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Global Population Growth, Technology, and Malthusian Constraints: A Quantitative Growth Theoretic Perspective*

Bruno Lanz † Simon Dietz ‡ Tim Swanson§

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Abstract

How much further will the global population expand, can all these extra mouths be fed, and what is the role in this story of economic growth? We study the interactions between global population, technological progress, per-capita income, demand for food and agricultural land expansion from 1960 to 2100. We structurally estimate a two-sector Schumpeterian growth model with endogenous fertility and finite agricultural land reserves, in which a manufacturing sector provides a consumption good and an agricultural sector provides food to sustain contemporaneous population. The model closely replicates 1960-2010 data on world population, GDP, productivity growth and crop land area, and we employ the model to make projections from 2010 to 2100. Results suggests a slowdown of technological progress, and, because it is the main driver of a transition to a regime with low population growth, significant population growth over the whole century. Global population is slightly below 10 billion by 2050, further growing to 12 billion by 2100. As population and per capita income grow, demand for agricultural output almost doubles over the century, but the land constraint does not bind because of capital investment and technological progress. This provides a first integrative view of future population development in the context of modern growth theory.

Keywords: Economic growth; Fertility transition; Technological progress; Global population; Land conversion; Structural estimation

JEL Classification numbers: O11, O13, J11, C53, C61, Q15, Q24.

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1 Introduction

World population has doubled over the last fifty years and quadrupled over the past century (United Nations, 1999). During this period and in most parts of the world, productivity gains in agriculture have confounded Malthusian-style predictions that population growth would outstrip food supply. Population and income have determined the demand for food, and thus agricultural production, rather than food availability determining population. However, the amount of land that can be brought into the agricultural system is physically finite, so the concern naturally emerges that a much larger world population cannot be fed. Our aim in this paper is to study how population and the demand for land interacted with technological progress over the past fifty years, and derive some quantitative implications for the years to come.

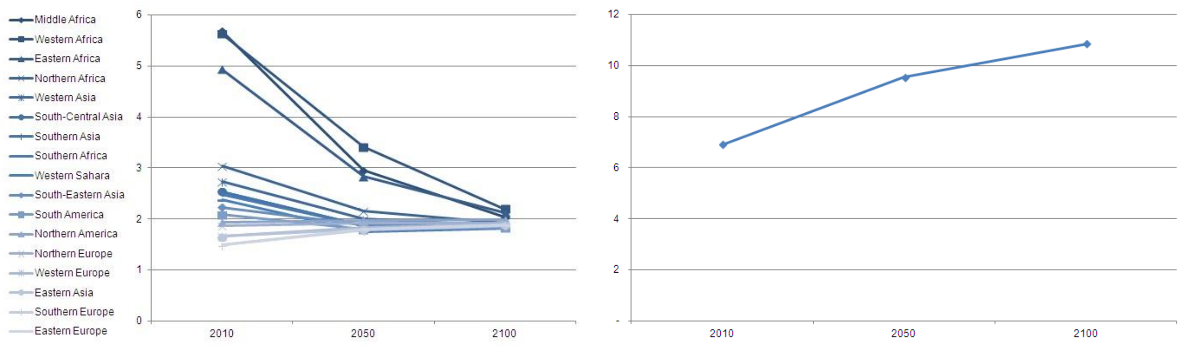
Despite the importance of understanding global population change and how fertility trends interact with per-capita income, food availability and the pace of technological progress, few economists have contributed to the debate about *future* population growth. This is especially surprising given the success of economic theories in explaining the demographic transition in developed countries, and in particular the role of technological progress (e.g. Galor and Weil, 2000; Jones, 2001; Bar and Leukhina, 2010; Jones and Schoonbroodt, 2010; Strulik et al., 2013, and other contributions reviewed below). Instead, the *de facto* standard source of demographic projections is the United Nations' series of *World Population Prospects*, updated every two years. The latest edition (United Nations, 2013) projects a global population, on a medium scenario, of 9.6 billion in 2050 and 10.9 billion in 2100, by which time the population growth rate is close to zero. The crucial assumption of the medium scenario, displayed in Figure 1, is that all countries around the world converge towards a replacement fertility rate of 2.1 over the next 100 years, irrespective of their starting point.[1]

The UN projections are highly sensitive to the assumed trajectory for fertility and small variations in the fertility trajectories for countries in Asia and Africa in particular account for most

[1] The UN uses a so-called 'cohort-component projection method', i.e. it works from the basic demographic identity that the number of people in a country at a particular moment in time is equal to the number of people at the last moment in time, plus the number of births, minus the number of deaths, plus net migration, all of this done for different age groups. This requires assumptions about fertility, mortality and international migration rates, which are exogenously determined.

Figure 1: United Nations population projections 2010 – 2100 (United Nations, 2013)

(a) Regional fertility (b) World population



of the variance in population projections.[2] These are precisely the regions for which uncertainty about the evolution of fertility is large, and empirical evidence in developing countries suggests no clear pattern of convergence towards a low fertility regime (Strulik and Vollmer, 2013). Interestingly, over the past ten years the bi-annual UN projections have been revised systematically upwards, with the 2008 projections of a steady state at around 9 billion still used in many policy discussions.

In essence the UN projections are based on a sophisticated extrapolation of the decline in fertility observed in developed countries. Because households' demand for education is closely linked to technology (Rosenzweig, 1990) and long run fertility development is associated with per-capita income (Herzer et al., 2012), it implicitly raises a question about future technological progress. Furthermore, there are concerns that an increase in the demand for food associated with sustained growth in population and income could not possibly be met (e.g. Godfray et al., 2010; Phalan et al., 2011; Tilman et al., 2011).

The purpose of this paper is to contribute a new, macro-economic approach to global population forecasting. We formulate a model of endogenous growth with an explicit behavioral representation linking child-rearing decisions to technology, per-capita income and availability of food, making the path for fertility an outcome rather than an assumption. More specifically, households in the model have preferences over own consumption, the number of children they

[2] Using the UN's cohort-component method, imposing the 'high' fertility scenario in these regions alone, so that they converge to a fertility rate of 2.6 rather than 2.1, implies a global population of around 16 billion by 2100.

have and the utility of their children, in the tradition of Barro and Becker (1989). Child-rearing is time intensive, and fertility competes with other labor-market activities. In order to capture the well-documented complementarity between human capital and the level of technology (Goldin and Katz, 1998), we include a positive relationship between the cost of fertility and technological progress. Thus technological progress implies a higher human capital requirement, so that population increments need more education and are thus more costly. As in Galor and Weil (2000), the opportunity cost of fertility increases over time, implying a gradual transition to low fertility and a decline in population growth.

Besides the time required for child-rearing and education, the other key constraint to population growth in our model is food availability. We make agricultural output a necessary condition to sustain population, and assume that food production requirements increase with both the size of the population and per-capita income, the latter capturing changes in diet as affluence rises (e.g. Subramanian and Deaton, 1996). An agricultural sector, which meets the demand for food, requires land as an input, and agricultural land has to be converted from a stock of natural land. Therefore, as population and income grow, the demand for food increases, raising the demand for agricultural land. In the model land is an treated as a scarce form of capital, which has to be converted from a finite resource stock of natural land, and substitution possibilities in agriculture are limited (Wilde, 2013). The cost of land conversion and the fact that it is physically finite generate a potential Malthusian constraint to long run population development.

Technology plays a central role for both fertility and land conversion decisions. On the one hand, technological progress raises the opportunity and human capital cost of children. On the other hand, whether land conversion acts as a constraint to population growth mainly depends on technological progress. We model the process of knowledge accumulation in the Schumpeterian framework of Aghion and Howitt (1992), where the growth rate of total factor productivity (TFP) increases with labor hired for R&D activities. A well known drawback of such a representation of technological progress is the population scale effect (see Jones, 1995a).[3] Following Chu et al. (2013), we 'neutralize' the scale effect by making the growth rate of TFP

[3] The population scale effect implies that productivity growth is proportional to population growth, which contradicts empirical evidence as reported by Jones (1995b) and Laincz and Peretto (2006). This is particularly important in a setting with endogenous population, as it would imply that population would be a fundamental driver of long run technology and income growth.

a function of the *share* of labor allocated to R&D. This implies that long run growth can occur without the need for the population to grow.[4]

To fix ideas, we start with a simple illustration of the theoretical mechanism underlying fertility and land conversion decisions in our model. However, the main contribution of our work is to structurally estimate the model and use it to make quantitative projections. More specifically, most of the parameters of the model are either imposed or calibrated from external sources, but those determining the marginal cost of population, labor productivity in R&D and labor productivity in agricultural land conversion are structurally estimated with simulation methods. We use 1960-2010 data on world population, GDP, TFP growth and crop land area to define a minimum distance estimator, which compares observed trajectories with those simulated by the model. Trajectories simulated with the estimated vector of parameters closely replicate observed data for 1960 to 2010, and we then employ the estimated model to make projections until 2100.

The key results are as follows. Our quantitative results suggest a population of 9.85 billion by 2050, further growing to 12 billion by 2100. These numbers are above the UN's current central projection (United Nations, 2013), and they lie on the upper limit of the 95 percent confidence interval implied by the probabilistic projections reported in Lutz and Samir (2010).[5] Although population *growth* declines over time, population does not reach a steady state over the period we consider. Indeed the pace of technological progress, which, given our assumptions, is the main driver of the demographic transition, declines over time, so that population growth remains positive over the horizon we consider. Despite what this implies for food demand, however, agricultural land expansion stops by 2050 at around 1.8 billion hectares, a 10 percent increase on 2010, which is about the same magnitude as projections by the Food and Agriculture

[4] As we further discuss below, Chu et al. (2013) show that the qualitative behavior of our Schumpeterian representation of R&D is in line with more recent representations of technological progress, put forward by Dinopoulos and Thompson (1998), Peretto (1998), and Young (1998) among others and thus provides a good basis to study growth in contemporary history.

[5] Probabilistic projections by Lutz and Samir (2010) use the same cohort-component method, but apply probabilities to the different fertility scenarios at the country level. Being based on the assumption that all countries converge to replacement fertility, population at the median stops growing by 2050 and remains around 9 billion. Probabilistic projections using the UN 2012 revision, however, suggest that there's a 95 percent chance that 2100 population will lie in between 9 and 13 billion (Gerland et al., 2014).

Organization (Alexandratos and Bruinsma, 2012).[6] A direct implication of our work is that the land constraint does not bind, even though (i) our population projections are higher than conventional wisdom; and (ii) our projections are rather conservative in terms of technological progress (agricultural TFP growth in both sectors is below one percent per year and declining from 2010 onwards).

As a corollary to its ambitions, our work necessarily relies upon a number of simplifications. Among these, perhaps the most important is that of a representative agent, which misses out on the age-structure and regional heterogeneity of population around the globe.[7] Another important simplification is implied by the fact that we solve for the social planner representation of the problem. While this allows to make a number of simplifications in the formulation of the problem and to exploit efficient solvers for constrained non-linear optimization, it abstracts from R&D externalities that would arise in a decentralized equilibrium (see Romer, 1994, for example). To some extent, however, both population heterogeneity and market imperfections prevailing over the estimation period will be reflected in the parameters that we estimate to rationalize observed trajectories, and hence be factored into our projections. Nevertheless, we advise against too literal an interpretation of our work, which should principally be regarded as an attempt to see population projections through the lens of endogenous growth theory, and thus to complement existing approaches.

1.1 Related literature

Population projections are mostly made by demographers working for governmental agencies, and work by economists in this area is scarce. However, this paper relates to at least three stands of research on economic growth. First, there is unified growth theory, which studies economic development and population over the long run. Seminal contributions include Galor and Weil (2000) and Jones (2001) (see Galor, 2005, for a survey). Jones (2003) and Strulik

[6] This corresponds to the conversion of a further 150 million hectares of natural land into agriculture, roughly the area of Mongolia or three times that of Spain. Because developed countries will likely experience a decline in agricultural land area (Alexandratos and Bruinsma, 2012), land conversion in developing countries will need to be more than that.

[7] See Mierau and Turnovsky (2014) for a more realistic treatment of age-structured population in a general equilibrium growth model. To keep the model tractable however, they have to treat the demographic structure as exogenous. Integrating a richer representation of population heterogeneity into a model with endogenous fertility decisions remains an important research topic.

(2005) analyze the joint development of population, technological progress and human capital (see also Tournemaine and Luangaram, 2012, for a recent investigation and comprehensive overview of the literature), while Hansen and Prescott (2002) and Strulik and Weisdorf (2008) consider the role of agriculture and manufacturing activities along the development path. The structure of our model, linking technology and economic growth with fertility decisions and human capital, is hence related to these papers, although an important aspect of our work is the potential constraint to growth represented by food and land availability.

In unified growth theory models, the initial phase of economic development relies on the scale effect to generate take off. However, a number of empirical studies using evidence from modern growth regimes refute the scale effect (e.g. Jones, 1995b; Laincz and Peretto, 2006), and Strulik et al. (2013) show how the transition between the two growth regimes can be achieved endogenously based on the accumulation of human capital. More recent growth theories circumvent the scale effect with 'product line' representations of technological progress (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for seminal contributions).[8] These recent models have been used to develop theories of endogenous population and resource constraints, most notably Peretto and Valente (2011) and Bretschger (2013), and these theoretical contributions are thus close in spirit to our work. Relative to these papers, we treat land as a scarce form of capital that is required to produce food, and may ultimately limit human development over time.

A final set of papers has in common with us the use of a quantitative macroeconomic model to study particular aspects of unified growth theory, especially economic development and the demographic transition. These include Mateos-Planas (2002), Doepke (2005), Strulik and Weisdorf (2008; 2014), Bar and Leukhina (2010), Jones and Schoonbroodt (2010), and Ashraf et al. (2013). These papers demonstrate that macroeconomic growth models are able to capture essential features of the demographic transition in countries where such a transition has already taken place. Our contribution is to show that models like these can not only closely replicate

[8] In a product-line representation of technological progress, the number of products grows over time, thereby diluting R&D inputs, so that long-run growth doesn't necessarily rely on the population growth rate, but rather on the share of labor in the R&D sector. Another strategy to address the scale effect involves postulating a negative relationship between labor productivity in R&D and the existing level of technology, giving rise to "semi-endogenous" growth models (Jones, 1995a). In this setup, however, long-run growth is only driven by population growth, which is also at odds with empirical evidence (Ha and Howitt, 2007).

recent history, they can also provide an alternative to existing methods of projecting future population and land use.

The remainder of the paper proceeds with a simple analytical model capturing the key features of our analysis (Section 2). The structure of our quantitative model and estimation strategy are presented in Section 3. Section 4 reports projections with the model. Sensitivity analysis is provided in Section 5, and we discuss some broader implication of our results in Section 6. Some concluding comments are provided in Section 7.

2 Simple analytics of household fertility, technology and land

In order to provide some intuition for the mechanisms driving the demographic transition in our quantitative model, this section studies the fertility decisions made by our representative household in a simplified set-up. In particular, we simplify the representation of technological progress so that technology in our two sectors changes exogenously, and we also omit capital. As we will show, this distills the problem into one of allocating labor between several competing uses. Population and land are the remaining state variables. Even with all this simplification, we still have a problem that is too complex to yield analytical solutions for the whole development path, but we can nonetheless obtain useful results relating to optimal fertility (and agricultural land expansion) between any two successive time-periods.

We assume that a representative household has preferences over its own consumption of a homogeneous, aggregate manufactured good c_t, the number of children it produces n_t, and the utility that each of its children experiences in the future $U_{i,t+1}$. We use the class of preference suggested by Barro and Becker (1989), which is defined recursively as

$$U_t = u(c_t) + \beta b(n_t) \sum_{i=0}^{n_t} U_{i,t+1} \tag{1}$$

where $u(.)$ is the per-period utility function and we assume that $u' > 0$, $u'' < 0$, and that $u(.)$ also satisfies the Inada conditions such that $\lim_{c\to 0} u' = \infty$ and $\lim_{c\to\infty} u' = 0$. The function $b(.)$ specifies preferences for fertility. We further assume that children are identical and, for ease of analytical exposition, just for now we assume that they survive only one period, so $N_{t+1} = n_t$. $\beta \in (0, 1)$ is the discount factor.

The recursive nature of Barro-Becker preferences allows us to the define the utility function of the dynastic household head as (Alvarez, 1999):[9]

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \tag{2}$$

This formulation is useful, because the household's planning horizon coincides with that of a social planner. In our quantitative model, we choose a constant-elasticity-of-substitution (CES) function for $u(c_t)$ and we set the elasticity consistent with empirical studies such that $U_t > 0$. In this case, a preference for fertility that is subject to diminishing returns, and in turn overall concavity of (2), requires that $b' > 0$ and $b'' < 0$ (Jones and Schoonbroodt, 2010). This also implies that fertility and the utility of children are complements in parents' utility (which is easiest to see in the context of (1), where our combination of assumptions implies that $\partial^2 U_t / \partial n_t \partial U_{t+1} > 0$). We further assume that $\lim_{N \to 0} b' = \infty$ and $\lim_{N \to \infty} b' = 0$.

Each agent is endowed with one unit of time in each period, which can be spent rearing and educating children, or working on a competitive market for manufacturing labor at wage w_t. Bringing up children hence competes with labor-market activities as it does in the standard model of household fertility choice (Becker, 1960; Barro and Becker, 1989). In addition, we characterize a complementarity between technology and skills (Goldin and Katz, 1998), by postulating an increasing relationship between the time-cost of rearing and educating children and the level of technology in the economy (specifically in manufacturing), where the latter is denoted $A_{t,mn}$.[10] Technological progress increases the returns to education, which increases the time needed to produce effective labor units. Formally, fertility is given by

$$n_t = \chi(L_{t,N}, A_{t,mn})$$

where $L_{t,N}$ is the absolute amount of labor time devoted by all agents to child-rearing and education. We assume that $\partial \chi(L_{t,N}, A_{t,mn})/\partial L_{t,N} > 0$, $\partial^2 \chi(L_{t,N}, A_{t,mn}) / \partial L^2_{t,N} < 0$, $\partial \chi(L_{t,N}, A_{t,mn})/\partial A_{t,mn} < 0$, $\partial^2 \chi(L_{t,N}, A_{t,mn}) / \partial A^2_{t,mn} > 0$ and $\partial^2 \chi(L_{t,N}, A_{t,mn}) / \partial L_{t,N} \partial A_{t,mn} < 0$.

[9] Because household preferences are defined recursively, the sequence of household decisions in period 0 will be the same as those in period *t*.

[10] In our quantitative model, the cost of children is proportional to an output-weighted average of TFP in manufacturing and agriculture, although the consequent weight on the former is much larger.

In our model there is an additional constraint bearing upon the household, which is that sufficient food must be available for it to eat at all times. The aggregate food requirement is the product of total population N_t and per-capita food requirements $\overline{f_t}$:[11]

$$\overline{f_t} N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \tag{3}$$

In this simple analytical model, food is directly produced by households by combining 'agricultural' labor $L_{t,ag}$ and land X_t, given agricultural TFP $A_{t,ag}$.[12] We assume strictly positive and diminishing returns to labor and land, and we assume the Inada conditions also hold on both.

There is a finite supply of land $\overline{X}$ that is in full, private ownership of households at all times. Non-agricultural land can be converted into agricultural land with the use of the household's labor $L_{t,X}$. The state equation for land is then

$$X_{t+1} = \psi(L_{t,X}), \quad X_t \leq \overline{X}$$

where $\psi' > 0$, $\psi'' < 0$ and the Inada conditions again hold.[13] Land that is prepared for agricultural use thus acts as a productive stock of capital that is physically finite.

Where $L_{t,mn}$ is the absolute amount of time spent by all agents working in the manufacturing sector, the household's budget constraint is $c_t N_t = w_t L_{t,mn}$. Combined with the food constraint (3) and the overall constraint on the household's allocation of labour $N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag}$, the dynastic head's optimization problem can be written as:

$$\begin{aligned}
\max_{\{L_{t,j}\}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \\
\text{s.t.} \quad & N_{t+1} = \chi(L_{t,N}, A_{t,mn}); \quad X_{t+1} = \psi(L_{t,X}); \quad X_t \leq \overline{X} \\
& c_t N_t = w_t L_{t,mn}; \quad N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag}; \quad \overline{f_t} N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \\
& N_0, X_0 \quad \text{given}
\end{aligned}$$

[11] An important simplification that will remain throughout is that food consumption does not enter the utility function of households, but is rather a complement to the consumption of other goods c_t. We return to this assumption below.

[12] We introduce agricultural firms in the quantitative analysis later.

[13] In this formulation agricultural land is "recolonized" by nature every period, i.e. the depreciation rate is 100 percent. This is obviously a simplification and we introduce a more realistic depreciation pattern in our quantitative analysis.

At the heart of the household's problem is therefore the allocation of labor between four competing uses $\{L_{t,j}\}$: (i) Supply of labor to the manufacturing sector, $L_{t,mn}$; (ii) Spending time rearing and educating children, $L_{t,N}$; (iii) Spending time producing food for own consumption, $L_{t,ag}$; and (iv) Spending time expanding the agricultural land area, $L_{t,X}$. Necessary and sufficient conditions for a maximum allow us to obtain the following useful result:

Lemma 1. *At the optimum, fertility and hence population growth are chosen to equate the marginal costs and benefits of increasing the population in the next period, specifically*

$$\underbrace{u'(c_t)b(N_t)w_t\left(\frac{\partial\chi(L_{t,N},A_{t,mn})}{\partial L_{t,N}}\right)^{-1}}_{A}+\underbrace{\beta u'(c_{t+1})b(N_{t+1})w_{t+1}f_{t+1}\left[A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}\right]^{-1}}_{B}$$
$$=\underbrace{\beta u(c_{t+1})\left[b'(N_{t+1})N_{t+1}+b(N_{t+1})\right]}_{C}+\underbrace{\beta u'(c_{t+1})b(N_{t+1})w_{t+1}}_{D} \tag{4}$$

Proof. See Appendix A. □

As Lemma 1 shows, the marginal costs of increasing the population in the next period are twofold. First, there is the opportunity cost of present consumption foregone (A), as time is spent rearing and educating children rather than working in the manufacturing sector. Second, there is the discounted opportunity cost of consumption foregone in the next period by having to provide additional food to sustain the extra mouths (B). On the other hand, the marginal benefits of increasing the population in the next period are also twofold: the discounted marginal utility of fertility (C), plus the discounted marginal utility of additional consumption, made possible by expanding the pool of labor that can work in manufacturing (D).

We can use this result to explore what happens when the level of technology in the economy is increased. This requires explicit characterization of the manufacturing sector. Identical, competitive manufacturing firms employ household labor and combine this with the exogenously given technology $A_{t,mn}$ to produce the composite good that households consume. Production of the representative firm is hence

$$Y_{t,mn} = A_{t,mn}\cdot Y_{mn}(L_{t,mn})$$

where $Y'_{mn} > 0$, $Y''_{mn} < 0$ and the Inada conditions hold.

Let the evolution of TFP in the manufacturing sector be described by $A_{t+1,mn} = (1 + g_{t,mn})A_{t,mn}$. Then the following proposition describes the resulting condition for an increase in the level of TFP in period t to reduce fertility.

Proposition 1. *An increase in the level of manufacturing TFP in period t results in a reduction in fertility and population growth if and only if*

$$\underbrace{u'(c_t)b(N_t)Y'_{mn}\left[\partial\chi(L_{t,N},A_{t,mn})/\partial L_{t,N} - A_{t,mn}\frac{\partial^2\chi(L_{t,N},A_{t,mn})}{\partial L_{t,N}\partial A_{t,mn}}\right]}_{A'} + \underbrace{\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})f_{t+1}\left[\frac{1}{A_{t+1,ag}}\frac{\partial Y_{ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}\right]^{-1}}_{B'} > \underbrace{\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})}_{D'}$$

Proof. See Appendix A. □

An increase in $A_{t,mn}$ increases the opportunity cost of present consumption foregone (A'), because an effective labor unit is more time-consuming to rear and educate, while it also increases the discounted opportunity cost of providing additional food in the next period (B'). On the other hand, an increase in $A_{t,mn}$ increases consumption in the next period (D'). In general, whether an increase in the level of manufacturing TFP reduces fertility thus depends on the positive effect on the marginal costs of fertility (A' + B') being larger than the positive effect on the marginal benefits of fertility (D').

Proposition 1 gives us a feel for the incentives at work at the household level in driving a decline in population growth linked to technological progress. Equally, however, it can be used to ask the question; what happens if the rate of technological progress itself slows down? This is an important question, because it is what we have observed in recent history (see Section 4). In this case, the opportunity costs at the margin of fertility will fall relative to a scenario of higher technological progress, as will the marginal benefits. Provided the former effect exceeds the latter, fertility will hold up and population growth will not slow down as much.

From Proposition 1 we can also extract a sufficient condition for population growth to slow in the face of technological progress that is linked to the food requirement.

Corollary 1. *A sufficient condition for an increase in the level of manufacturing TFP in period t to reduce fertility and population growth is that it is not too cheap to meet food requirements, specifically*

$$\frac{\overline{f}_{t+1}}{A_{t+1,ag}\frac{\partial Y_{t+1,ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}} > 1$$

Proof. Given our assumptions,

$$u'(c_t)b(N_t)Y'_{mn}\left[\partial\chi(L_{t,N},A_{t,mn})/\partial L_{t,N} - A_{t,mn}\left(\partial^2\chi(L_{t,N},A_{t,mn})\right)/\partial L_{t,N}\partial A_{t,mn}\right] > 0$$

The Corollary follows immediately. □

Corollary 1 is more likely to hold the larger is the per-capita food requirement $\overline{f}_{t+1}$ and the less productive is agricultural labor, $A_{t+1,ag}\ \partial Y_{t+1,ag}(L_{t+1,ag};X_{t+1})/\partial L_{t+1,ag}$.

We turn now to the link between population growth and technological progress in agriculture. Over the period 1961-2005, agricultural productivity as measured by output per unit area – agricultural yield – increased by a factor of 2.4, or nearly 2% per year (Alston et al., 2009). The following proposition establishes that an increase in the level of agricultural TFP unambigously increases fertility in this model, by relaxing the food contraint and therefore one of the marginal costs of fertility.

Proposition 2. *An increase in the level of agricultural TFP results in an increase in fertility and population growth.*

Proof. Let the evolution of TFP in the agricultural sector be described by $A_{t+1,ag} = (1+g_{t,ag})A_{t,ag}$. The partial derivative of (4) with respect to $A_{t,ag}$ is

$$-\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,A,mn})\overline{f}_{t+1}\left[(1+g_{t,ag})(A_{t,ag})^2\frac{\partial Y_{ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}\right]^{-1} < 0$$

□

On the other hand, as Alston et al. (2009) further point out, agricultural productivity grew at a slower pace from 1990 to 2005 (1.82% per year) than it did from 1961 to 1990 (2.03% per year), so we can view Proposition 2 from the opposite angle as supplying intuition for how a sustained slowdown in the pace of technological improvements in agriculture might start to put a brake on population growth.

The Malthusian constraint on the expansion of agricultural land also has the potential to affect population growth. In the extreme case where the constraint binds, there are no improvements to agricultural TFP and labor and land are perfect complements in food production, no further increase in the population can take place. More generally, the extent to which the population can grow despite the constraint binding depends on technological improvements in agriculture and the substitutability of labor and land.

It is in fact useful to briefly inspect the optimal dynamics of agricultural land:

Remark 1*.* Optimal expansion of agricultural land in period t, under the assumption that the land constraint does not bind, requires that

$$u'(c_t)b(N_t)w_t \left[\psi'(L_{t,X})\right]^{-1} = \beta u'(c_{t+1})b(N_{t+1})w_{t+1}f_{t+1} \frac{\partial Y_{ag}(L_{t,+1ag},X_{t+1})}{\partial X_{t+1}} \left(\frac{\partial Y_{ag}(L_{t+1,ag},X_{t+1})}{\partial L_{t+1,ag}}\right)^{-1} \tag{5}$$

The term on the left-hand side is the marginal cost of land conversion, in terms of present consumption foregone by diverting labor away from manufacturing. In the case where the land constraint binds, so that $X_t = \overline{X}$, the shadow price of the constraint will appear as a cost in the form of a scarcity rent. The term on the right-hand side is the discounted marginal benefit of land conversion – expanding agricultural land relaxes the food constraint and, since food and manufacturing goods are perfect complements in consumption in our model, this enables additional consumption. Notice that the marginal benefit of land conversion is higher, the higher is the marginal productivity of land in agriculture relative to the marginal productivity of labor in the same sector.

One important implication of (5) is associated with the fact that labor used to invest in the stock of agricultural land is subject to decreasing returns. Therefore as the agricultural land area expands, the land input becomes relatively more expensive. In our simulation this will be the main driver of a slow-down in land conversion. Investing in the stock of land becomes relatively less attractive and hence land as a factor of production becomes relatively less important over time.

3 Quantitative model

In our quantitative model, we add capital to the set of factor inputs to manufacturing and agriculture described above. Therefore the problem facing the representative dynastic household is now to maximize discounted utility by allocating not only labor but also capital across sectors, as well as by selecting the savings/investment rate. The second major addition is that sectoral technological progress is determined by the allocation of labor to R&D activities, in the framework of Aghion and Howitt (1992). This implies that the demographic transition will occur endogenously.

This section first describes the structure of the quantitative model. We then explain how we take the model to the data in order to draw quantitative implications about future demographic evolution.

3.1 The economy

3.1.1 Production

In agriculture and manufacturing aggregate output is represented by a constant-returns-to-scale production function with endogenous, Hicks-neutral technological change.[14] In manufacturing, aggregate output in period t is given by a standard Cobb-Douglas production function:

$$Y_{t,mn} = A_{t,mn} K_{t,mn}^{\vartheta} L_{t,mn}^{1-\vartheta}, \tag{6}$$

where $K_{t,mn}$ is capital allocated to manufacturing and $\vartheta \in (0,1)$ is a share parameter. Conditional on technical change being Hicks-neutral, the assumption that output is Cobb-Douglas is consistent with long-term empirical evidence (Antràs, 2004).

In agriculture, we posit a two-stage constant-elasticity-of-substitution (CES) functional form (e.g. Kawagoe et al., 1986; Ashraf et al., 2008):

$$Y_{t,ag} = A_{t,ag} \left[(1-\theta_X) \left(K_{t,ag}^{\theta_K} L_{t,ag}^{1-\theta_K} \right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

[14] Assuming technological change is Hicks-neutral, so that improvements to production efficiency do not affect the relative marginal productivity of input factors, considerably simplifies the analysis at the cost of abstracting from a number of interesting issues related to the direction or bias of technical change (see Acemoglu, 2002).

where $\theta_{X,K} \in (0, 1)$, and σ is the elasticity of substitution between a capital-labor composite factor and agricultural land. This specification provides flexibility in specifying how capital and labor can be substituted for land, and it nests the Cobb-Douglas specification as a special case ($\sigma = 1$). While a Cobb-Douglas function is often used to characterize aggregate agricultural output (e.g. Mundlak, 2000; Hansen and Prescott, 2002), it is quite optimistic in that, in the limit, land is not required for agricultural production, and long-run empirical evidence reported in Wilde (2013) indeed suggests that $\sigma < 1$.

3.1.2 Innovations and technological progress

The evolution of sectoral TFP is based on a discrete-time version of the Schumpeterian model by Aghion and Howitt (1992). In this framework innovations are drastic, so that a firm holding the patent for the most productive technology temporarily dominates the industry until the arrival of the next innovation. The step size of productivity improvements associated with an innovation is denoted $s > 0$, and we assume that it is the same in both sectors.[15] Without loss of generality, we assume that there can be at most $I > 0$ innovations over the length of a time period, so that the maximum growth rate of TFP each period is $S = (1 + s)^I$. For each sector $j \in \{mn, ag\}$, the growth rate of TFP is then determined by the number of innovations arriving within each time-period, and this rate can be specified in relation to maximum feasible TFP growth S:[16]

$$A_{t+1,j} = A_{t,j} \cdot (1 + \rho_{t,j} S) \ , \quad j \in \{mn, ag\} \, . \tag{8}$$

where $\rho_{t,j}$ is the arrival *rate* of innovations each period, in other words how many innovations are achieved compared to the maximum number of innovations.

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[15] In general, the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure a right over the proposed innovation. For the purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not a marginal change), which is usually represented as a minimum one-time shift.

[16] The arrival of innovations is a stochastic process, and we implicitly make use of the law of large numbers to integrate out the random nature of growth over discrete time-intervals. Our representation is qualitatively equivalent, but somewhat simpler, to the continuous time version of the model where the arrival rate of innovations is described by a Poisson process.

Innovations in each sector are a function of labor hired for R&D activities:

$$\rho_{t,j} = \bar{\lambda}_{t,j} \cdot L_{t,A_j}, \quad j \in \{mn, ag\},$$

where L_{t,A_j} is labor employed in R&D for sector j and $\bar{\lambda}_{t,j}$ measures labor productivity. As mentioned above, the standard Aghion and Howitt (1992) framework implies the scale effect and is at odds with empirical evidence on modern growth. Instead we work with the scale-invariant formulation proposed by Chu et al. (2013), where $\bar{\lambda}_{t,j}$ is specified as a decreasing function of the scale of the economy. In particular, we define

$$\bar{\lambda}_{t,j} = \lambda_j L_{t,A_j}^{\mu_j - 1} / N_t^{\mu_j}$$

where $\lambda_j > 0$ is a productivity parameter and $\mu_j \in (0, 1)$ is an elasticity. Including population N_t in the denominator, so that innovation depends on the share of labor allocated to R&D, neutralizes the scale effect and is in line with more recent representations of technological change (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for example). In particular, using the share of employment in R&D can be seen as a proxy for average employment allocated to a growing number of product varieties (see Laincz and Peretto, 2006). Furthermore, our representation of R&D implies decreasing returns to labor in R&D through the parameter μ_j, which captures the duplication of ideas among researchers (Jones and Williams, 2000).

3.1.3 Land

As in the simple analytical model above, land used for agriculture has to be converted from a finite stock of reserve land $\overline{X}$. Converting land from the available stock requires labor, therefore there is a cost in bringing new land into the agricultural system. Once converted, agricultural land gradually depreciates back to the stock of natural land with a linear depreciation rule. Thus the allocation of labor to convert land determines the amount of land available for agriculture each period, and over time the stock of land used in agriculture develops according to:

$$X_{t+1} = X_t(1 - \delta_X) + \psi \cdot L_{t,X}^{\varepsilon}, \quad X_0 \text{ given}, \quad X_t \leq \overline{X}, \tag{9}$$

where $\psi > 0$ measures labor productivity in land clearing activities, $\varepsilon \in (0, 1)$ is an elasticity, and the depreciation rate δ_X measures how fast converted land reverts back to natural land.

One important aspect of equation (9) is the decreasing returns to labor in land-clearing activities, which imply that the marginal cost of land clearing increases with the amount of land already converted. More specifically, as the amount of land used in agriculture increases, labor requirements to avoid it depreciating back to its natural state increase more than proportionally. Intuitively, this captures the fact that the most productive agricultural plots are converted first, whereas marginal land still available at a later stage of land conversion is less productive. Labor can be used to bring these marginal plots into agricultural production, although the cost of such endeavors increases as the total land area is exhausted.

3.1.4 Preferences and population dynamics

We now further specialize the households preferences described in Section 2. We again use the dynastic representation that is associated with Barro and Becker (1989) preferences, so that the size of the dynasty coincides with the total population N_t. We use the standard constant elasticity function $u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}$ where $1/\gamma$ is the intertemporal elasticity of substitution, and specify $b(n_t) = n_t^{-\eta}$, where η is an elasticity determining how the utility of parents changes with n_t. The utility of the dynasty head in terms of aggregate quantities is then:

$$U_0 = \sum_{t=0}^{\infty} \beta^t N_t^{1-\eta} \frac{c_t^{1-\gamma} - 1}{1-\gamma}, \tag{10}$$

Parametric restrictions ensuring overall concavity of the objective and in turn existence and uniqueness of the solution are easy to impose. For $\gamma > 1$, which is consistent with empirical evidence on the intertemporal elasticity of substitution, concavity of Equation (10) in (c_t, N_t) requires $\eta \in (0, 1)$. This implies that, depending on η, preferences of the dynastic head correspond with both classical and average utilitarian objectives, in terms of social planning, as limiting cases.[17]

Aggregate consumption $C_t = c_t N_t$ is derived from the manufacturing sector. Given a social-

[17] See Baudin (2010) for a discussion of the relationship between dynastic preferences and different classes of social welfare functions.

planner representation, manufacturing output can either be consumed by households or invested into a stock of capital:

$$Y_{t,mn} = C_t + I_t , \tag{11}$$

The accumulation of capital is then given by:

$$K_{t+1} = K_t(1 - \delta_K) + I_t , \quad K_0 \text{ given} , \tag{12}$$

where δ_K is a per-period depreciation rate. In this formulation investment decisions mirror those of a one-sector economy (see Ngai and Pissarides, 2007, for a similar treatment of savings in a multi-sector growth model).

In each period, fertility n_t determines the change in population together with mortality d_t:

$$N_{t+1} = N_t + n_t - d_t , \quad N_0 \text{ given} . \tag{13}$$

We make the simplifying assumption that population equals the total labor force, so that n_t and d_t represent an increment and decrement to the stock of effective labor units, respectively. The mortality rate is assumed to be constant, so that $d_t = N_t\delta_N$, where $1/\delta_N$ captures the expected working lifetime.

As described above, the cost of fertility consists of time spent both rearing and educating children in order to produce effective labor. We again exploit the social-planner representation, which allows us to treat these as a single activity:

$$n_t = \bar{\chi}_t \cdot L_{t,N} ,$$

where $\bar{\chi}_t$ is an inverse measure of the time cost of producing effective labor units. Treating child-rearing and education as an activity implies that there is an opportunity cost, which is increasing in the level of technology through the following function:

$$\bar{\chi}_t = \chi L_{t,N}^{\zeta-1} / A_t^{\omega}$$

where $\chi > 0$ is a productivity parameter, $\zeta \in (0, 1)$ is an elasticity representing scarce factors required in child-rearing and education,[18] A_t is an economy-wide index of technology and $\omega > 0$ measures how the cost of children increases with the level of technology.

Population dynamics are further constrained by food availability, as measured by agricultural output. As in our analytical model, we have the following constraint:

$$Y_t^{ag} = N_t \overline{f}_t$$

Per-capita demand for food determines the quantity of food required for maintaining an individual in a given society, and captures both physiological requirements (e.g. minimum per-capita caloric intake) and the positive relationship between food demand and per-capita income, reflecting changing diet. The relationship between food expenditures and per-capita income is not linear, however, so we specify food demand as a concave function of per-capita income: $\overline{f}_t = \xi \cdot \left(\frac{Y_{t,mn}}{N_t}\right)^{\kappa}$, where ξ is a scale parameter and $\kappa > 0$ is the income elasticity of food consumption. Therefore, while food consumption does not directly enter the utility function of households, food availability will affect social welfare through its impact on population dynamics.[19]

3.2 Optimal control problem and empirical strategy

We consider a social planner choosing paths for C_t, $K_{t,j}$ and $L_{t,j}$ by maximizing the utility of the dynastic head (10) subject to technological constraints (6), (7), (8), (9), (11), (12), (13) and market-clearing conditions for capital and labor:

$$K_t = K_{t,mn} + K_{t,ag}, \quad N_t = L_{t,mn} + L_{t,ag} + L_{t,A_{mn}} + L_{t,A_{ag}} + L_{t,N} + L_{t,X}.$$

[18] More specifically, ζ captures the fact that the costs of child-rearing over a period of time may increase more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p.412, Moav, 2005, and Bretschger, 2013).

[19] Formally, household food consumption is effectively proportional (and thus a perfect complement) to the consumption of other goods, and hence could be modeled with standard Leontiev preferences. However, preferences for food are non-homothetic, and this would significantly complicate the specification and calibration of the utility function. One drawback of making per-capita food demand proportional to income is that, in a decentralized setting, we are effectively creating an externality. However, since we solve for the central-planner problem, the demand for food and associated production costs are fully internalized by the planner.

Aggregate consumption is provided by allocating capital and labor to the manufacturing sector, as well as labor to manufacturing R&D. Increases in the population require time to be spent rearing and educating children. In addition, sufficient food must be provided at all times to feed the population, by allocating capital, labor and land to agriculture, as well as labor to agricultural R&D. Insofar as increasing agricultural production requires greater inputs of land, labor must also be allocated to converting reserves of natural land into agricultural land.

Since consumption grows over time and since fertility and the welfare of children are complements in parents' utility, the main driver of any slowdown in fertility will be the cost of fertility itself and how it evolves over time. Building on Section 2, we can identify several components to this evolution. First, there are diminishing returns to labor in the production of children, implying that the marginal cost of fertility with respect to labor is an increasing and convex function. This is the usual assumption for the cost of education (Moav, 2005), and can also represent a form of congestion (see Bretschger, 2013). Second, technological progress increases human-capital requirements and in turn lowers the marginal productivity of labor in the production of children, because more time is required for their education. Third, as the economy develops the marginal productivity of labor in rearing and educating children changes relative to the marginal productivity of labor in other activities. This implies among other things that technological progress, which will raise labor productivity in the two production sectors, will tend to increase the opportunity cost of labor in child-rearing and education. Fourth, a cost of fertility is meeting food requirements, and the demand for food increases with per-capita income (at a decreasing rate). Thus growth in population and per-capita income are associated with an increasing demand for agricultural output. This can be achieved either through technological progress, or by allocating primary factors, i.e. labor, capital and land, to agriculture. However, agricultural land is ultimately fixed, either because it is constrained by physical availability or because its conversion cost increases with the area already converted. Thus over time the cost of agricultural output will increase, adding a further break to population growth.

3.2.1 Numerical solution concept

The optimization problem is an infinite horizon optimal control problem, and we use mathematical programming techniques to solve for optimal trajectories. In particular, we employ a solver for constrained non-linear optimization problems, which directly mimics the welfare maximization program: the algorithm searches for a local maximum of the concave objective function (the discounted sum of utility), starting from a candidate solution and improving the objective subject to maintaining feasibility as defined by the technological constraints.[20]

A potential shortcoming of direct optimization methods, as compared to dynamic programming for example, is that they cannot explicitly accommodate an infinite horizon.[21] However, as long as $\beta < 1$ only a finite number of terms matter for the solution, and we consider a finite-horizon problem truncated to the first T periods. The truncation may induce differences between the solution to the infinite-horizon problem and its finite-horizon counterpart, because the shadow values of the stock variables are optimally zero in the terminal period T, whereas they will be so only asymptotically if the planning horizon is infinite. Since we are interested in trajectories over the period from 1960 to 2100, we check that the solution over the first $T^l = 200$ periods are not affected by the choice of T, finding that $T = 300$ is sufficient to make computed trajectories over the first $T^l = 200$ periods independent of T. Similarly, re-initializing the model in $T^l = 200$ and solving the problem onwards, we remain on the same optimal path with a precision of 0.1 percent for all the variables in the model. Given the truncation over 300 periods and appropriate scaling of variables, the model solves in a matter of seconds.

3.2.2 Empirical strategy

Having defined the numerical optimization problem, our empirical strategy proceeds in three steps. First, a number of parameters are determined exogenously. Second, we calibrate some of the parameters to match observed quantities, mainly to initialize the model based on 1960 data. Third, we estimate the remaining parameters with simulation methods. These are the crucial parameters determining the cost of fertility (χ, ζ, ω), technological progress (μ_{mn}, μ_{ag}) and land conversion (ψ, ε). We now discuss each step in turn. The full set of parameters of the model is listed in Table 1.

[20] The program is implemented in GAMS and solved with KNITRO (Byrd et al., 1999, 2006), which alternates between interior-point and active-set methods.

[21] By definition, the objective function is a sum with an infinite number of terms, and the set of constraints includes an infinite number of elements, which is incompatible with finite computer memory.

Table 1: List of parameters of the model and associated numerical values

Imposed parameters		
ϑ	Share of capital in manufacturing	0.3
θ_K	Share of capital in capital-labor composite for agriculture	0.3
θ_X	Share of land in agriculture	0.25
σ	Elasticity of substitution between land and the capital-labor composite	0.6
δ_K	Yearly rate of capital depreciation	0.1
S	Maximum increase in TFP each year	0.05
$\lambda_{mn,ag}$	Labor productivity parameter in R&D	1
γ	Inverse of the intertemporal elasticity of substitution	2
η	Elasticity of altruism towards future members of the dynasty	0.001
κ	Income elasticity of food demand	0.25
β	Discount factor	0.99
Initial values for the stock variables and calibrated parameters		
N_0	Initial value for population	3.03
X_0	Initial the stock of converted land	1.35
$A_{0,mn}$	Initial value for TFP in manufacturing	4.7
$A_{0,ag}$	Initial value for TFP in agriculture	1.3
K_0	Initial value for capital stock	20.5
ξ	Food consumption for unitary income	0.4
δ_N	Exogenous mortality rate	0.022
δ_X	Rate of natural land reconversion	0.02
Estimated parameters (range of estimates for relaxed goodness-of-fit objective in parenthesis)		
χ	Labor productivity parameter in child-rearing	0.153 (0.146 – 0.154)
ζ	Elasticity of labor in child-rearing	0.427 (0.416 – 0.448)
ω	Elasticity of labor productivity in child-rearing w.r.t. technology	0.089 (0.082 – 0.106)
μ_{mn}	Elasticity of labor in manufacturing R&D	0.581 (0.509 – 0.585)
μ_{ag}	Elasticity of labor in agricultural R&D	0.537 (0.468 – 0.545)
ψ	Labor productivity in land conversion	0.079 (0.078 – 0.083)
ε	Elasticity of labor in land conversion	0.251 (0.238 – 0.262)

Exogenous parameters

Starting with production technology, we need to select values for the share parameters ϑ, θ_K and θ_X, and for the elasticity of substitution σ. In manufacturing, the Cobb-Douglas functional form implies that the output factor shares (or cost components of GDP) are constant over time, and we use a standard value of 0.3 for the share of capital (see for example Gollin, 2002). In agriculture, the CES functional form implies that the factor shares are not constant, and we choose θ_X to approximate a value for the share of land in global agricultural output of 0.25 in 1960. While there are no data on the global land factor share in 1960, it has been shown to be negatively correlated with income (Caselli and Feyrer, 2007), and 25 percent is in line with recent data for developing countries reported in Fuglie (2008). For the capital-labor composite, we follow Ashraf et al. (2008) and also use a standard value of 0.3 for the share of capital. Taken together, these estimates of the output value shares in agriculture are also in agreement with factor shares for developing countries reported in Hertel et al. (2012).[22]

As mentioned previously, the long-run elasticity of substitution between land and the capital-labor composite input is expected to be less than one, which is confirmed by empirical evidence reported in Wilde (2013). Using long-run data on land and other inputs in pre-industrial England, he finds robust evidence that $\sigma \sim 0.6$. While external validity of these estimates may be an issue (in particular for the currently developing countries with rapidly growing population), it reflects long-run substitution possibilities that are consistent with our CES functional form (7). While we consider $\sigma = 0.6$ to be the best estimate available, in the sensitivity analysis we derive trajectories assuming $\sigma = 0.2$ and $\sigma = 1$.

The yearly rate of capital depreciation δ_K is set to 0.1 (see Schündeln, 2013, for a survey and evidence for developing countries), and maximum TFP growth per year S is set to 5 percent. The latter number is consistent with evidence on yearly country-level TFP growth rates from Fuglie (2012), which do not exceed 3.5 percent. The labor productivity parameter in R&D λ_j is not separately identified from S, and we set it to 1 without affecting our results.

The next set of imposed parameters determines preferences over consumption and fertility. First, the income elasticity of food demand is 0.25, which is consistent with evidence across countries and over time reported in Subramanian and Deaton (1996), Beatty and LaFrance (2005), and Logan (2009). Second, the elasticity of intertemporal substitution is set to 0.5 in line with estimates from Guvenen (2006), which corresponds with $\gamma = 2$. Given the constraint on η to maintain concavity of the objective function, we initially set it to 0.01 so that the planner effectively has a classical utilitarian objective. Intuitively, this implies that parents' marginal utility of fertility is almost constant, or that altruism towards the welfare of children remains constant as the number of children increases. Correspondingly, we also assume a high degree of altruism by setting the discount factor to 0.99, which implies a pure rate of time preference of 1

[22] For 2007, the factor shares for the global agricultural sector reported in Hertel et al. (2012) are 0.15 for land, 0.47 for labor and 0.37 for capital. However, shares for developing countries are probably a better estimates of the value shares prevailing at the global level in 1960. It turns out that our results are not significantly affected by variations in the estimated value shares within a plausible range.

percent per year. This will tend to produce relatively high population projections, and we report sensitivity analysis for the case where altruism declines with n_t, in particular $\eta = 0.5$, and for a discount factor of 0.97.[23]

Initial values and external calibration

Starting values for the state variables are calibrated to observed quantities in 1960. Initial population N_0 is set to an estimate of the world population in 1960 of 3.03 billion (United Nations, 1999). Initial crop land area X_0 is set to 1.348 billion hectares (Goldewijk, 2001) and the total stock of natural land reserves that can be converted for agriculture is 3 billion hectares (see Alexandratos and Bruinsma, 2012). For the remaining state variables, sectoral TFP $A_{0,ag}, A_{0,mn}$ and the stock of capital K_0, there are no available estimates, and we target three moments. First, we use an estimate of world GDP in 1960 of 9.8 trillion 1990 international dollars (Maddison, 1995; Bolt and van Zanden, 2013). Second, we obtain an estimate of world agricultural production by assuming that the share of agriculture in total GDP in 1960 is 15% (see Echevarria, 1997). Third, we assume that the marginal product of capital in 1960 is 15 percent. While this may appear relatively high, it is not implausible for developing economies (see Caselli and Feyrer, 2007). Solving for the targeted moments as a system of three equations with three unknowns gives initial values of 4.7 and 1.3 for TFP in manufacturing and agriculture respectively, and a stock of capital of 20.5.

Three other parameters of the model are calibrated to observed quantities. First, the parameter measuring food consumption for unitary income (ξ) is calibrated such that the demand for food in 1960 represents about 15% of world GDP, which is consistent with the calibration targets for initial TFP and capital stock. This implies $\xi = 0.4$. Second, the mortality rate δ_N is calibrated by assuming an average adult working life of 45 years (United Nations, 2013), which implies $\delta_N = 0.022$. We vary that assumption in the sensitivity analysis, using $\delta_N = 0.015$ instead, in other words a 65 year working life. Finally we set the period of regeneration of natural land to 50 years so that $\delta_X = 0.02$.

Estimation of the remaining parameters

The seven remaining parameters $\{\mu^{mn,ag}, \chi, \zeta, \omega, \psi, \varepsilon\}$ are conceptually more difficult to tie

23 In fact, as we show below, the estimation error is significantly higher if we assume $\eta = 0.5$, and only slightly better for $\beta = 0.97$.

down using external sources, and we therefore estimate them using simulation-based structural methods. The moments we target are taken from observed trajectories over the period 1960 to 2010 for world GDP (Maddison, 1995; Bolt and van Zanden, 2013), world population (United Nations, 1999, 2013), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and sectoral TFP (Martin and Mitra, 2001; Fuglie, 2012).[24] For each time series, we target one data point for each five-year interval, denoted τ, yielding 11 data points for each targeted quantity (55 points in total).[25] The data are reported in Appendix B.

The targeted quantities in the model are respectively $Y_{t,mn} + Y_{t,ag}$, N_t, X_t, $A_{t,mn}$ and $A_{t,ag}$, and we formulate a minimum distance estimator as follows. For a given vector of parameters v, we solve the model and obtain the values for each targeted quantity, which we denote $Z^*_{v,k,\tau}$. We then compute the squared deviations between the solution of the model and observed data points $Z_{k,\tau}$, and sum these both over k and τ to obtain a measure of the estimation error over time and across targeted variables. Formally the error for a vector of parameters v is given by:

$$error_v = \sum_k \left[\sum_\tau (Z^*_{k,\tau} - Z_{k,\tau})^2 \Big/ \sum_\tau Z_{k,\tau} \right], \tag{14}$$

where the error for each variable is scaled to make these comparable. Our procedure is therefore essentially a non-linear least squares estimation procedure defined over several model outcomes. Importantly the error for each vector of parameters is computed for all targeted variables in one run of the model, so that all the parameters are jointly rather than sequentially estimated.

In order to select the vector of parameters that minimizes the goodness-of-fit objective (14), we simulate the model over the domain of plausible parameter values, starting with bounds of a uniform distribution, which is our initial 'prior' for the plausible values of the parameters. For elasticity parameters, these bounds are 0.1 and 0.9 and for the labor productivity parameters we use 0.03 and 0.3. We then solve the model for 10,000 randomly drawn vectors of parameters

[24] Data on TFP is derived from TFP growth estimates and are thus more uncertain than other trajectories. Nevertheless, a robust finding of the literature is that the growth rate of economy-wide TFP and agricultural TFP is on average around 1.5-2% per year. To remain conservative about the pace of future technological progress, we assume TFP growth was at 1.5 percent between 1960 and 1980, declined to 1.2 percent from 1980 to 2000, and was equal to 1 percent over the last decade of the estimation period.

[25] Considering five-year intervals smooths year-on-year variations and allows us to focus on the long-run trends in the data. Using yearly data would not change our results. Similarly, we use the level of TFP rather than its growth rate to mitigate the impact of discontinuities implied by the TFP growth rates.

and evaluate the error between the simulated trajectories and those observed. Having identified a narrower range of parameters for which the model approximates observed data relatively well, we reduce the range of values considered for each parameter and draw another 10,000 vectors to solve the model. This algorithm gradually converges to the estimates reported in Table 1.[26]

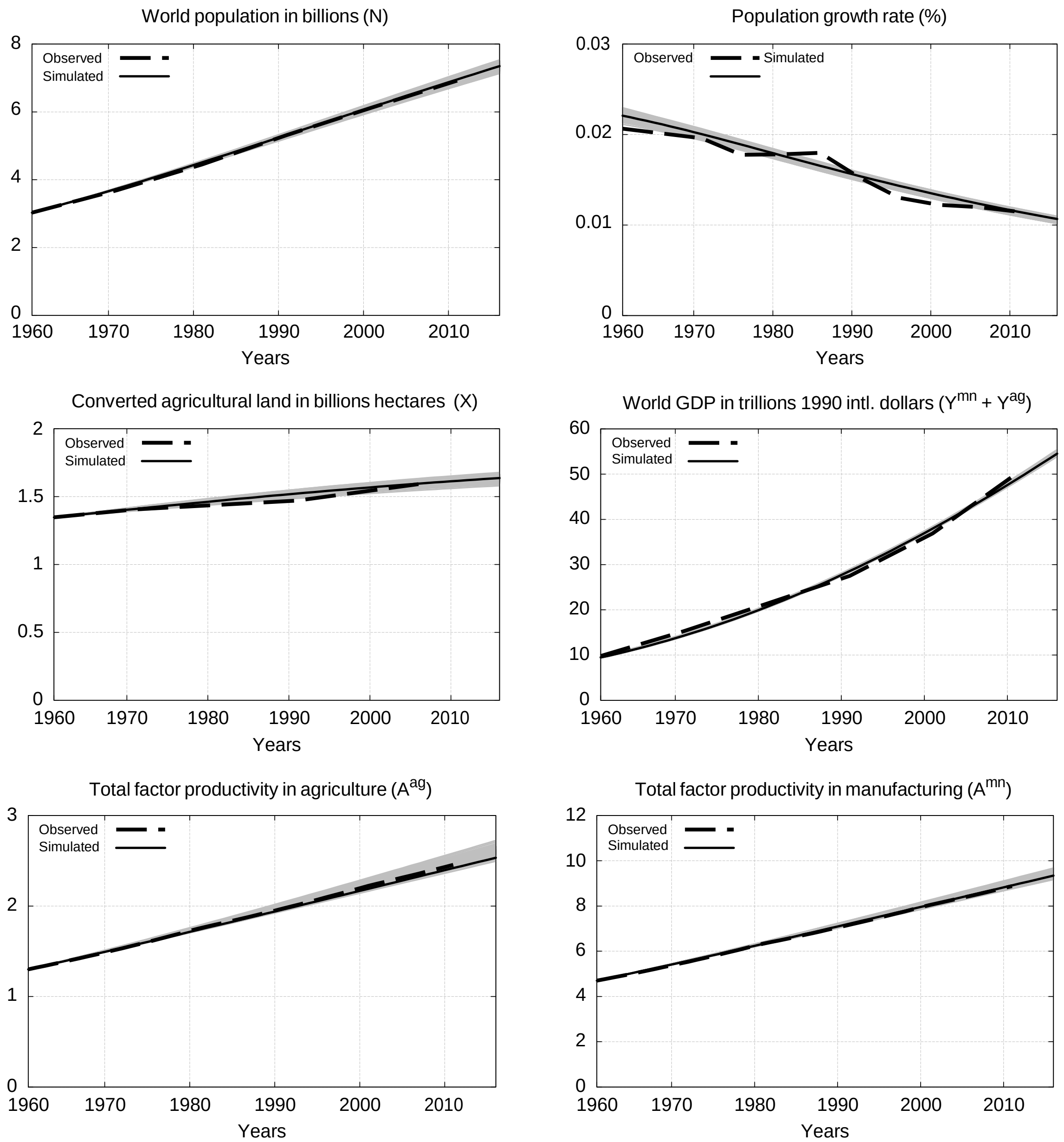
The resulting simulated trajectories are reported in Figure 2 and compared to observed ones. The model closely fits observed trajectories, with a relative squared error of 3.52 percent across all variables. The size of the error is mainly driven by the error on output (3.3 percent), followed by land (0.1 percent) and population (0.03 percent). In Figure 2 we also report runs for which the goodness-of-fit objective is relaxed by 10% relative to the best fit achieved, as represented by the shaded area. In other words, the shaded area reports the set of simulated trajectories with an error of 3.9 percent at most. The associated range of parameters is reported in Table 1.

Because of our focus on fertility and population, the model should also provide a good fit of the population growth rate even though it is not directly targeted by the estimation. This is shown in the top right panel of Figure 2. While this series naturally shows more volatility than the level of population, the model replicates well the decline of population growth in the past fifty years. A second measure not directly targeted in the estimation that is important for the analysis is the evolution of agricultural output over time. According to Alexandratos and Bruinsma (2012), agricultural output has grown by 2 percent per year on average from 1960 to 2010, or an equivalent 269 percent over that period. In our model agricultural output over the same period increased by 279 percent.

While the estimated parameters provide a very good fit to targeted quantities, the fact that they are conditional on the structure of the model and on the value of the imposed parameters makes them difficult to compare with evidence from other sources. First, in our specification the cost of effective labor units is increasing and convex, whereas other quantitative studies typically assume a constant cost of children (and report different assumptions about these costs). However, evidence from two papers can be quite meaningfully compared to our estimates. Jones

[26] As for other simulation-based estimation procedures involving highly non-linear models, the uniqueness of the solution to the estimation of the parameters cannot be formally proved (see Gourieroux and Monfort, 1996). Our experience with the model suggests however that the solution is unique, with no significantly different vector of parameters providing a comparable goodness-of-fit objective. In other words, estimates reported in Table 1 provide a global solution to the estimation objective. The fact that we simultaneously estimate the whole vector of parameters makes the criteria highly demanding, as changing one parameter will impact trajectories across all variables in the model.

Figure 2: Estimation of the model 1960-2010



and Schoonbroodt (2010) report the cost of children in terms of years of output for developing countries between 1970 and 1990, which ranges from 4.5 to 17.4. Jones and Schoonbroodt (2014) further estimate the cost of children in terms of both time and goods. The time cost amounts to 15 percent of work time, while the goods cost amounts to around 20 percent of household income. In our model the *implied* time cost of children varies from 7.5 years (χ_t =

0.133) in 1960 to 17.9 years in 2010 (χ_t = 0.056). While our 2010 estimate then appears to be high, remember that it combines the time and goods costs of children.

Our estimate of the elasticity of technology on fertility can be compared to the empirical results of Herzer et al. (2012). In particular, they estimate that the long-run elasticity of fertility with respect GDP growth of one percent is around -0.0018.[27] In our model, a one percent increase in TFP (and hence GDP) reduces fertility by -0.00089 *in the same period*, or about half of the long-term impact. Our estimate is hence in the same ballpark. Finally, the elasticity of labor in R&D activities (μ_j) is also discussed in the literature. However, there is disagreement on what this parameter should be. In particular, Jones and Williams (2000) argue that it is around 0.75, while Chu et al. (2013) use a value of 0.2. These two papers however rely on thought experiments to justify their choices. According to our results, a doubling of the share of labor allocated to R&D would increase TFP growth by around 50%.[28]

4 Global projections from the quantitative model

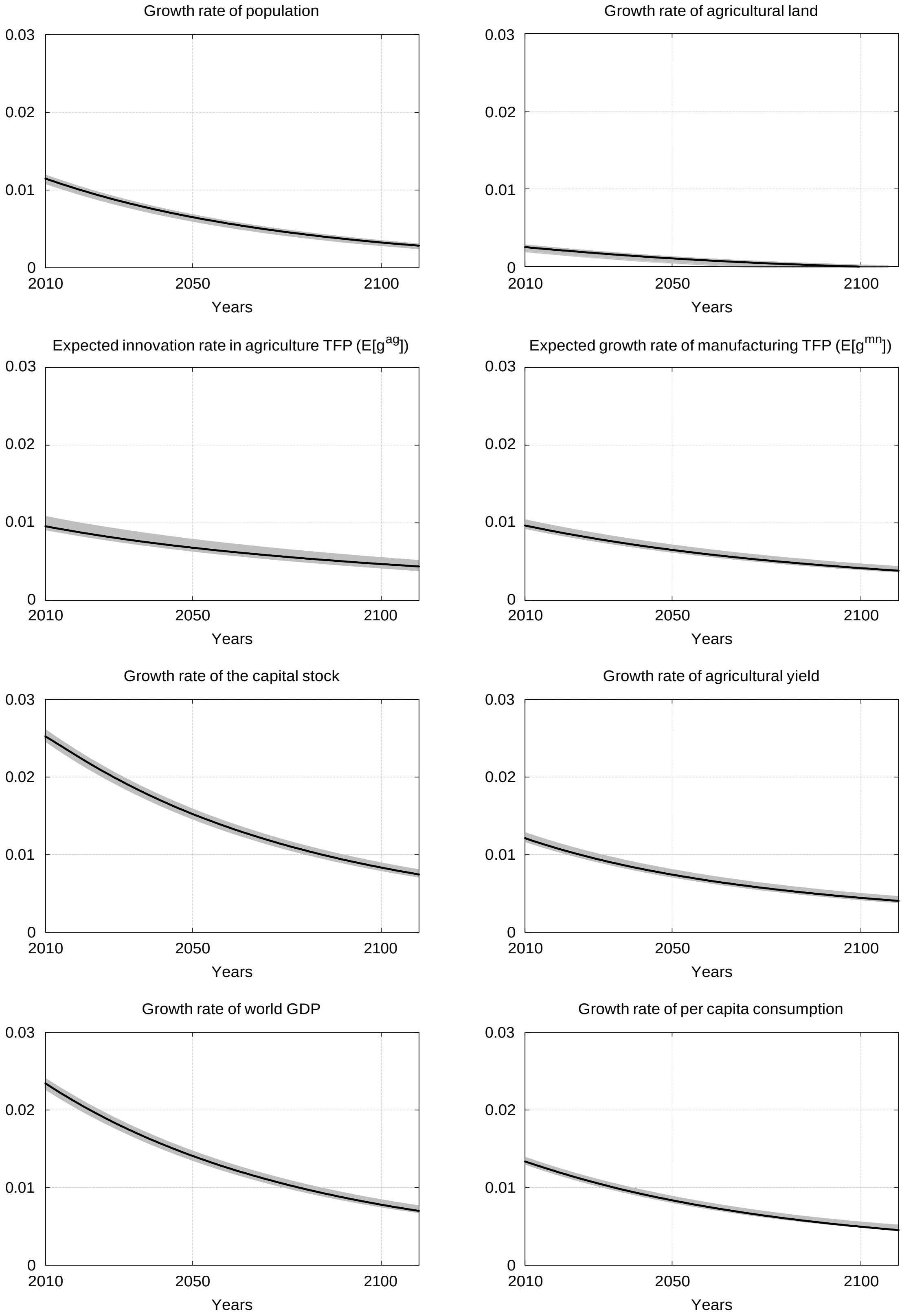
Figure 3 displays projections of the growth rate of key variables from 2010 to 2100. The main feature of these paths is that they all decline towards a balanced growth trajectory where population, land and capital reach a steady state. Agricultural land area is the first state variable to reach a steady state as its growth rate becomes negligible by 2050. Thus the total amount of land that can be used for agriculture is never exhausted. Population growth on the other hand remains significantly above zero over the whole century, being around 0.3 percent by 2100. Thus the model is far from predicting a complete collapse of population growth over the coming fifty years. Nevertheless population growth continues to decline after that, being around 0.1 percent in 2150.

The pace of technological progress also declines over time, starting at around one percent per

[27] More specifically they estimate a long-run cointegrating relationship between the crude birth rate and the log of GDP, with their central estimate being -5.83. For a one percent increase in GDP, this implies a reduction of the crude birth rate of -0.058, or -0.0018 percent at the mean fertility level of 33. In a model with country-specific time trends, they report an elasticity of -3.036, which is associated with an elasticity of -0.0009, which is almost identical to our own estimate.

[28] We are not aware of comparable evidence for our estimates related to land clearing. These estimates rationalize the relatively slow development of agricultural land area as compared to agricultural output and thus integrate forces determining the allocation of land, such as the demand for pastures and urban areas.

Figure 3: Estimated model: Growth rate of selected variables



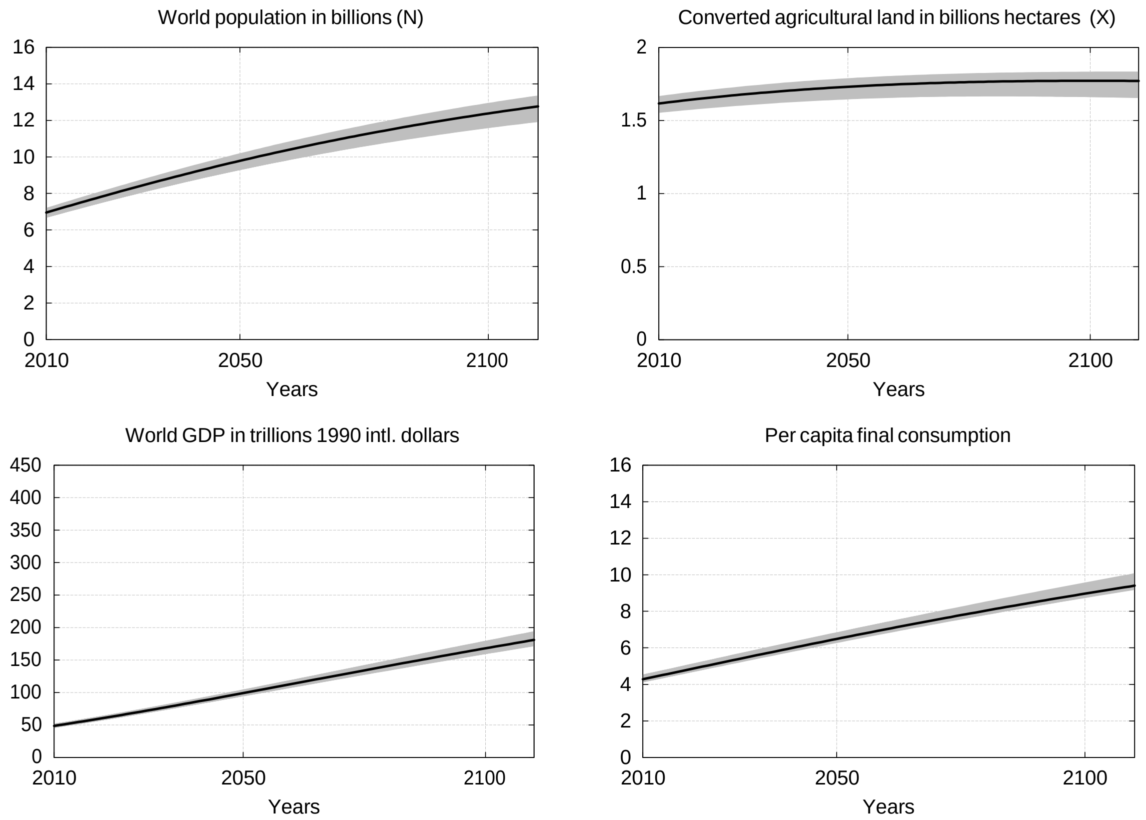
year and reaching about half of one percent by the end of the century. This has the consequence that, over time, labor productivity and the educational costs of children grow less quickly than in the period 1960-2010. This is the main explanation for why population growth does not fall more quickly, which in turn implies a relatively high population *level* reported in Figure 4 (see also Appendix B). In particular, world population is around 9.85 billion by 2050, which is broadly consistent with the UN's new projections (United Nations, 2013), but not with its previous projections, where global population peaked at 9 billion and remained stable from 2050 onward. Our model further suggests that population growth continues over the entire century, so that the global population reaches more than 12 billion by 2100. This estimate lies towards the upper bound of the probabilistic forecasts recently made by Gerland et al. (2014).

Interestingly the shaded band for the population growth rate, which represents a range of alternative pathways for vectors of parameters with a slightly lower fit, shrinks over time. This demonstrates that the estimation of the parameters does not affect the long-run growth rate of population, whereas different transition paths imply a range of possible population levels between 11 and 13 billion by 2100.

Our model indicates that a significant increase of population over the century is compatible with food production possibilities. Between 1960 and 2010, agricultural output in the model increased by 279 percent, and increases by a further 67 percent between 2010 and 2050. These figures are very close to the 72 percent increase in global agricultural output projected by Alexandratos and Bruinsma (2012) for the period 2010 to 2050. After 2050, our model suggests a further increase in agricultural output of 31 percent by 2100, so that by the end of the century agricultural output roughly doubles relative to the current level. This can be compared to 80 percent growth in population and a 95 percent increase in per-capita income.

In light of these results, the fact that agricultural land area stabilizes at around 1.77 billion hectares is an important finding. First, this number is slightly higher than land conversion projections by Alexandratos and Bruinsma (2012), in which cropland expansion is expected to stop at around 1.66 billion hectares. As with population growth, land conversion will mostly occur in developing countries, while agricultural area in developed countries has declined and presumably will continue to do so on economic grounds. Second, TFP growth in agriculture remains below 1 percent, which is a fairly conservative assumption. In other words the pace

Figure 4: Estimated model: Projections for selected variables



of technological progress does not need to be very high to allow for sustained growth in agricultural output. Third, the halt of agricultural land expansion suggests that the elasticity of substitution (σ) is high enough to allow agricultural output to grow from the accumulation of capital (we return to the role of σ in the sensitivity analysis). Indeed, although the share of capital allocated to agriculture declines over time, the stock of capital in agriculture almost doubles between 2010 and 2050.[29] This would mainly represent improvements to irrigation facilities. Both technology improvement and capital accumulation are reflected in the growth rate of agricultural yield (Figure 3), measuring growth in agricultural output per hectare used in agricultural production.[30]

[29] As expected, both the share and the quantity of labor allocated to agriculture decline over time.

[30] Projections in the growth rate of agricultural yields, derived from projections on agricultural output and agricultural land, are thus broadly in agreement with Alexandratos and Bruinsma (2012). In particular, they report a decline in agricultural yield growth from 1.9 percent per year from 1960 to 1985, 1.4 from 1985 to 2007, and 0.7 percent per year from 2007 to 2050.

Finally, the growth rate of GDP falls from more than two percent in 2010 to less than one percent in 2100, which implies that world GDP doubles by 2050 and more than triples by 2100. Similarly, per-capita consumption more than doubles by 2100 relative to 2010.

5 Sensitivity analysis

We now report the results of sensitivity analysis with respect to a number of assumptions we have made: substitution possibilities in agriculture (σ), the elasticity of utility with respect to fertility (η), the discount factor (β) and the expected working lifetime ($1/\delta_N$). For each change in the value of a parameter, it is necessary to re-estimate the vector of parameters to match observed data over the period 1960 – 2010. We plot results for our two main variables of interest, population and agricultural land, against our baseline results discussed above and we report the vector of estimates associated with each sensitivity run in Table 2.

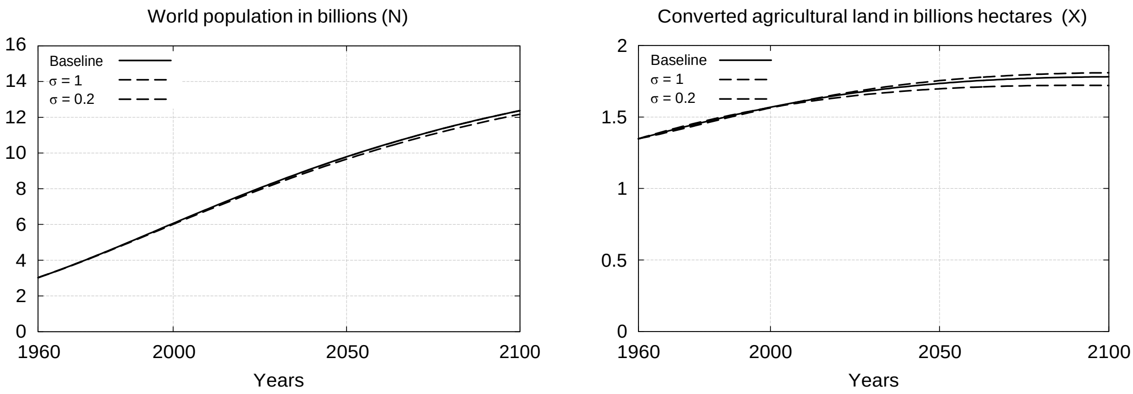
The parameter σ determines the elasticity of substitution between land and the capital-labor composite input in the agricultural production function. Our baseline case is obtained under the assumption that $\sigma = 0.6$, which follows empirical evidence by Wilde (2013). However, evidence with regard to this parameter remains scarce, and it is the main determinant of the demand for agricultural land, and in turn the ability to produce food and sustain the population.

We therefore re-estimate the parameters of the model assuming that $\sigma = 1$, so that agricultural production is Cobb-Douglas, and $\sigma = 0.2$, which provides a lower bound on substitution possibilities in agriculture. The results reported in Figure 5 demonstrate that the choice of σ has a small impact on land conversion and virtually no impact on population. As expected, a high value of σ implies less land conversion, since other factors can be more easily substituted when the marginal cost of land conversion increases. Conversely, a lower σ makes land more important in agriculture, so that the overall area of agricultural land is higher. However, estimating the model over 50 years of data largely ties down the trajectory for land use in a robust manner, irrespective of the choice of σ. Estimates of labor productivity in land conversion imply a higher (lower) conversion cost under $\sigma = 0.2$ ($\sigma = 1$). Estimates of the marginal productivity of labor in agricultural R&D also adjust, implying lower productivity for $\sigma = 0.2$, exemplifying inter-dependencies in our estimation procedure. The fit of the model remains very similar.

Table 2: Estimates supporting the sensitivity analysis

Parameter	Baseline	$\sigma = 0.2$	$\sigma = 1$	$\eta = 0.5$	$\beta = 0.97$	$\delta_N = 0.015$
χ	0.153	0.155	0.151	0.205	0.155	0.104
ζ	0.427	0.417	0.426	0.399	0.460	0.516
ω	0.089	0.085	0.088	0.161	0.087	0.091
μ_{mn}	0.581	0.575	0.580	0.751	0.523	0.525
μ_{ag}	0.537	0.549	0.509	0.482	0.383	0.512
ψ	0.079	0.063	0.083	0.078	0.083	0.077
ε	0.251	0.174	0.256	0.239	0.243	0.186
Estimation error	0.035	0.033	0.035	0.189	0.029	0.045

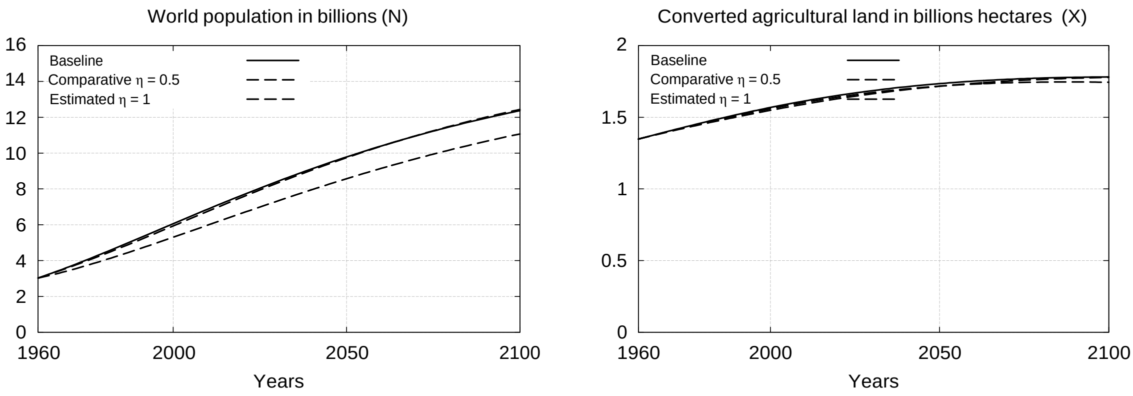
Figure 5: Sensitivity analysis on substitution possibilities in agriculture



The second sensitivity test we conduct targets η, the elasticity of utility with respect to fertility. We consider the case of $\eta = 0.5$, so that the marginal utility of fertility (and population) declines more rapidly than under our baseline assumption of $\eta = 0.01$.[31] We re-estimate the parameters of the model so that the model fits observed trajectories given alternative assumptions about η, and report the resulting trajectories in Figure 6. In addition, we also report trajectories obtained with $\eta = 0.5$ but where the baseline parameter estimates are retained. This can be thought of as a comparative-static experiment (we label these trajectories "comparative"). As Figure 6 shows, when the model is not re-estimated trajectories over 1960 to 2010 differ significantly.

[31] Note that in our setting an average utilitarian objective corresponds to $\eta = 0$, but it implies that the objective function is not globally concave.

Figure 6: Sensitivity analysis on altruism towards children

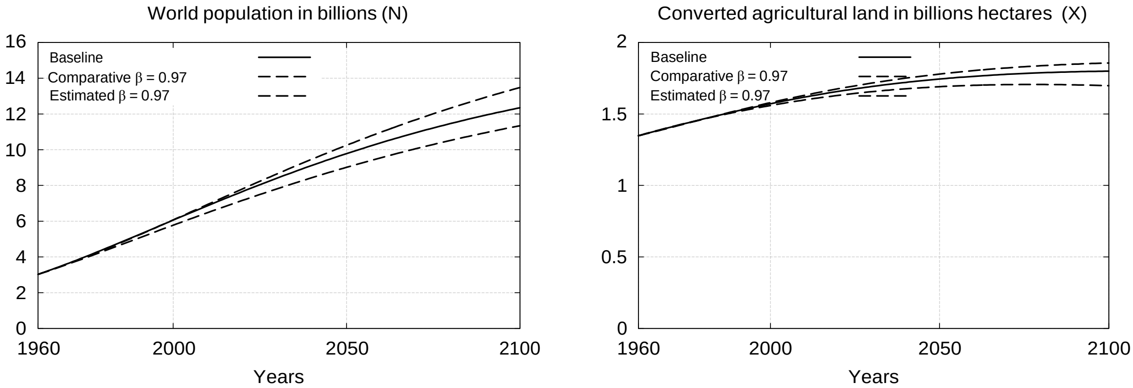


As expected, reducing η while keeping the estimated parameters to their baseline values implies lower population growth. This results from putting less weight on the welfare of future members of the dynasty, so that the dynastic head reallocates resources to increase its own consumption at the expense of population growth. However, once we re-estimate the model to observed trajectories over 1960 to 2010, the population path is virtually identical to the baseline trajectory. Note that the estimated parameters under $\eta = 0.5$ are very different from those in the baseline case, and the estimation error is significantly higher (see Table 2).

The third parameter we vary is the discount factor. The baseline value of $\beta = 0.99$ implies a relatively low discount rate, and we instead use $\beta = 0.97$. We report a trajectory where we re-estimate the model to 1960-2010 data under the assumption that $\beta = 0.97$, as well as a comparative-static exercise in which we set $\beta = 0.97$ while keeping other parameters to their baseline values. Results are reported in Figure 7.

Reducing β gives less weight to the welfare of future members of the dynasty, thus reducing the demand for children and lowering population growth. This implies that the comparative-static trajectory for population is lower than the baseline trajectory. Moreover reducing the discount factor implies a lower saving rate, so that there is less capital available for agricultural production, and more land is needed to compensate. However, by re-estimating the model to 1960-2010 data under the assumption $\beta = 0.97$, we find that the opposite is true. As compared to the baseline, a lower discount factor implies a higher long-run population, while the agricultural land area is smaller. Indeed estimates of the cost of fertility imply higher labor productivity and more weakly decreasing returns to labor, and hence a lower marginal cost of fertility both

Figure 7: Sensitivity analysis for the discount factor



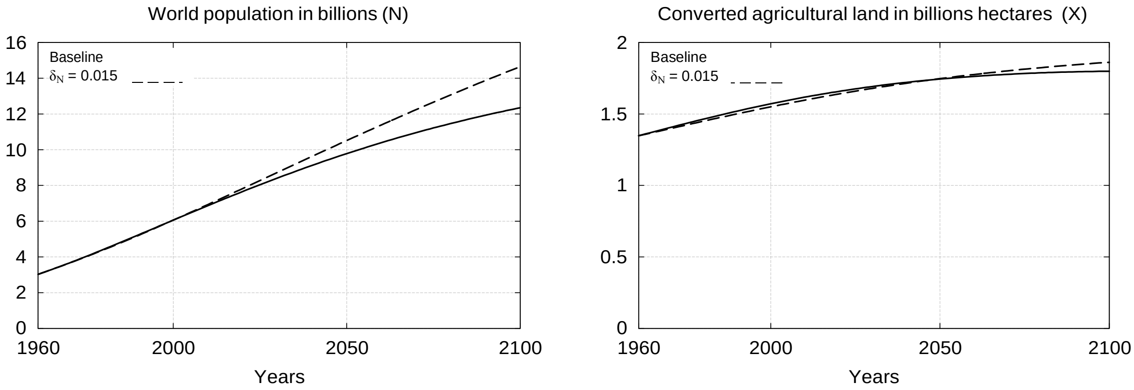
within and across periods. In turn, the accumulation of labor becomes cheap relative to capital and land, incentivizing the accumulation of population as a substitute for the accumulation of capital and land. This result contrasts with changes in η, which did not directly affect the accumulation of capital and land.

The final sensitivity test is on the death rate δ_N, or equivalently the expected working lifetime $1/\delta_N$. We illustrate the effect of this parameter by using a somewhat extreme value of 65 years, corresponding to $\delta_N = 0.015$. Trajectories are reported in Figure 8. As expected this implies a larger long-run population, reaching more than 10 billon in 2050 and around 15 billion by 2100. The impact of this parameter is mostly felt in the long run, as it implies that the growth rate of population declines less rapidly over time. This result confirms the importance of δ_N as a driver of population dynamics, as demonstrated by Jones and Schoonbroodt (2010) and Strulik and Weisdorf (2014).

6 Discussion

Our approach distinguishes itself from existing population projections in two main ways. First, fertility decisions are endogenous. While our representation of preferences for fertility implies some restrictions and is open to debate, it contrasts with the current practice in demography of assuming a fixed trajectory for fertility. In fact, the rapid decline of population growth towards zero implied by existing UN projections is an outcome of the assumed convergence of fertility to its replacement level. The basis of this assumption is the observed convergence of

Figure 8: Sensitivity analysis on the expected working lifetime



developed countries to a low fertility regime. A rapid decline of fertility in developing countries has strong implications in terms of technological progress and economic convergence, which are only implicit in projections from other sources.

Second, our integrated representation endogenizes the evolution of quantities that are jointly determined with fertility choices, notably technological progress, income per capita and agricultural output. Given the structure of the model, the dynamic relationship between these variables is informed by structurally estimating the model to observed trajectories. Our model thus treats the representation of preferences and technology as fixed, with the dynamics being driven exclusively by structural assumptions. This contrasts with existing projections that are carried out in isolation from each other, yet mutually rely on one another. For example, projections of food demand and agricultural land use reported in Alexandratos and Bruinsma (2012) rely on population projections by United Nations (2013), the latter obviously assuming that the projected population can be fed.

Overall, our results confirm the widespread expectation that the long-standing processes of growth in population and land conversion are in decline. This stems from a quality-quantity trade-off: shifting from a quantity-based economy with rapid population growth and associated land conversion towards a quality-based economy with investments in technology and education, and lower levels of fertility. Land is the first quantity to reach a steady state, doing so in the coming half-century. We find, however, that a steady state in land conversion is consistent with sustained growth in food demand and agricultural output as well as mildly optimistic assumptions about technological progress in the future.

In our projections population growth declines over time but remains positive (and significantly so) in 2100. While uncertainty over such a time horizon cannot be overstated, a key finding of our analysis is therefore that population does not reach a steady state in the foreseeable future. Population growth falls more slowly than in the existing population projections of the United Nations (2013) and Lutz and Samir (2010). We think this is plausible, because of the amount of inertia in the system, and because better economic prospects will sustain the demand for children despite an increasing cost associated with child-rearing and education. In our framework the slowdown of technology accumulation implies a slowdown in the decline in fertility, so to speak, so that the decline in population growth itself slows down.[32]

7 Concluding comments

We have studied the implications of using a macroeconomic growth model to make projections of world population and land use over the long run. Our model integrates fertility decisions in a wider framework where technological progress, per-capita income, food production and land conversion are jointly determined. Once the model is calibrated to data from recent history, the resulting projections are surprisingly robust to different parametric assumptions.

Our results suggest that sustained population growth over the coming century is compatible with an evolution of agricultural output close to what has been observed in the past, mainly on account of technological change and capital accumulation. Of course, we take a highly aggregated view of the problem, and food security is very likely to remain of concern at the regional level. That is to say, our results should perhaps be interpreted in terms of *potential* food security. Other constraints may also become important over the next century, in particular the availability of water, itself partly linked to climate change. This may be particularly important in Asia and Africa, where population and economic growth are expected to be high.

One virtue of our integrated model is that it provides a rich empirical framework to study interactions among key drivers of growth over the long run. In particular, it can be used to

[32] By the same mechanism driving land to a steady state, namely linear depreciation and decreasing returns to labor in the expansion of agricultural land, the dynamic equation describing population implies a steady state. However, the data suggests that convergence towards a steady state for population is slow, and the growth rate will drop to zero only in the very long run.

evaluate policies affecting different drivers such as the cost of children or constraints to land conversion. In this paper our aim has rather been to study implications of the model with respect to long-term population development. This provides a novel perspective on widely used projections from a small number of sources that use the same assumptions regarding convergence of fertility to its replacement level and the halt of population growth over the coming century.

Appendix A

Proof of Lemma 1

Write the dynastic household's optimization problem as

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t)b(N_t)N_t + \mu_{t,N}\,[N_{t+1} - \chi(L_{t,N}, A_{t,mn})] + \mu_{t,X}\,[X_{t+1} - \psi(L_{t,X})] \\ +\theta_{t,X}[\bar{X} - \psi(L_{t,X})] + \theta_{t,N}\,[N_t - L_{t,mn} - L_{t,N} - L_{t,X} - L_{t,ag}] \\ +\theta_{t,ag}\left[A_{t,ag}Y_{ag}(L_{t,ag}, X_t) - \bar{f}_t N_t\right] \end{array} \right\}$$

Substituting in the budget constraint, $c_t = 1/N_t\, w_t L_{t,mn}$, the necessary first-order conditions for a maximum include that

$$\frac{\partial L}{\partial L_{t,mn}} = u'(c_t)b(N_t)w_t - \theta_{t,N} = 0$$

$$\frac{\partial L}{\partial L_{t,N}} = -\mu_{t,N}\frac{\partial \chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}} - \theta_{t,N} = 0$$

$$\frac{\partial L}{\partial L_{t,ag}} = \theta_{t,ag}A_{t,ag}\frac{\partial Y_{ag}(L_{t,ag}, X_t)}{\partial L_{t,ag}} - \theta_{t,N} = 0$$

$$\frac{\partial L}{\partial L_{t,X}} = (-\mu_{t,X} - \theta_{t,X})\psi'(L_{t,X}) - \theta_{t,N} \le 0$$

The marginal effect on household welfare of fertility in period t, at the optimum, can be characterized as

$$\frac{\partial L}{\partial N_{t+1}} = \beta u(c_{t+1})\left[b'(N_{t+1})N_{t+1} + b(N_{t+1})\right] + \mu_{t,N} + \beta\theta_{t+1,N} - \beta\theta_{t+1,ag}\bar{f}_{t+1} = 0$$

We now proceed by using the first-order conditions on the controls to eliminate the shadow prices. It is straightforward to verify that –

$$\mu_{t,N} = \left[-u'(c_t)b(N_t)w_t\right]\left[\frac{\partial \chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}}\right]^{-1},$$

$$\theta_{t+1,N} = u'(c_{t+1})b(N_{t+1})w_{t+1} \text{ and}$$

$$\theta_{t+1,ag} = \left[u'(c_{t+1})b(N_{t+1})w_{t+1}\right]\left[A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag}, X_{t+1})}{\partial L_{t+1,ag}}\right]^{-1}$$

The Lemma follows immediately. D

Proof of Proposition 1

Partially differentiate (4) with respect to $A_{t,mn}$, i.e. compute $\frac{\partial^2 L}{\partial N_{t+1}\partial A_{t,mn}}$, where $A_{t+1,mn} = (1+g_{t,mn})A_{t,mn}$.

Using the condition that maximizes firm profits, $A_{t,mn}Y'_{mn}(L_{t,mn}) = w_t$, the partial derivative of part (A) with respect to $A_{t,mn}$ is

$$u'(c_t)b(N_t)Y'_{mn}\left[\partial\chi(L_{t,N},A_{t,mn})/\partial L_{t,N} - A_{t,mn}\partial^2\chi(L_{t,N},A_{t,mn})/\partial L_{t,N}\partial A_{t,mn}\right]$$

Since we assume that $\partial^2\chi(L_{t,N},A_t)/\partial L_{t,N}\partial A_t < 0$, this term is positive.

The partial derivative of part (B) with respect to $A_{t,mn}$ is

$$\left[\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})\bar{f}_{t+1}\right] A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag},X_{t+1})}{\partial L_{t+1,ag}}$$

This term is also positive. Part (C) is not a function of $A_{t,mn}$.

The partial derivative of part (D) with respect to $A_{t,mn}$ is

$$\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})$$

This term is again positive. Combining these three terms yields the Proposition. D

Appendix B Observed and simulated data

The table below reports both observed and simulated data from 1960 to 2100, by 10-year intervals. Note that agricultural area is only available for 2005.

Year	Population (billion)		Population growth (%)		Crop land area (billion ha)		GDP (trillions 1990 intl. $)	
	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated
1960	3.03	3.03	0.021	0.022	1.37	1.35	9.8	9.5
1970	3.69	3.74	0.020	0.020	1.41	1.41	15.3	14.3
1980	4.45	4.51	0.018	0.018	1.43	1.47	21.3	20.6
1990	5.32	5.32	0.015	0.015	1.47	1.52	27.5	28.5
2000	6.13	6.14	0.012	0.013			36.9	38.0
2005					1.59	1.60		
2010	6.92	6.95	0.011	0.011		1.62	50.0	48.6
2020		7.74		0.010		1.65		60.5
2030		8.49		0.009		1.69		73.2
2040		9.19		0.007		1.71		86.6
2050		9.85		0.006		1.73		100.5
2060		10.46		0.006		1.75		114.5
2070		11.02		0.005		1.76		128.5
2080		11.53		0.004		1.77		142.4
2090		12.00		0.004		1.77		156.1
2100		12.42		0.003		1.77		169.3

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