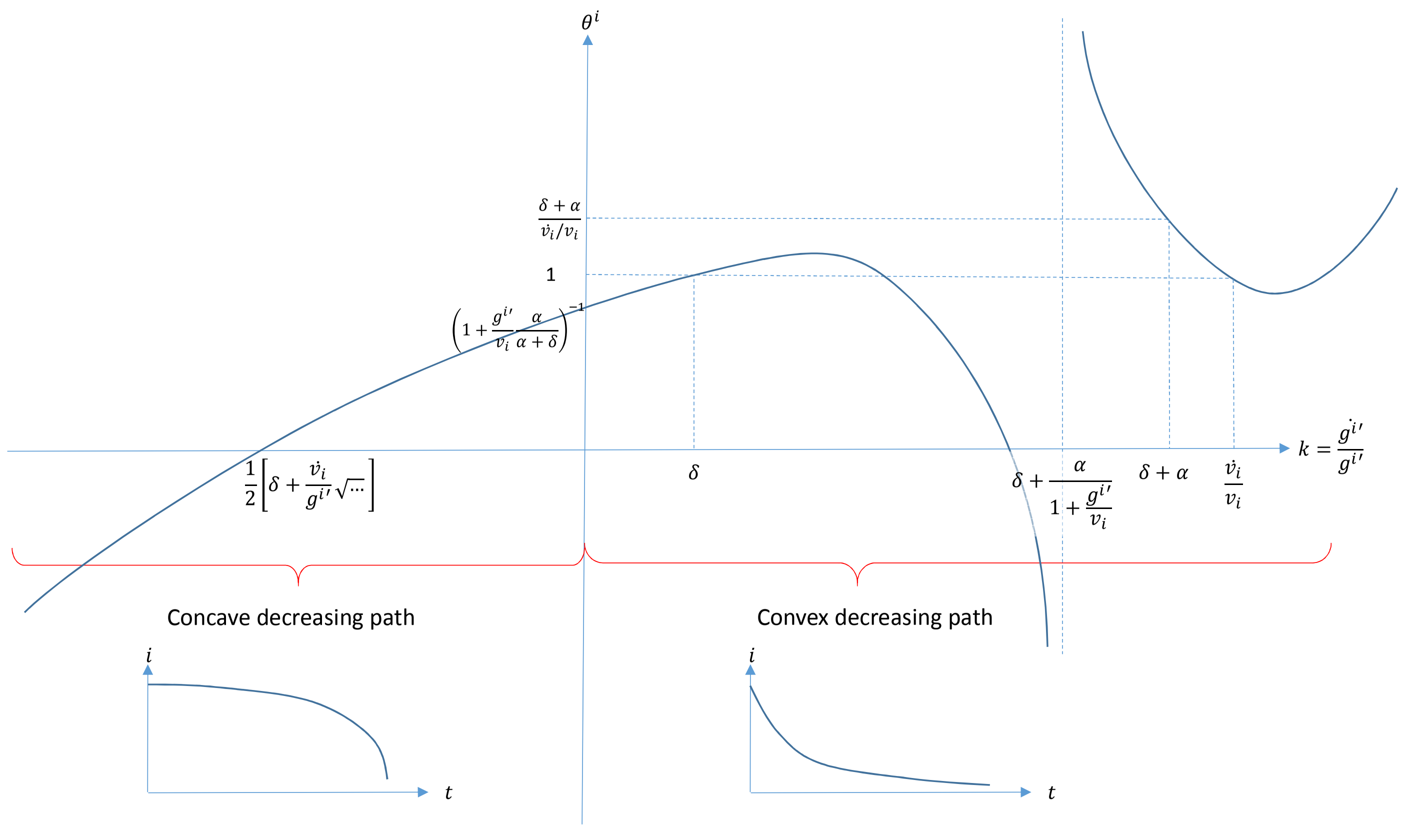
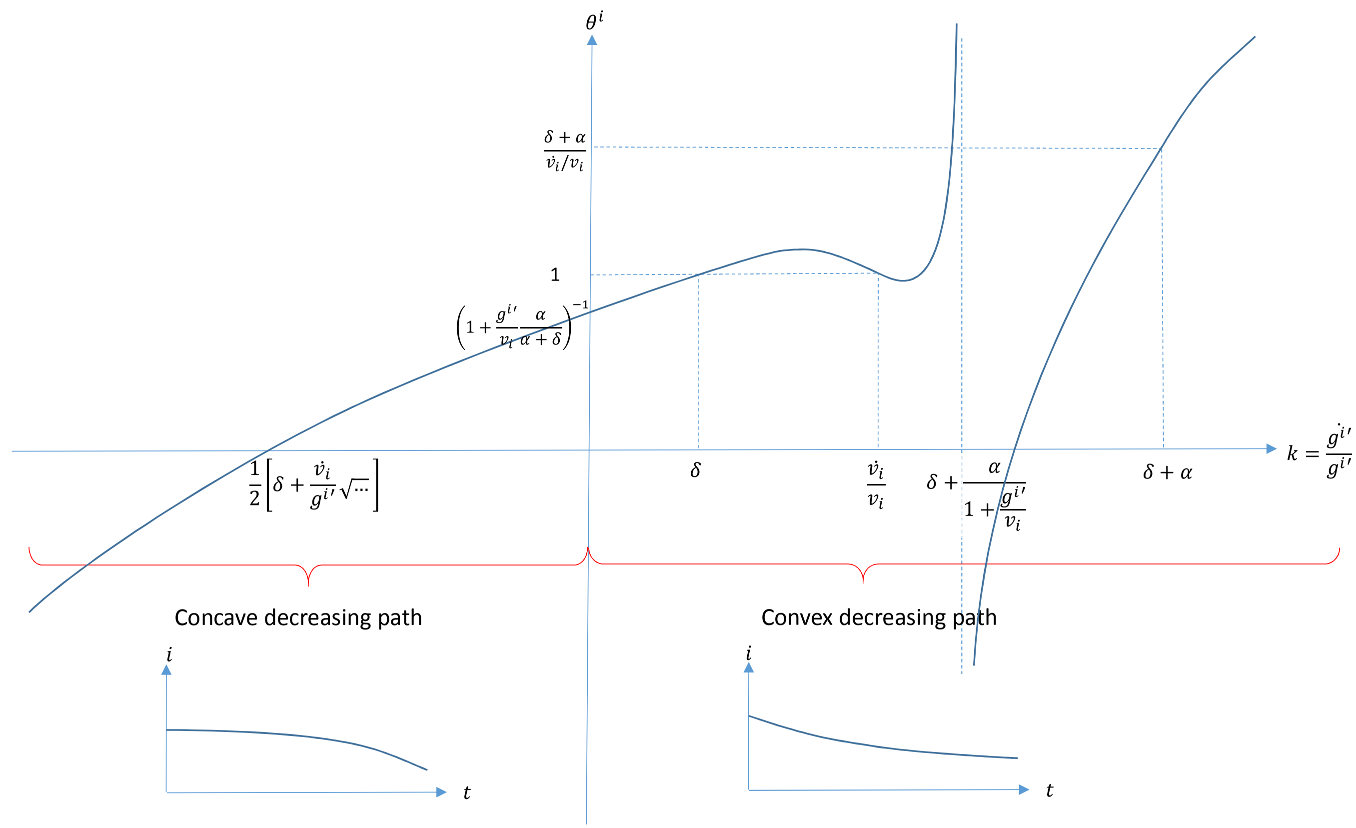


Figure 3: The endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v}_i/v_i < \delta + \alpha/\left(1 + g^{i\prime}/v_i\right)$.



effect fully cancels out the sum of marginal consumption-level utility and the instantaneous endowment effect. If consumption has no value today (tomorrow), the discount rate is infinitely large (small).

5 Numerical illustrations

Some numerical examples will be helpful at this point. They will enable us to quantify the endowment effect on the discount rate. We can also inspect cases similar to those above, where the sign and size of the endowment factor was found to be conceptually ambiguous. Finally, we can for the first time look at non-monotonic growth paths, where loss aversion plays a prominent role.

5.1 Single-good setting

Functional forms and parameter scheme In the single-good setting, we specify a consumption-level utility function that is isoelastic, i.e.

$$v(C_t) = \frac{1}{1-\phi} C_t^{1-\phi}, \tag{21}$$

where $\phi > 0$ is the elasticity of marginal consumption-level utility. For gain-loss utility $g^i(x)$, $i = C$, we use a generalisation of the functional form proposed by Tversky and Kahneman (1992), which is consistent with Assumptions 1-3:

$$g^i(x) = \begin{cases} (x+\psi)^\beta - \psi^\beta, & x \geq 0 \\ -\lambda \left[(-x+\psi)^\beta - \psi^\beta \right] & x < 0 \end{cases} , \tag{22}$$

where $\beta \in (0, 1]$ and $\lambda \geq 1$. Compared with Tversky and Kahneman (1992), we introduce the parameter $\psi > 0$ to ensure marginal gain-loss utility is bounded from above as $x \to 0$ in the limit, in a similar fashion to the bounding parameter in harmonic absolute risk aversion (HARA) functions (Gollier, 2001). The parameter ψ enters twice in order to also satisfy the property that $g^i(0) = 0$. Bounding marginal gain-loss utility becomes important when we consider non-monotonic paths later in this section. It does mean that $g^i(x)$ exhibits a non-constant elasticity, which in turn means that k is not constant, but for $x \gg 0$ it will be approximately constant.

A weighted sum of (21) and (22) makes up the instantaneous utility function. Assuming consumption is increasing, this would be written as

$$U_t(C_t, \underline{C}_t) = \tfrac{\zeta}{1-\phi} C_t^{1-\phi} + (1-\zeta) \left[(C_t - \underline{C}_t + \psi)^\beta - \psi^\beta \right]. \tag{23}$$

The parameter $\zeta \in [0, 1]$ governs the value share of consumption-level utility relative to gain-loss utility. In order to calibrate ζ, we target the initial value share of consumption-level utility, Z. We initialise the model twenty years in the past, so that by the time our discounting analysis begins (at $t = 0$), reference consumption levels have formed, which are consistent with historical data. Therefore

$$Z \approx \frac{\zeta \left(v_{C_{-20}} C_{-20} + v_{E_{-20}} E_{-20} \right)}{U_{C_{-20}} C_{-20} + U_{E_{-20}} E_{-20}}.$$

Table 1: Default parameter values

Parameter	Value
Z	0.75
ϕ	1.5
γ	0.9
σ	0.5
β	0.9
λ	2.25
δ	1.5%
α	0.5
ψ	$1

Table 1 lists the default parameter values chosen in order to populate (23), as well as the pure rate of time preference δ, the reference-level decay parameter α, and some further parameters that will not come into use until we analyse the two-good setting below. We choose typical values from empirical studies for the elasticity of marginal utility $\phi = 1.5$ (Groom and Maddison, 2013) and the parameters of the gain-loss function; $\beta = 0.9$ and $\lambda = 2.25$ (Barberis, 2013). Choosing the pure rate of time preference is particularly controversial, so we opt for a middle-of-the-road value of 1.5%. Z is hard to pin down with empirical evidence, so we conservatively give a 3/4 share of instantaneous utility to consumption-level utility.

Convex increasing consumption Figure 4 plots the mean discount rate in an illustration, in which annual consumption per capita grows at 1.5% per annum. This is the growth rate of global average household final consumption expenditure per capita over the last 30 years.[12] Discount rates with and without the endowment effect are shown.

Without the endowment effect, it is well known that the discount rate 'r^C std.' is given by the Ramsey rule; $r^C = \delta + \phi \cdot \dot{C}/C$, so $1.5 + 1.5 * 1.5 = 3.75\%$.[13] But, when the endowment effect is present, the discount rate 'r^C endow.' is initially just 3.24%, and falls further to 2.73% in 100 years. Therefore the endowment effect makes a big difference in this empirically plausible example, resulting in a much more patient decision-maker who would be willing to pay more to improve the environment in the future, in a single-good setting.

Since r^C endow. is lower than r^C std. and falling relative to it, the material endowment factor $0 < \theta^C < 1$ and $\dot{\theta^C} < 0$. Consumption of the produced good is on a convex increasing path, so the implication is that we have an example here that is similar to $\dot{v_C}/v_C < k < 0$ above (Figure 1 and Proposition 2). Marginal gain utility falls slowly enough that the endowment effect dampens our preference to smooth consumption of the produced good.

[12]The initial value, which is also the reference value, is $3467. All these data are taken from the World Bank *World Development Indicators*.

[13]Technically the Ramsey rule will only be an approximation with a discrete time step; actually r^C std. $= 3.79\%$.

Figure 4: Material discount rates with and without the endowment effect

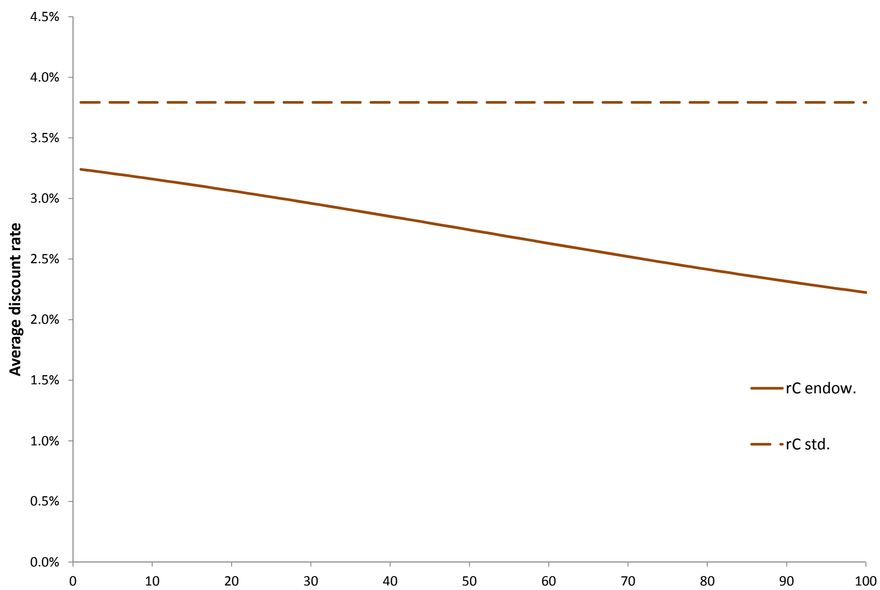
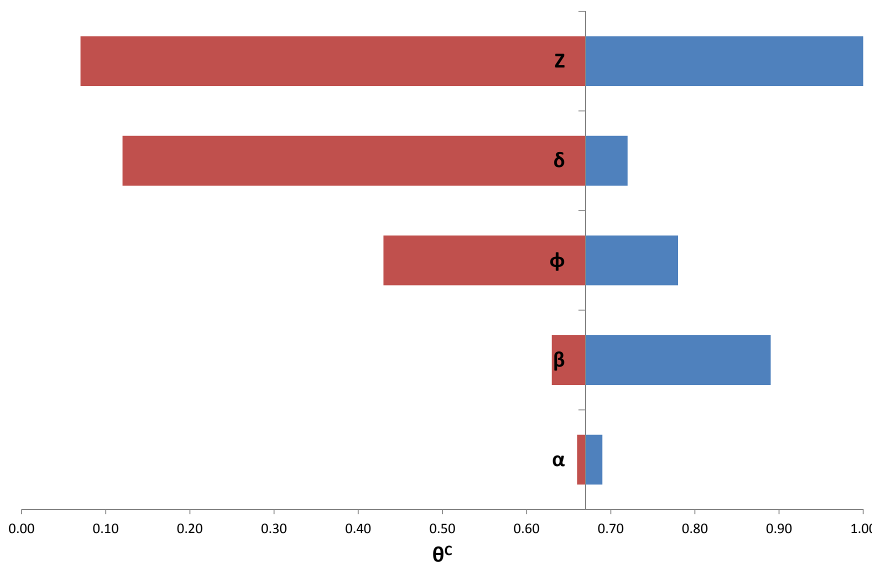


Figure 5: Sensitivity of θ^C to parameters at $t = 50$.



Sensitivity analysis Figure 5 analyses the sensitivity of θ^C to variation in the preference parameters on this consumption path at a maturity of 50 years, a typical horizon for a long-run environmental project. The gain-loss parameters α, β and Z are varied over their entire possible ranges, while $\delta \in [0.001, 0.02]$ and $\phi \in [0.5, 4]$. The value of θ^C that corresponds with the default parameter settings is 0.66 at $t = 50$. The material endowment factor is most sensitive to Z, the initial value share of consumption-level utility, followed by the pure rate of time preference δ. If $Z = 0$ so that preferences only depend on gains and losses, $\theta^C = 0.07$ at $t = 50$, and the discount rate is close to the pure rate of time preference.[14] Observe that $0 < \theta^C < 1$ for all values of ϕ that we investigated, despite the fact that ϕ bears upon $\dot{v_C}/v_C$, which we know to be important in the case of convex increasing consumption.

5.2 Two-good setting

Functional forms In the two-good setting, we specify a consumption-level utility function that exhibits both a constant elasticity of intertemporal substitution and a constant elasticity of substitution between the produced good and environmental quality (like Hoel

[14]Recall Z is the initial value share of consumption-level utility, so by $t = 50$ the contemporaneous value share of consumption-level utility has increased slightly.

and Sterner, 2007; Traeger, 2011):

$$v(C_t, E_t) = \frac{1}{1-\phi}\left[\gamma C_t^{1-1/\sigma} + (1-\gamma)\, E_t^{1-1/\sigma}\right]^{\frac{(1-\phi)\sigma}{\sigma-1}}, \tag{24}$$

where σ is the elasticity of substitution between the two goods. The value share of the produced good relative to environmental quality is determined by $\gamma \in [0,1]$, so the instantaneous utility function in the two-good setting is

$$\begin{aligned} U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) \quad = \quad & \tfrac{\zeta}{1-\phi}\left[\gamma C_t^{1-1/\sigma} + (1-\gamma)\, E_t^{1-1/\sigma}\right]^{\frac{(1-\phi)\sigma}{\sigma-1}} \\ & + (1-\zeta)\,\gamma\left[(C_t - \underline{C}_t + \psi)^{\beta} - \psi^{\beta}\right] \\ & - (1-\zeta)\,(1-\gamma)\,\lambda\left[(-E_t + \underline{E}_t + \psi)^{\beta} - \psi^{\beta}\right], \end{aligned} \tag{25}$$

where consumption of the produced good is increasing and environmental quality is falling. By normalising the initial level of environmental quality such that $E_{-20} = C_{-20}$, and the initial reference levels such that $\underline{E}_{-20} = \underline{C}_{-20} = C_{-20}$,

$$\gamma_{-20} \approx \frac{U_{C_{-20}} C_{-20}}{U_{C_{-20}} C_{-20} + U_{E_{-20}} E_{-20}}.$$

The default values of σ and γ can be found in Table 1.

Convex increasing consumption together with convex decreasing environmental quality Figure 6 plots the environmental discount rate that results when annual material consumption per capita grows at 1.5% per year, as above, and when environmental quality falls at 0.5% per year. Environmental quality is hence on a convex decreasing path, which was a conceptually ambiguous case in Section 4.

Without the endowment effect, the average environmental discount rate 'r^E std.' begins at -0.12% and nudges upwards to 0.06% in 100 years. If $U_t(E_t) = {}^{1}/_{1-\phi} E_t^{1-\phi}$, then according to the Ramsey rule the environmental discount rate would be $1.5 + 1.5 * -0.5 = 0.75\%$, so the effect of including consumption-level utility from the produced good, $-\eta^{EC}\dot{C}/C$, is to pull the discount rate on environmental quality significantly downwards. Increases in environmental quality are more valuable when the produced good is relatively abundant; the relative prices story. For this to be the case, it must be that $\eta^{EC} > 0$, which can be verified for our parameter scheme.[15]

When the endowment effect is present, the average environmental discount rate 'r^E endow.' is around 0.3% throughout, so the endowment effect does indeed increase the rate at which we would discount an environmental project, when environmental quality directly enters our utility function. The environmental endowment factor $0 < \theta^E < 1$, which implies the growth rate of marginal loss utility is less than the pure rate of time preference, similar to the case highlighted in Proposition 5.[16]

[15] In particular, given (24),

$$\eta^{EC} = \frac{(\gamma-1)(\phi+\rho-1)E^{\rho}}{[\gamma C^{\rho} + (1-\gamma)\, E^{\rho}]},$$

where $\rho = 1 - 1/\sigma$. Hence $\eta^{EC} > 0 \iff (\gamma-1)(\phi+\rho-1) > 0$.

[16] Indeed marginal gain utility $g^{i\prime}$ grows at only about 0.05% per year.

Figure 6: Discount rates with and without the endowment effect

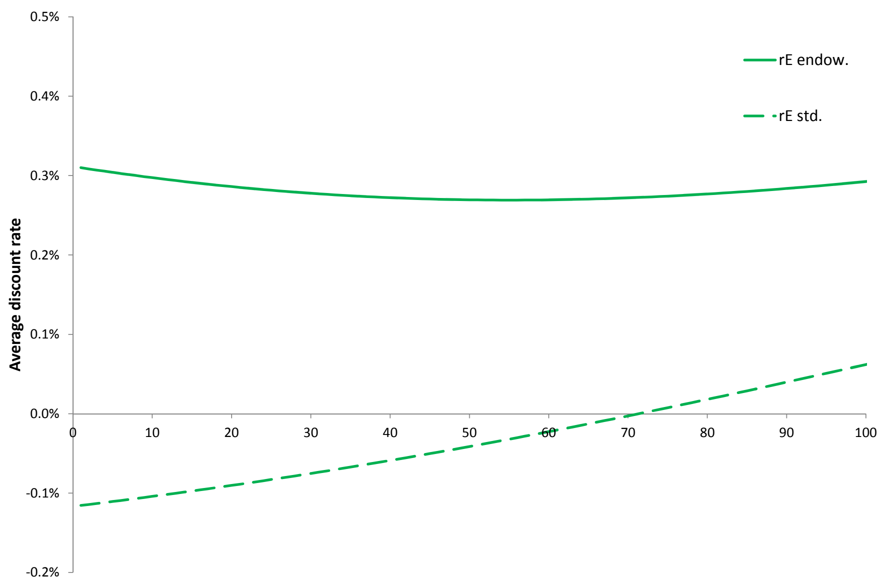
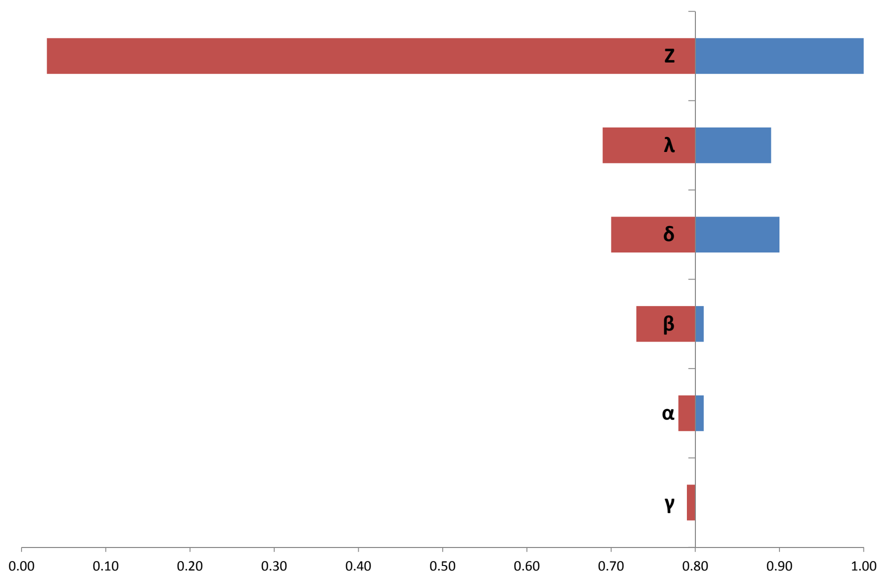


Figure 7: Sensitivity of θ^E to parameters at $t = 50$.



Sensitivity analysis Figure 7 analyses the sensitivity of θ^E to preference parameters at $t = 50$. The range of elasticity of substitution between the two goods is $\sigma \in [0.02, 100]$, running from approximately perfect substitutes to perfect complements, while the loss aversion parameter $\lambda \in [1, 5]$. Again the endowment factor is most sensitive to Z, followed by λ and then δ. Two important cases to point out are that when $Z = 0$, $\theta^E = 0.03$, while when $\delta = 0.001$, the smallest rate we consider, $\theta^E = 0.89$.

5.3 A non-monotonic path for environmental quality

Figure 8 focuses on the average environmental discount rate on an alternative path for environmental quality, whereby the initial growth rate is 0.5%, but the growth rate falls by 0.01 ppts. per year. This has the result that environmental quality grows for the first thirty years, and then falls. Therefore, for the first time, we move out of the framework of strictly increasing/decreasing consumption, and bring into play the discontinuity in the gain-loss function. As well as the default parameterisation of r^E endow., and as well as r^E std., we aid interpretation of the results by providing plots of r^E endow., in which loss aversion is omitted ($\lambda = 1$), and/or constant sensitivity is assumed ($\beta = 1$).

While r^E std. decreases over time, along with the average growth rate of environmental quality, default r^E endow. ($\lambda = 2.25$; $\beta = 0.9$) exhibits striking, non-monotonic and discontinuous behaviour. As $t \to 30$, it increases sharply to over 7%, before suddenly dropping to about -0.2%, and then increasing again to become close to r^E std. at the end of the time horizon. Under loss aversion, the instantaneous endowment effect is discontinuous on a non-monotonic consumption path. In this case, it will jump upwards when consumption growth turns negative. This in turn has an influence on the reference-level effect, which is of course the discounted and memory-adjusted sum of future instantaneous endowments. Moreover this influence will start to be seen prior to the turning point in consumption.

So, what is happening to r^E endow. ($\lambda = 2.25$; $\beta = 0.9$) in Figure 8 is that the reference-level effect starts increasing rapidly in size as $t \to 30$. Since the reference-level effect reduces overall marginal utility, the instantaneous discount rate increases, as does the average discount rate. But at exactly $t = 30$, the reference-level effect ceases its ascent, while the instantaneous endowment effect suddenly jumps. Since the instantaneous endowment effect increases overall marginal utility, this accounts for the sudden fall in the discount rate. Notice that in the absence of loss aversion ($\lambda = 1$), there is no jump in the discount rate. Under diminishing sensitivity but without loss aversion – in other words when the marginal gain-loss function is a smooth sigmoid – there is a trough in the discount rate around the turning point in consumption, just because marginal gain-loss utility becomes large when the change in consumption is small. But the effects of gains and losses are symmetrical. Moreover the discount rate only appears discontinuous due to the effect of the bounding parameter ψ. As one would expect, when neither loss aversion nor diminishing sensitivity is present ($\lambda = 1$; $\beta = 1$), so that the marginal gain-loss function is linear, the discount rate does not deviate from its declining path.

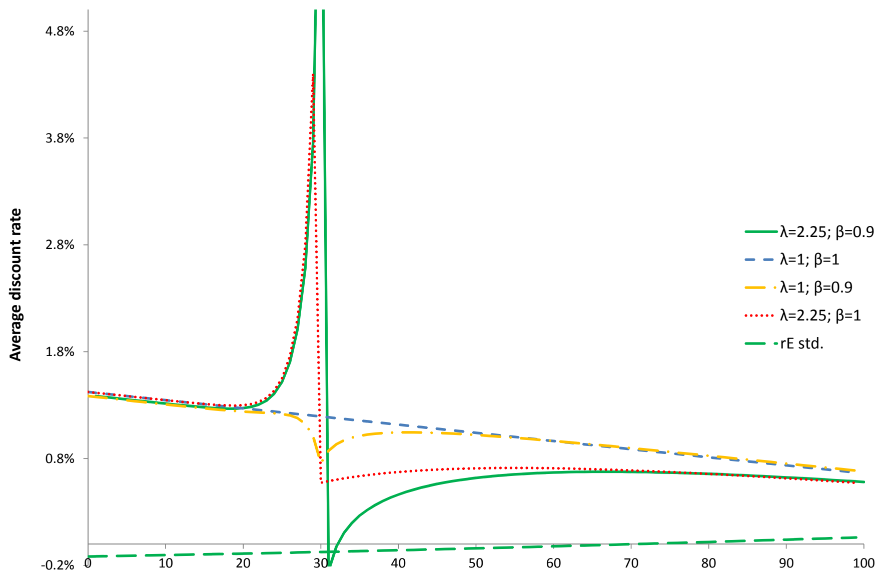
6 Discussion

Our analysis has shown that the endowment effect can make a substantial difference to the discount rate. In particular, we have shown that in many cases the endowment effect dampens our preference to smooth consumption over time, or indeed goes as far as reversing it. This is formalised in the idea that the endowment factor $\theta^i < 1$, $i \in \{C, E\}$. In summary, we have found that $\theta^i < 1$ in at least the following cases:

1. If gain/loss utility conforms to constant sensitivity (Proposition 1);
2. If consumption of good i is arithmetically increasing or decreasing (Proposition 1);
3. If consumption is convex increasing such that marginal gain utility is decreasing at a constant rate k, and k is greater than the rate of change of marginal consumption-level utility (Proposition 2 and Section 5.1);
4. If consumption is concave increasing or convex decreasing and $k < \delta$ (Propositions 3 and 5, and Section 5.2);
5. If consumption is concave decreasing (Proposition 4).

We have also shown that the implications of this result may differ fundamentally, depending on whether the environment is simply a factor of production of material goods, or has amenity value and directly enters the utility function. In the former case, it is appropriate to discount improvements in the future environment at the material discount

Figure 8: The environmental discount rate when E follows an inverse-U shaped path.



rate. Assuming material consumption is increasing, the preference to smooth consumption between dates contributes positively to the discount rate. That $\theta^i < 1$ makes us more patient if this is so. In the latter case, it is appropriate to discount improvements in the future environment at the environmental discount rate. Assuming environmental quality is falling, the preference to smooth consumption between dates contributes negatively to the discount rate. That $\theta^i < 1$ makes us *less* patient.

Especially the latter implication is perhaps surprising. One might have thought that the endowment effect would increase the value placed on an investment on a path where environmental quality is being lost. But it must be remembered that the exercise here is not to value the path itself, rather the discounting literature engages with the valuation of a marginal investment along a path. What matters is that, on a strictly decreasing path, environmental quality is being lost not only in the future, it is being lost today. If the marginal utility of losses today weighs more heavily on our welfare than the marginal utility of losses tomorrow, the endowment effect makes us less willing to postpone consumption to the future. It must also be borne in mind that, of the two elements of the endowment effect – reference dependence and loss aversion – only the former is at work on a strictly increasing/decreasing path. Hence we become accustomed – habituated – to lower environmental quality, such that future losses decrease our utility less.

Where loss aversion comes to the foreground is in valuing marginal investments along non-monotonic consumption paths. Section 5.3 illustrated that the endowment effect on the discount rate can be very large along such paths. This is because loss aversion introduces a discontinuity or kink in the gain/loss utility function when the change in consumption $x = 0$. On a non-monotonic path, which itself can be smooth as in our example, there will be a point in time when growth hits zero on its way from positive to negative territory, and *vice versa*. Around this point, the instantaneous endowment effect jumps, and the reference-level effect changes rapidly in advance of the jump. The chief implication is that valuation of environmental investments, which incur net benefits in the region of a turning point in consumption growth, is likely to be substantially modified by the endowment effect. It is clear, however, that the effect on valuations is context-specific.

Moving beyond a summary of our results to broader issues, there is naturally the question of whether the endowment effect ought to be considered in evaluating public environmental investments in the first place. There are at least two dimensions to this. First, there is the question of how strong the evidence behind the endowment effect is. Second, there is the question of whether preferences that represent the endowment effect should be afforded normative status, insofar as they are included in public/social decision-making.

On the first question, there is indeed much empirical evidence that demonstrates the endowment effect both in laboratory and field settings (e.g. Camerer and Loewenstein, 2004; DellaVigna, 2009). This is not to deny the existence of dissenting evidence. Most famously, List (2003) showed that experienced traders of a good do not exhibit the endowment effect with respect to that good, a result that is consistent either with those traders not being loss averse, or with those traders forming different reference points to inexperienced traders (DellaVigna, 2009). However, the preferences of people who trade baseball cards at least half a dozen times a month (i.e. an experienced trader) seem a poor analogy for those preferences of interest here, which are over future levels of overall material consumption and over environmental quality, something that is general not traded in markets.

On the second question, a simple application of the doctrine of consumer sovereignty would have it that, if the endowment effect characterises people's preferences, then the preferences of a social planner should include it too. However, objections can be raised to this position. It might be argued that the endowment effect is irrational even from the point of view of individual consumer choice. Since the requirements of preferences are usually axioms or primitives, the yardstick of rationality is difficult to establish. Nonetheless one can find comparable objections to affording normative status to related phenomena, such as hyperbolic discounting (e.g. Hepburn et al., 2010) and ambiguity aversion (Al-Najjar and Weinstein, 2009; Gilboa et al., 2009). With hyperbolic discounting, the concern is that preferences are time-inconsistent and therefore explain patterns of behaviour, such as addiction and procrastination, which are fairly obviously not in the best interests of those who hold these preferences. However, it is important to highlight that models of habit formation such as ours do not lead to time-inconsistency, even though the utility function is not time-separable (Végh, 2013). A different objection might be based on the ethical implications of our results. In the case of the environmental discount rate and falling environmental quality, $\theta^i < 1$ implies that we should be less inclined to improve the environment for future generations, because they are accustomed to poor environmental quality. This might appear immoral. We feel that a proposed resolution to this debate is clearly beyond the scope of the present paper. At the very least, our results indicate how consumers who exhibit the endowment effect really do value future consumption. And if the endowment effect is judged not to be a legitimate feature of the social planner's preference, then there is a wedge between private and social discount rates that may require policy intervention.

Lastly, there are at least three extensions to the present work, which are worthwhile considering. First, Appendix 2 points the way towards an analysis of optimal control of pollution under the endowment effect. This will not be simple, however, given the large number of state variables implied by having reference levels and more than one good. Second, our results assume perfect foresight, a natural consequence of minimally extending standard preferences. In fact, this is likely to have important implications for our results, because the strength of the reference-level effect rests on our anticipating the effect on future gain/loss utility of increments in consumption today. But what if we don't fully anticipate this effect, i.e. what if we succumb to projection bias? This would be worth looking into. Third, we have only examined the endowment effect in a riskless choice setting, in the tradition of Tversky and Kahneman (1991), even though reference dependence and loss aversion were first invoked to explain risky choices (Kahneman and Tversky, 1979). Therefore we could allow consumption of the two goods to follow a stochastic process. Again, this will not be wholly trivial, because the state space of future consumption levels could span the kink in marginal gain/loss utility that is implied by loss aversion. Under such circumstances, not only will there be familiar-looking results about the expectation of marginal gain/loss utility that derive from application of Jensen's inequality, there will also be a 'kink effect', so to speak.

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Appendix 1. Derivation of the discount factor and rate

A Fréchet or functional derivative describes the change in the welfare functional J with respect to a change in the consumption *path*. Following Karp and Traeger (2009), the functional derivative $\widehat{J(C)}$ with respect to a perturbation in the consumption path $\tilde{C}$ is

$$\begin{aligned} J\left(\widehat{C;\tilde{C}}\right) &= \frac{d}{d\epsilon} J\left(C_t + \epsilon \tilde{C}_t\right)\Big|_{\epsilon=0}, \\ &= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t} U\left[C_t + \epsilon\tilde{C}_t, \underline{C}\left(C_t + \epsilon\tilde{C}_t\right)\right] dt\Big|_{\epsilon=0}. \end{aligned}$$

Given utility function (2) and Eq. (3) describing the formation of the reference level,

$$\begin{aligned} J\left(\widehat{C;\tilde{C}}\right) &= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t}\left[v(C_t) + g\left(C_t - \underline{C}_t\right) + \left(v'(C_t) + g_C\left(C_t - \underline{C}_t\right)\right)\epsilon\tilde{C}_t\right. \\ &\quad \left. + \, g_{\underline{C}}\left(C_t - \underline{C}_t\right)\frac{d}{d\epsilon}\left(\underline{C}_t\left(C_t + \epsilon\tilde{C}_t\right)\epsilon\right)\right] dt\Big|_{\epsilon=0}, \\ &= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t}\left[\left(v'(C_t) + g_C\left(C_t - \underline{C}_t\right)\right)\epsilon\tilde{C}_t\ |_{\epsilon=0}\right. \\ &\quad + \int_0^\infty e^{-\delta t}\, g_{\underline{C}}\left(C_t - \underline{C}_t\right)\frac{d}{d\epsilon}\left.\left(\alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}\left(C_\tau + \epsilon\tilde{C}_\tau\right)d\tau\right)\right] dt\Big|_{\epsilon=0}. \end{aligned}$$

In view of the fact that $\frac{d}{d\epsilon}\left(\alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}C_\tau d\tau\right)\Big|_{\epsilon=0} = 0$,

$$\begin{aligned} J\left(\widehat{C;\tilde{C}}\right) &= \int_0^\infty e^{-\delta t}\left[\left(v'(C_t) + g_C\left(C_t - \underline{C}_t\right)\right)\tilde{C}_t\right. \\ &\quad \left. + \, g_{\underline{C}}\left(C_t - \underline{C}_t\right)\left(\alpha\int_{-\infty}^\infty 1_{\tau\le t}e^{-\alpha(t-\tau)}\tilde{C}_\tau d\tau\right)\right] dt. \end{aligned}$$

Since $g_{\underline{C}}\left(C_t - \underline{C}_t\right)$ is independent of τ,

$$\begin{aligned} J\left(\widehat{C;\tilde{C}}\right) &= \int_{-\infty}^\infty \left[1_{t>0}e^{-\delta t}\left(v'(C_t) + g_C\left(C_t - \underline{C}_t\right)\right)\tilde{C}_t\right] dt \\ &\quad + \left[\alpha\int_{-\infty}^\infty\int_0^\infty e^{-\delta t}g_{\underline{C}}\left(C_t - \underline{C}_t\right)1_{\tau\le t}e^{-\alpha(t-\tau)}\tilde{C}_t dt d\tau\right], \\ &= \int_{-\infty}^\infty \left[1_{t>0}e^{-\delta t}\left(v'(C_t) + g_C\left(C_t - \underline{C}_t\right)\right)\right. \\ &\quad \left. + \alpha\int_0^\infty e^{-\delta\tau}g_{\underline{C}}\left(C_t - \underline{C}_t\right)1_{\tau\ge t}e^{\alpha(t-\tau)}d\tau\right]\tilde{C}_t dt. \end{aligned} \tag{26}$$

Being a linear operator, the Fréchet derivative can also be written as the inner product of the consumption perturbation $\tilde{C}$ and a density function $J_C\left(C,t\right)$, which is defined by the relationship

$$J\left(\widehat{C;\tilde{C}}\right) = \int_{-\infty}^\infty J_C\left(C,t\right)\tilde{C}dt. \tag{27}$$

The density function $J_C(C,t)$ is also known as the Volterra derivative. While the Fréchet derivative is a functional that takes in two time paths as its arguments (i.e. C and $\tilde{C}$), the Volterra derivative has the time path C and date t as its arguments. The value of the Volterra derivative at t can also be understood as the welfare effect of a marginal increase in the consumption path at t. Therefore it can be written as $J_C(C,t) = J(\widehat{C;\Delta_t})$, where the delta distribution Δ_t is defined by $\int_{-\infty}^{\infty} \tilde{C}_\tau \Delta_\tau d\tau = \tilde{C}_t \forall \tilde{C} \in C^\infty$, i.e. a functional that concentrates full weight on time t.

Combining (26) with (27) and considering only consumption perturbations after $t = 0$,

$$\begin{aligned} J_C(C,t) &= e^{-\delta t}\left[v'(C_t) + g_C(C_t - \underline{C}_t)\right] + \alpha \int_0^\infty e^{-\delta\tau} g_{\underline{C}}(C_t - \underline{C}_t) 1_{\tau \geq t} e^{\alpha(t-\tau)} d\tau, \\ &= e^{-\delta t}\left[v'(C_t) + g_C(C_t - \underline{C}_t) + \alpha \int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g_{\underline{C}}(C_t - \underline{C}_t)\, d\tau\right]. \end{aligned}$$

Since $g_C = -g_{\underline{C}}$,

$$J_C(C,t) = e^{-\delta t}\left[v'(C_t) + g'(C_t - \underline{C}_t) - \alpha \int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g'(C_t - \underline{C}_t)\, d\tau\right]. \tag{28}$$

The discount factor for transferring a unit consumption from time 0 to t is

$$D^C \equiv \frac{J_C(C,t)}{J_C(C,0)}. \tag{29}$$

Substituting (28) into (29) gives the expression for the discount factor in the main body of the paper, Eq. (4).

The corresponding discount rate at a given point in time is defined as

$$\begin{aligned} r^C &\equiv -\frac{d}{dt}\ln D^C(t,0), \\ &= \frac{d}{dt}\delta t - \frac{d}{dt}\ln\left[v'(C_t) + g'(C_t - \underline{C}_t) - \alpha \int_{\tau=t}^\infty e^{-(\alpha+\delta)(\tau-t)} g'(C_\tau - \underline{C}_\tau)\, d\tau\right] \quad (30) \\ &\quad + \frac{d}{dt}\left[v'(C_0) + g'(C_0 - \underline{C}_0) - \alpha \int_{\tau=0}^\infty e^{-(\alpha+\delta)(\tau-0)} g'(C_\tau - \underline{C}_\tau)\, d\tau\right]. \quad (31) \end{aligned}$$

The third term is independent of t. Using the chain rule to take the derivative of the second term we find that

$$r^C = \delta - \frac{\dot{v}' + \dot{g}' - \alpha\dot{\mu}^{\underline{C}}}{v' + g' - \alpha\mu^{\underline{C}}}.$$

Finally, by again applying the chain rule we find that

$$\begin{aligned} \alpha\dot{\mu}^{\underline{C}} &= \frac{d}{dt}\alpha e^{(\alpha+\delta)t}\int_{\tau=t}^\infty e^{-(\alpha+\delta)\tau} g'(C_\tau - \underline{C}_\tau)\, d\tau \\ &= \alpha(\alpha+\delta)e^{(\alpha+\delta)t}\int_{\tau=t}^\infty e^{-(\alpha+\delta)\tau} g'(C_\tau - \underline{C}_\tau)\, d\tau + \alpha e^{(\alpha+\delta)t} e^{-(\alpha+\delta)t} g'(C_t - \underline{C}_t), \end{aligned}$$

so that we obtain Eq. (5), i.e.

$$r^C = \delta - \frac{v''\dot{C} + g''(\dot{C} - \alpha C + \alpha \underline{C}) + \alpha g' - \alpha(\alpha+\delta)\int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}{v' + g' - \alpha \int_t^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}.$$

Appendix 2. Derivation of the discount rate from an optimal control problem

The purpose of this Appendix is to link the analysis of discount rates in arbitrary economies, which is the focus of the main body of our paper, with discount rates in optimal economies. Suppose environmental quality is inversely related to the flow of pollution, $S = -E$. Following Brock (1973), we write production of the material good as a positive function of the flow of pollution. The production function is

$$Y = F(K, S),$$

where K is capital. We assume that $F_K > 0$ and $F_{KK} < 0$. For a given capital stock, production is also an increasing and strictly concave function of the pollution intensity of the capital stock, i.e. $F_S > 0$ and $F_{SS} < 0$. Production is either consumed or re-invested, so capital is accumulated according to

$$\dot{K} = F(K, S) - C.$$

Population and the production technology are assumed to be constant for simplicity, and for the same reason we omit capital depreciation.

The single-good setting

The single-good planning problem corresponding with this setting is

$$\max_{\{C,S\}} J = \int_0^{\infty} e^{-\delta t} \left[v(C_t) + g(C_t - \underline{C}_t) \right] dt \tag{32}$$

$$\text{s.t.} \qquad \dot{K} = F(K, S) - C, \tag{33}$$

$$\dot{\underline{C}} = \alpha (C - \underline{C}), \tag{34}$$

and initial K and $\underline{C}$. The current value Hamiltonian is defined as

$$\mathcal{H} = v(C) + g(C - \underline{C}) + \mu^K [F(K, S) - C] + \check{\mu}^{\underline{C}} [\alpha (C - \underline{C})].$$

Notice that the costate variable on reference consumption of the material good $\check{\mu}^{\underline{C}} = -\mu^{\underline{C}}$ in Eq. (6), as mentioned in Section 2.

Necessary conditions for a maximum include

$$\mu^K = v' + g' + \check{\mu}^{\underline{C}} \alpha, \tag{35}$$

$$\frac{\dot{\mu}^K}{\mu^K} = \delta - F_K, \tag{36}$$

$$\frac{\dot{\check{\mu}}^{\underline{C}}}{\check{\mu}^{\underline{C}}} = \delta + \alpha - \frac{g'}{\check{\mu}^{\underline{C}}}. \tag{37}$$

Combining these leads to an extended version of the standard Euler equation, which shows that in an optimal economy the material discount rate in Eq. (7) must be equal to the marginal product of capital:

$$r^C = F_K = \delta - \frac{\dot{v'} + \dot{g'} + \alpha \mu^{\dot{\check{\underline{C}}}}}{v' + g' + \alpha \mu^{\check{\underline{C}}}} = \delta - \frac{\dot{v'} + \dot{g'} - \alpha \dot{\mu}^{\underline{C}}}{v' + g' - \alpha \mu^{\underline{C}}}.$$

The two-good setting

The two-good planning problem is

$$\max_{\{C,S\}} J = \int_0^{\infty} e^{-\delta t} \left[v\left(C_t, E_t\right) + g^C\left(C_t - \underline{C}_t\right) + g^E\left(E_t - \underline{E}_t\right) \right] dt$$

subject to (33), (34), $\dot{\underline{E}} = \alpha\left(E - \underline{E}\right)$ and initial K, E, $\underline{C}$ and $\underline{E}$. The current value Hamiltonian in this case is

$$\begin{aligned} \mathcal{H} \quad = \quad & v\left(C, E\right) + g^C\left(C - \underline{C}\right) + g^E\left(E - \underline{E}\right) + \\ & \mu^K\left[F(K,S) - C\right] + \mu^{\check{\underline{C}}}\left[\alpha\left(C - \underline{C}\right)\right] + \mu^{\check{\underline{E}}}\left[\alpha\left(E - \underline{E}\right)\right]. \end{aligned}$$

Necessary conditions for a maximum include (35)-(37),

$$\mu^K F_S = v_E + g^{E\prime} + \alpha \mu^{\check{\underline{E}}} \text{ and} \tag{38}$$

$$\frac{\mu^{\dot{\check{\underline{E}}}}}{\mu^{\check{\underline{E}}}} = \delta + \alpha - \frac{g^{E\prime}}{\mu^{\check{\underline{E}}}}. \tag{39}$$

Since we are dealing with a flow pollutant, the current-valued shadow price of environmental quality is just $-\mu^K F_S$. Therefore the environmental discount rate is

$$r^E = \delta - \frac{\dot{\mu^K F_S}}{\mu^K F_S}.$$

Combined with Eq. (38), this gives

$$r^E = \delta - \frac{\dot{v_E} + \dot{g^{E\prime}} + \alpha \mu^{\dot{\check{\underline{E}}}}}{v_E + g^{E\prime} + \alpha \mu^{\check{\underline{E}}}} = \delta - \frac{\dot{v_E} + \dot{g^{E\prime}} - \alpha \dot{\mu}^{\underline{E}}}{v_E + g^{E\prime} - \alpha \mu^{\underline{E}}}, \tag{40}$$

which is equivalent to (15) with (16).

In the case of a stock pollutant, where $\dot{E} = -S - \omega E$ and ω is the decay rate of the pollutant in the environment, stock pollution requires an additional costate equation,

$$\frac{\mu^{\dot{\check{\underline{E}}}}}{\mu^{\check{\underline{E}}}} = \delta + \omega - \frac{v_E + g^{E\prime} + \mu^{\check{\underline{E}}}\alpha}{\mu^E}, \tag{41}$$

and Eq. (38) becomes just

$$\mu^E \quad = \mu^K F_S. \tag{42}$$

The appropriate discount rate to trade off a marginal unit of stock pollution over time is therefore defined as

$$r^E = \delta - \frac{\dot{\mu}^E}{\mu^E}.$$

Combined with Eq. (42), this gives the following environmental discount rate:

$$r^E = -\omega + \frac{v_E + g^{E\prime} + \check{\underline{\mu}}^E \alpha}{\mu^E},$$

which differs from Eq. (40), because it includes the fact that adding a unit of pollution at a given date will affect the quality of the environment at future dates.

Appendix 3. Proof of Proposition 1

Proof. We begin by proving the endowment factor is given by (17). It does not matter whether this is done with respect to material consumption or environmental quality. In the case of diminishing sensitivity, but where the consumption path is linear decreasing, $E_t = E_{t_0} + \kappa(t - t_0),\ k < 0$ for any arbitrary date in the past $t_0 \in (-\infty, t]$ and we can write Eq. (11) as

$$\begin{aligned}
\underline{E}_t &= \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} E_\tau d\tau + \alpha \int_{t_0}^{t} e^{-\alpha(t-\tau)} [E_{t_0} + \kappa(\tau - t_0)] d\tau, \\
&= \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} E_\tau d\tau + \alpha e^{-\alpha t} \int_{t_0}^{t} e^{\alpha \tau} E_{t_0} d\tau + \alpha \kappa e^{-\alpha t} \int_{t_0}^{t} e^{\alpha \tau} (\tau - t_0) d\tau, \\
&= \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} E_\tau d\tau + E_{t_0}(1 - e^{-\alpha(t-t_0)}) + \kappa(t - t_0) - \frac{\kappa}{\alpha} + \frac{1}{\alpha} e^{-\alpha(t-t_0)}. \qquad (43)
\end{aligned}$$

Taking the limit as t_0 goes to minus infinity we obtain

$$\lim_{t_0 \to -\infty} \underline{E}_t = E_{t_0} + \kappa(t - t_0) - \frac{\kappa}{\alpha} = E_t - \frac{\kappa}{\alpha}.$$

Therefore $g^E\left(E_t - \underline{E}_t\right) = g^E\left(\frac{\kappa}{\alpha}\right)$, which is constant over time. If consumption follows a linear increasing path instead, t_0 is taken to be the time when consumption was zero, $E_{t_0} = 0$. This eliminates the first two terms in Eq. (43). Since we cannot take the limit as t_0 goes to minus infinity, we approximate the same result if t_0 is sufficiently far in the past: $\underline{E}_t \approx E_{t_0} + \kappa(t - t_0) - \frac{\kappa}{\alpha}$. In this case $g^E\left(E_t - \underline{E}_t\right) = g^E\left(\frac{\kappa}{\alpha}\right)$ too. Therefore on a linear path the instantaneous endowment effect $g^{E\prime}$ is constant, which also means that $\dot{g}^{E\prime} = 0$. This is self-evidently true if preferences obey constant sensitivity, as long as consumption is strictly increasing or decreasing, in other words the increase/decrease need not be linear. Either way, since $g^{E\prime}$ is constant over time, the reference-level effect

$$\alpha \int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g^{E\prime} d\tau = \alpha g^{E\prime} \left[\frac{e^{-(\alpha+\delta)(\tau-t)}}{-\alpha-\delta} \right]_t^\infty = \frac{\alpha}{\alpha+\delta} g^{E\prime}.$$

Substituting this result into (16) results in Eq. (17). From (17), it is clear that $\delta > 0$ is a necessary and sufficient condition for $0 < \theta^i < 1$. $\square$

Appendix 4. Interpreting k

Convex exponential paths

Suppose that consumption grows exponentially at rate h:

$$C_t = C_{t_0} e^{h(t-t_0)}. \quad (44)$$

When $h > 0$ consumption is convex increasing; when $h < 0$ it is convex decreasing. Setting the current time to t_0 and substituting Eq. (44) into the definition of the reference level yields

$$\underline{C}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} C_t e^{-h(t-\tau)} d\tau = \alpha C_t \int_{-\infty}^{t} e^{-(\alpha+h)(t-\tau)} d\tau. \quad (45)$$

Let us first consider $h > -\alpha$. Given that $\alpha \in [0, 1]$, this covers all convex increasing paths, and convex decreasing paths as long as the rate of decrease is not too large. If $h > -\alpha$, (45) simplifies to

$$\underline{C}_t = \frac{\alpha}{h+\alpha} C_t, \quad (46)$$

such that the current reference level is a fixed proportion of current consumption and it exhibits the same exponential growth as consumption:

$$C_t - \underline{C}_t = \frac{h}{h+\alpha} C_t = \frac{h}{h+\alpha} C_{t_0} e^{h(t-t_0)}$$
$$\Rightarrow h = \frac{\dot{C}_t - \dot{\underline{C}}_t}{C_t - \underline{C}_t}.$$

The elasticity of marginal gain-loss utility is

$$\eta_t^g \equiv -\frac{\partial g^{i\prime}/g^{i\prime}}{\partial x/x} = -\frac{g^{i\prime\prime}}{g^{i\prime}}(x),$$

where as before $x \equiv C_t - \underline{C}_t$. In Section 4 we used k to denote the rate of change of marginal gain-loss utility, therefore in general $k_t = \dot{g}^{i\prime}/g^{i\prime}$. This immediately implies that

$$k_t = -\eta_t^g h,$$

in other words the rate of change of marginal gain-loss utility at time t is the product of the negative of the elasticity of marginal gain-loss utility and the consumption growth rate.

Section 4 specifically deals with paths that conform to constant k over the time period $[t, \infty)$. It turns out that if $h > 0$ and the gain-loss function $g^i(.)$ exhibits constant elasticity on the domain $[C_t - \underline{C}_t; \infty[$, then within the time interval $[t, \infty)$ we have the special case of

$$k = -\eta^g h.$$

This is also true if $\alpha < h < 0$, but only if $g^i(.)$ exhibits constant elasticity on the domain $[C_t - \underline{C}_t; 0]$, which would imply $\lim_{x \to 0} g^{i\prime}(x) = \infty$. In case this is felt to be undesirable, we might consider convex decreasing paths that are approximated by (44), but where $\dot{h} > 0$ for $t > \tau$. Another option is to assume that the reference-level effect is not infinitely forward-looking, rather it only extends to a finite date T arbitrarily far in the future.

What about if $h \leq -\alpha$? From Eq. (45) we can see that $\underline{C}_t = \infty$. Since we 'forget' our past consumption levels at a slower rate than consumption is falling, the reference level is determined by the infinite consumption we once enjoyed on this strictly decreasing path.

Concave exponential paths

Suppose instead that consumption evolves according to

$$C_t = \Upsilon - \Upsilon e^{h(t-T)}. \tag{47}$$

When $h > 0$, consumption is concave decreasing from a horizontal asymptote at $C_{-\infty} = \Upsilon$ to $C_T = 0$. On the whole path, $t - T < 0$. When $h < 0$, consumption is concave increasing from $C_T = 0$ and converges asymptotically to $C_\infty = \Upsilon$. On the whole path, $t - T > 0$.

Consider $h > 0$. Substituting (47) into the definition of the reference level, this time we obtain

$$\underline{C}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} \left[\Upsilon - \Upsilon e^{h(\tau-T)}\right] d\tau = \Upsilon - \frac{\alpha}{\alpha + h} \Upsilon e^{h(t-T)}, \tag{48}$$

which again means that $h = \left(\dot{C}_t - \dot{\underline{C}}_t\right) / \left(C_t - \underline{C}_t\right)$ and so, if the gain-loss function is isoelastic on the domain $]-\infty; C_t - \underline{C}_t]$, $k = -\eta^g h$.

Now consider $-\alpha < h < 0$. The path begins at $C_T = 0$, in which case

$$\underline{C}_t = \alpha \int_{T}^{t} e^{-\alpha(t-\tau)} \left[\Upsilon - \Upsilon e^{h(\tau-T)}\right] d\tau = \Upsilon \left[1 - e^{-\alpha(t-T)}\right] - \frac{\alpha}{\alpha + h} \Upsilon \left[e^{h(t-T)} - e^{-\alpha(t-T)}\right].$$

For $t >> T$ this solution converges to Eq. (48) (see also Appendix 3), so again $k = -\eta^g h$. However, this case requires that the gain-loss function is isoelastic on the domain $[0; C_t - \underline{C}_t]$, which encounters the same possible objection and proposed solutions as the case of convex decreasing paths above.

There is no tractable relation between k and g when $h \leq -\alpha$ and consumption evolves according to (47).

Appendix 5. Asymptotic behaviour of θ^i on convex decreasing paths

Equation (19) can also be written in the following way:

$$\theta^i = \frac{\alpha + \delta - k + \frac{g^{i\prime}}{v_i} k(\delta - k)}{\alpha + \delta - k + \frac{g^{i\prime}}{v_i}(\delta - k)}.$$

To understand the sign of the denominator, consider θ^i in the neighbourhood of the vertical asymptote at $k = \delta + \frac{\alpha}{1 + g^{i\prime}/v_i} + \epsilon$ with arbitrarily small ϵ. Substituting this value of k into the denominator gives

$$\alpha - \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}} - \epsilon - \frac{g^{i\prime}}{v_i}\left(\frac{\alpha}{1+\frac{g^{i\prime}}{v_i}} + \epsilon\right) = -\epsilon - \frac{g^{i\prime}}{v_i}\epsilon.$$

The denominator will therefore be positive to the left of the asymptote and negative to the right of it.

The numerator is positive in the neighbourhood of the asymptote at $k = \delta + \frac{\alpha}{1+g^{i\prime}/v_i}$ if

$$\alpha - \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}} + \frac{g^{i\prime}}{\dot{v}_i}\left(\delta + \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}}\right)\left(\frac{-\alpha}{1+\frac{g^{i\prime}}{v_i}}\right) > 0$$

$$\Leftrightarrow \frac{v_i}{\dot{v}_i}\left(\delta + \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}}\right) < 1.$$

Therefore the numerator is positive if $\frac{\dot{v}_i}{v_i} < 0$ or $\frac{\dot{v}_i}{v_i} > \delta + \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}}$. As a result, θ^i jumps from infinity to minus infinity as k increases beyond the asymptote. On the contrary, the numerator is negative if $0 < \frac{\dot{v}_i}{v_i} < \delta + \frac{\alpha}{1+\frac{g^{i\prime}}{v_i}}$, with the result that θ^i jumps from minus infinity to infinity as k passes the asymptote.