Breaking Up May Not Be Hard to Do:
Terminating Links between Emission Trading Programs∗

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Abstract

We consider the termination of a compliance link between regional emission trading programs, a topic highlighted by New Jersey’s decision to exit the multi-state Regional Greenhouse Gas Initiative at the end of 2011. We consider two ‘delinking’ policies. One treats currently circulating allowances differently, with each program’s allowances reverting to compliance only in its own region. The other treats currently circulating allowances the same, with all such allowances being split into a fraction of an allowance in each region. Using a two-region, two-period model, we describe the price dynamics and relative cost-effectiveness of each policy, with and without uncertainty about whether delinking will occur. We find that treating circulating allowances the same tends to reduce costs, highlighted through a numerical example of the proposed EU-Australian link. We conclude with a discussion of broader concerns and the value of addressing the delinking question at the time a link is announced.

1 Introduction

Despite significant effort and multiple rounds of negotiations, there is no coordinated global program to regulate carbon emissions. Rather than waiting for these efforts to bear fruit, various super-national, national, and sub-national entities have developed independent carbon-trading programs. Conceptually, each of these program features its own denomination of carbon permits and a registry in which permits are established, tracked, and ultimately

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cancelled when surrendered for compliance purposes. Within this framework, some of these trading programs have decided to link together, meaning one program accepts another program’s permits for compliance in its system (and typically vice-versa). For example, Quebec and California have chosen to link their programs, as have Australia and the European Union. The Regional Greenhouse Gas Initiative (RGGI) in the United States is effectively a system of linked state programs.

In this paper we focus on the termination of a link, because it is unreasonable to expect all links to be immutable. Recent experience points to the salience of this issue, as New Jersey effectively delinked itself from RGGI when it withdrew at the end of 2011. This implies that when links are formed, consideration should be given to the possibility of delinking in the future, and whether and how provisions for delinking might be included at the outset. To date, this issue has tended to be ignored if not outright avoided.¹

To analyze these issues, we consider a two-period model with two regions. Firms in both regions face a regulation that requires them to surrender a tradable permit for each ton of pollution they emit. In the first period, the trading programs are linked, that is, firms in both regions can use permits from either region. Permits may also be saved (banked) for use in the second period. We consider several policies governing the trade of permits in the second period. We first consider the fully linked baseline case in which the markets remained linked in the second period so firms can continue to use permits from either region. We then turn to the issue of delinking. When markets are delinked, firms can only comply using permits denominated by their home region. To complete the description of the delinked case, we must also specify what happens to permits that were saved in the first period. In particular, do they revert to the region where they were issued or are they “split” into a fraction of an allowance in each region?

Our main point is that this delinking policy matters. If the delinking policy specifies that permits saved at the end of the first period revert to the region where they were issued,

¹During an August 2012 press conference on linking with the EU, Australian Minister Greg Combet repeatedly ducked questions about delinking or a “get-out” provision. See Combet (2012).
then first period prices can diverge and costs are usually higher than the fully linked case. In fact, even mere speculation about delinking can lead to such price dispersion. In contrast, if the delinking policy specifies that all permits saved at the end of the first period are split such that a fraction $\pi$ goes to one region and $1 - \pi$ goes to the other region, regardless of origin, then first period prices never diverge. For a particular choice of $\pi$, prices and costs can frequently match the fully linked case. And as a result, speculation about delinking may not have any market consequences.

These results suggest an important trade-off exists for jurisdictions contemplating a link: Ignore the possibility of delinking and perhaps communicate a greater commitment to a permanent link—but also risk that a future decision to delink, or mere speculation about such a decision, could be disruptive to market prices and raise costs. Or, plan for delinking—which might be as simple as stipulating that future policy changes will always treat permits in public circulation identically, regardless of origin. Such an exercise might create more uncertainty about the durability of the link, but could ensure that any future delinking event, or speculation about such an event, would be less disruptive and less costly.

To put these ideas firmly into context, the next section briefly reviews the policy history and literature on linking. We then present our model and results in section 3. Section 4 discusses some additional considerations—such as the possibility that cost minimization may not be the only objective and that delinking provisions create additional uncertainty about the link. Section 5 concludes.

2 History and Literature on Linking

During much of the 1990s, public debate focused largely on how to design a single global market for trading carbon permits as “the” vehicle to address global climate change. Because one ton of a greenhouse gas emitted anywhere in the world has the same climate change consequences for everyone, a single global market would be an economically desirable outcome,
equalizing the marginal cost of reducing emissions everywhere. A single market is also more resil-
ient to regional disruptions, spreading any imbalance over a larger volume of supply and demand. The Kyoto Protocol was widely viewed as a first step in this direction.

However, this 1990s perspective has turned out to be a practical impossibility, at least for the time being, as participation in the Kyoto Protocol has declined to a largely symbolic gesture among countries with well-aligned domestic policies. Instead, we see a multiplicity of distinct regional, national and even sub-national trading programs emerging, most notably the Emissions Trading System set up by the European Union (EU-ETS) in 2005, but also including the Regional Greenhouse Gas Initiative in the northeastern United States, the New Zealand Emissions Trading Scheme, California, Quebec, and, on the near-term horizon, Australia. Here, we view distinct trading programs as those with separate legal authorities establishing a compliance obligation for firms.\(^2\)

Alongside the emergence of multiple trading programs, we have also witnessed a range of linkages among trading programs—by linkages, we mean the adoption of mechanisms by which credits from one trading program are recognized for compliance in another program and typically vice-versa. At one extreme is RGGI, where participating states jointly developed their emission trading programs through negotiation of a model rule, including automatic recognition of other participating states’ emission allowances, that was then the basis of each state’s legislation and/or regulation. At the other extreme is Australia and the EU-ETS, where Australia is only now going through a process of harmonizing features with the EU-ETS in advance of linking. California and Quebec lie somewhere in the middle—there was a great deal of cooperation as they were designed and implemented, but linking was not automatic.

To date, we have one example of delinking. In May 2011, Governor Chris Christie announced that New Jersey would pull out of RGGI, effective at the end of 2011. At that

\(^2\)For example, the EU-ETS was established in all EU Member States by a single directive of the European Council. In contrast, RGGI was established by separate statutory and/or regulatory authority in each participating state.
point New Jersey would delink from the other states and end the compliance obligations for facilities in New Jersey. Almost immediately, the other states announced that they would continue to allow their regulated firms to make use of all current vintage (e.g., 2011 and earlier) New Jersey allowances that were already in circulation. They later announced that firms could make use of all future vintage (e.g., 2012 and later) New Jersey allowances that were already in circulation.\(^3\) New Jersey ultimately ended their emission trading program as announced.

### 2.1 The literature on linking and delinking

There has been little discussion of delinking to date. Mehling and Haites (2009a) note that linking agreements should require a procedure for terminating the link that addresses the status and validity of unused units—a point ignored by agreements to date. There is, however, an extant literature on linking. Mehling and Haites catalog existing links between trading programs and discuss potential future links. They also describe several different mechanisms by which links may be formed: unilateral, bilateral, and multiple (i.e., reciprocally adopted) unilateral. The differences between the latter two are subtle and unimportant for our analysis. We assume that permit from one program can be used in the other program, and vice versa, which encompasses both the bilateral and multiple unilateral cases.

Jaffe et al (2009) consider whether linking may act as substitute for, complement of, or precursor to a potential international climate policy agreement. They also discuss the benefits and concerns about linking, a topic that is also covered by Flachsland et al (2009). Tuerk et al (2009) focus on barriers that may prevent links from forming. Finally, a series of papers look at the details of forming specific links between countries in North America (Haites and Mehling 2009b), between EU and USA (Sterk and Kruger 2009), between Australia and the EU (Jotzo and Betz 2009), and the particular problem of international aviation and shipping emissions (Haites 2009).

\(^3\)The RGGI design arranged for each auction to sell a combination of current and future vintage allowances from each participating state. See various documents at http://www.rggi.org/design/history/njparticipation.
2.2 To link or not to link?

An accounting of the costs and benefits of forming a link naturally starts with the costs associated with abating pollution. As described by Jaffe et al (2009), there are three mechanisms by which linking will lower joint abatement costs. First, immediate cost savings are realized by two regions who choose to link: the initially low-price region sells allowances at prices above their local cost; the initially high-price region replaces high-cost local mitigation with cheaper imported allowances. Second, and regardless of any initial or expected price differences, both regions will also tend to experience reduced volatility. Local shocks to allowance demand—particularly weather and business cycles—lead prices to diverge from their expected value. Under very basic assumptions, it is easy to show that volatility raises compliance costs. By linking systems, however, volatility should decline as local shocks are spread over a larger market. Third, increased liquidity—particularly for a small region linking to a large one—can reduce transaction costs by increasing access to derivatives and other hedging tools as well as reducing bid-ask spreads.

While the focus of the economic literature (and this paper) tends to be cost-effectiveness, it is useful to recognize the wider variety of reasons that may affect a jurisdiction’s decision about linking and potentially delinking. On the “pro-linking” side, political strengthening may be lurking behind the enthusiasm of some linking proponents. This reflects the idea that the more linked and integrated a particular ETS becomes, the more resilient it becomes to weakening or dismantling in the future. The flipside of this view is that delinking may be seen as a precursor to dismantling—as it was in the case of New Jersey.

The greatest obstacle to linking tends to the need to harmonize programs in advance of linking. Often, differences in market design reflect different preferences that may be hard to reconcile. California, for example, has a price floor that limits low prices along with a

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4 Imagine $C(\bar{q} - q)$ is the cost of reducing emissions from $\bar{q}$ to $q$, where $q$ is an emission cap and $\bar{q}$ is the uncontrolled emission level. If $\bar{q}$ is uncertain and costs are a convex function of emission reductions, Jensen’s inequality states that $E[C(\bar{q} - q)] > C(E[\bar{q}] - q)$. That is, increased uncertainty about $\bar{q}$ raises costs.

5 This is analogous to the idea that a free trade agreement can lock in market reforms. See Hufbauer and Schott (2005).
system of allowance reserves that attempt to manage the risk of high prices. The EU has no automatic mechanism tied to prices and might find such features unpalatable. Fully integrating their markets, however, would expose the European market to those features. In many ETSs, choices about such features have been critical to achieving an internal political agreement, making subsequent adjustments difficult.

Related to this, linking to another program means accepting current and future choices by that jurisdictions about their future ambition toward decreasing carbon emissions. This has interesting consequences, potentially pitting the economic gains from integration against broader equity concerns. On the one hand, the largest economic benefit comes from integrating a high price market with a low price market. On the other hand, large price discrepancies among jurisdictions might easily raise “red flags” about the potential acceptability of each other’s ambition among countries with similar economic status. That is, while economic benefits from linking are shared by both parties, they rest on top of the prior-to-linking costs imposed by each jurisdiction unilaterally. Linking means, for a high-price market especially, accepting what is (or what appears to be) lower ambition in a low-price market. This may not be politically acceptable in the high-price region and, ultimately, could adversely affect choices about future ambition in other, yet-unregulated jurisdictions by suggesting that lower ambition is acceptable.

Finally, regardless of concerns about ambition, different jurisdictions may simply prefer different prices, reflecting preferences about both the social cost of carbon and domestic distributional consequences. A jurisdiction may view a high price as desirable, for example, because carbon pricing is also a powerful tool to drive a domestic agenda, such as changing the structure of a coal-based economy to a low carbon future, moving toward broader sustainability goals, addressing local pollution, raising revenues, stimulating new investment, etc. Market integration that lowers CO₂ prices may achieve near-term cost savings with regard to climate change mitigation, but reduce progress towards other goals as well as create higher longer-term mitigation costs if higher prices are ultimately justified (by the social cost of car-
bon). Or a jurisdiction might feel that high market prices have unpalatable and unavoidable distributional consequences—across regions, income classes, or other dimensions—and prefer lower market prices in concert with other regulations and technology incentives to achieve their mitigation goals. While market integration and higher prices from selling allowances abroad could generate net benefits for the jurisdiction as a whole, the within-jurisdiction distributional effects of higher prices may be undesirable.

After we develop results concerning the cost-effectiveness associated with delinking, we will consider how policy might be designed to account for some of these broader issues at the end of the paper.

3 A Model of Linking and Delinking

We consider a two-period, two region-model with pollution permit markets. The government in each region creates an endowment of permits in each period $w_{rt}$. Here and throughout, subscripts $t \in \{1, 2\}$ and $r \in \{H, L\}$ indicate period and region. As discussed in more detail below, the labels $H$ and $L$ signify that one region can be viewed, without loss of generality, as a high-cost region and the other as a low-cost region. Firms that emit pollution are required to surrender one permit for each ton of emissions. We assume a representative agent seeking to minimize abatement costs across regions, time, and possibly uncertainty about delinking, subject to the relevant permit market constraints. Region and time-specific aggregate abatement cost functions $C_{rt}(e_{rt})$ are defined with respect to emissions of pollution $e_{rt}$ and are assumed to be well-behaved.\(^6\) This leads to a general problem,

$$
\min_{e_{rt}} E \left[ \sum_{r \in \{H, L\}} \sum_{t \in \{1, 2\}} C_{rt}(e_{rt}) \right]
$$

\(^6\)Unlike our children. Formally we assume the abatement cost functions are twice-differentiable, convex, and $\lim_{e_{rt} \to 0} C_{rt}(e_{rt}) \to \infty$. The latter assumption rules out the possibility that $e_{rt} \leq 0$. This is not an important assumption, but simplifies exposition.
such that

$$F(e_{rt}, w_{rt}, \cdot) = 0$$

where $F$ represents a set of market constraints that may involve the (soon to be defined) flexibility variables ($\cdot$) that govern the shifting of emissions across regions and periods.\(^7\) Under our assumptions about abatement costs, the optimization problem (1) is equivalent to a competitive market outcome with many firms where permit prices equal the Lagrange multipliers on the appropriate equations in $F$. It is these permit prices along with the aggregate cost implications associated with different delinking provisions that motivate our interest: Do different delinking provisions lead to lower costs and/or more predictable prices?

Without banking and linking, the market constraints $F$ would amount to $e_{rt} = w_{rt}$ in each period and region, making the above optimization trivial. Banking and linking create flexibility to shift emissions, potentially lowering costs while keeping aggregate emissions constant. Correspondingly, we define two sets of flexibility variables, one set for banking and one set for linking. Banking implies inter-temporal flexibility. In particular, a first-period permit may either be used in the first period or be saved or “banked” for use in the second period. We use $B_r$ to denote the quantity of permits banked by region $r$. Typically, permit markets allow for banking but not borrowing. This corresponds to a market constraint $B_r \geq 0$ that we call the “no borrowing” constraint.\(^8\)

Linking implies inter-regional flexibility. When markets are linked, permits from one region can be used for compliance in the other region and vice-versa. This “links” the markets across regions. We use $\Delta_t$ to denote the (net) use of region $L$ permits for compliance in region $H$ in period $t$. As typically implemented, such linking is limited in that allowances moving between regions in the first period have to be used for compliance in that period—they cannot be banked and used for compliance next period in the other region. Another way of saying this is that linking does not allow conversion of a permit in one region into a

\(^7\)For simplicity we assume the discount factor is one.

\(^8\)Some programs have allowed a limited amount of borrowing, which would create an alternate constraint $B_r \geq -\bar{B}_r$, where $\bar{B}_r$ is the borrowing limit in region $r$. 
permit in another region that can then be banked for later use in the other region. Rather, linking in a given period allows firms to meet a current compliance obligation with permits from another region—and that is it. This implies the maximum flow of permits through a link equals the total emissions in the destination region; that is, \( \Delta_1 \leq e_{H1} \) and \( -\Delta_1 \leq e_{L1} \). Without without loss of generality, we assume that \( \Delta_1 \) is positive; that is, \( H \) is the high-price region prior to linking in period one and \( L \) is the low-price region (if not, simply relabel the regions). This allows us to ignore the constraint \( -\Delta_1 \leq e_{L1} \) throughout the analysis. In what follows we refer to \( \Delta_1 \leq e_{H1} \) as the “maximum link” constraint.\(^9\)

Our ultimate goal is to understand the effects of different delinking policies when there is uncertainty about whether or not the markets will remain linked in the second period. As a preliminary step, however, it is useful to assume at first there is no uncertainty about the second period. Under this assumption, we analyze three different policies: no delinking, asymmetric delinking, and symmetric delinking.

### 3.1 No Delinking – First Best

Here markets are linked in both the first and second period. The set of market constraints \( F \) is written as

\[
\begin{align*}
\epsilon_{H1} - \Delta_1 + B_H &= w_{H1} \\
\epsilon_{L1} + \Delta_1 + B_L &= w_{L1} \\
\epsilon_{H2} - \Delta_2 &= w_{H2} + B_H \\
\epsilon_{L2} + \Delta_2 &= w_{L2} + B_L \\
B_r &\geq 0
\end{align*}
\]

\(^9\)Another way to think about the maximum link constraint is to consider it in conjunction with the first period market equilibrium constraint in region \( H \): \( \epsilon_{H1} - \Delta_1 + B_H = w_{H1} \). Combining this with \( \Delta_1 \leq e_{H1} \) gives \( B_H \leq w_{H1} \). In other words, the amount of banked region \( H \) permits cannot exceed the period 1 endowment of \( H \) permits.
The first four constraints are the market equilibrium constraints that equate demand and supply for permits in each region and period. For example, in the first period, the demand for region $H$ permits is equal to the emissions in this region minus the influx of permits from region $L$, plus permits banked for use in the next period. The last set of constraints simply encompass the no borrowing constraint for each region. We refer to this as the first best because all other policy scenarios will involve additional constraints leading to, at best, costs equal to those in this scenario.

Notice that we do not include the maximum link constraint $\Delta_1 \leq e_{H1}$ in the set of market constraints (2). Because markets are linked in the second period, there is no need to try to move allowances between regions in the first period in order to bank them for compliance in the second period—one can simply move allowances between regions in the second period. So without loss of generality we can assume that the maximum link constraint will always be satisfied under no delinking.\(^{10}\)

Now consider maximizing the objective function (1) subject to the market constraints described by (2). We can characterize the solution to this problem with the first-order conditions. Let $p_{rt}$ be the Lagrange multiplier on the region $r$ and period $t$ market equilibrium constraint and let $\lambda_r$ be the Lagrange multiplier on the region $r$ no borrowing constraint. Then the first-order conditions can be written as

\begin{align*}
\text{FOC for } e_{rt}: & \quad -C'_{rt} = p_{rt}^* \\
\text{FOC for } \Delta_t: & \quad p_{Ht}^* = p_{Lt}^* \equiv p_t^* \\
\text{FOC for } B_r: & \quad \lambda_r^* = p_{r1}^* - p_{r2}^* \equiv p_1^* - p_2^* \equiv \lambda_t^* \quad \text{(Rubin-Schennach banking equation)}
\end{align*}

where we have used superscript $\ell^*$ to indicate the solution to the no delinking problem.

\(^{10}\)Suppose that the optimal solution to the no delinking problem called for $\Delta_1$ to be greater than $e_{H1}$. Then let $k = \Delta_1 - e_{H1}$. Decrease $\Delta_1$ by $k$, decrease $B_H$ by $k$, increase $B_L$ by $k$, and increase $\Delta_2$ by $k$. Then we have a new feasible solution that satisfies the maximum link constraint, and in addition, has the exact same values for $e_{rt}$ as the optimal solution. So it is optimal as well. Note the implication that $B_r$ and $\Delta_t$ are not necessarily uniquely identified at the optimum.
Because the markets are linked in both periods, the solution collapses to the standard result for a permit market with banking (Rubin 1996, Schennach 2000). There is a single price in both regions, and that price is either (a) the same across periods or (b) falling from period 1 to 2. If prices are the same in both periods, then, relative to the initial endowments, we undertake additional emission reduction in the first period, bank some permits, and then emit higher amounts in the second period. If prices are higher in the first period, then we would like to generate additional emissions in the first period, borrow permits from the second period, and then reduce emissions in the second period. But this would violate the no borrowing constraint. When there is a price difference between the two periods, the discrepancy \( \lambda_r^* = \lambda_r^* = p_r^1 - p_r^2 \) reflects the hypothetical cost savings from shifting one ton of emissions from second period to the first, if it were allowed.

Another point to notice is that linking in the second period may be unnecessary to generate equal second period prices. If linking in the first period can equate \( p_{1H} = p_{1L} \) and banking in both regions can equate \( p_{1r} = p_{2r} \), then transitivity yields \( p_{2H} = p_{2L} \). This suggests that even with delinked second-period markets, it may be possible to achieve the same cost minimization achieved with “no delinking.” This leads right into a discussion of delinking.

### 3.2 Asymmetric Delinking

In our delinking cases, the permit markets are linked in the first period but not in the second. We need not be concerned at this point about why this occurs, because we simply want to study the effects of delinking on prices and abatement costs. To close the model, we need to specify how allowances that are banked at the end of period 1 are treated. The first policy we consider is called asymmetric delinking. As we discuss below, this is perhaps the most natural “default” policy that would govern delinked markets. Here banked region \( H \) allowances are only valid in region \( H \) and banked region \( L \) allowances are only valid in region
Under this policy, the market constraints are now:

\[ e_{H1} - \Delta_1 + B_H = w_{H1} \]
\[ e_{L1} + \Delta_1 + B_L = w_{L1} \]
\[ e_{H2} = w_{H2} + B_H \]
\[ e_{L2} = w_{L2} + B_L \]
\[ B_r \geq 0 \]
\[ \Delta_1 \leq e_{H1} \]

Because markets are not linked in the second period, \( \Delta_2 \) is constrained to be zero and has been dropped from all the equations. As we previously noted and shall see below, that does not mean that second period prices cannot be equalized—the remaining three flexibility variables, \( B_H, B_L, \) and \( \Delta_1 \), can be sufficient if the inequality constraints are not binding. Speaking of which, we now include the maximum link constraint \( \Delta_1 \leq e_{H1} \). Because markets are delinked in the second period, it is possible that the markets may now want to move more allowances from \( L \) to \( H \) in the first period in order to bank more \( H \) permits for compliance in the second period. This is limited by the maximum link constraint.

Denoting the Lagrange multiplier on the maximum link constraint by \( \gamma \), the first-order conditions can be written as

\[ \text{FOC for } e_{rt \neq H1}: \quad -C'_{rt} = p^a_{rt} \]

\[ \text{(MAC equals price except maybe } H1) \]

\[ \text{FOC for } e_{H1}: \quad -C'_{H1} = p^a_{H1} - \gamma^a \]

\[ (4) \]

\[ \text{FOC for } \Delta_1: \quad p^a_{H1} - \gamma^a = p^a_{L1} \]

\[ \text{(Linking equates } H \text{ and } L \text{ MAC in period 1 but perhaps not prices)} \]

\[ ^{11} \text{It has not escaped our notice that the single example of delinking that we cite did not involve this policy. However, Governor Christie’s announcement was accompanied by an almost immediate announcement by the RGGI authority that symmetric delinking would occur, which we discuss next. This suggests that RGGI authorities may well have worried that absent such an announcement, market participants would have assumed asymmetric treatment.} \]
FOC for $B_r$: \[ \lambda_r^{a*} = p_r^{a*} - p_r^{a*} \] (Rubin-Schennach banking equation)

where superscript $a^*$ indicates the solution to the asymmetric delinking problem. These look similar to the first-order conditions for the no delinking case, except there is no longer an equation equating regional prices in period 2, and $\gamma$ appears in the first-order condition for $e_{H1}$ and $\Delta_1$.

As noted earlier, this asymmetric delinking policy can at best achieve the minimum costs associated with the no delinking case. Relative to the first-best, no delinking optimization problem, the asymmetric delinked optimization problem has the added constraints $\Delta_2 = 0$ and $\Delta_1 \leq e_{H1}$. If these constraints can be satisfied with same the emission solution to the no delinking optimization problem, then costs associated with no delinking and asymmetric delinking are indeed the same. If the constraints cannot satisfied while maintaining the emissions associated with the solution to the no delinking optimization problem, then costs will be higher in the asymmetric delinked case.

We would now like to describe conditions under which the asymmetric delinking optimization problem does and does not obtain the first-best outcome. Equivalently, we want to analyze conditions where the constraints $\Delta_2 = 0$ and $\Delta_1 \leq e_{H1}$ are, and are not, satisfied by the first-best outcome. At first glance, it may appear to be quite unlikely that equality constraint $\Delta_2 = 0$ would hold except in special "knife-edge" cases. But, due to the redundancy of $\Delta_2$ described earlier, it is often possible to obtain the first-best outcome by fixing $\Delta_2 = 0$ and adjusting the other flexibility variables. For this reason, it is a bit cleaner to defined the desired conditions indirectly with respect to marginal abatement costs and prices, rather than directly using the constraints on $\Delta_1$ and $\Delta_2$.

In particular, we compare the initial conditions of the model—the marginal abatement costs evaluated at the permit endowments—with the first-best permit prices. Accordingly, the marginal costs in period 2 at the permit endowment are $C'_{r2}(w_{r2})$ and the first-best period 2 permit price is $p_2^{b*}$. Now, is the former higher or lower than the later? That is, relative to
first-best, are we starting at a point in period 2 where we need to move permits into, or out of, each region?

There are three relevant cases.

Case 1. \( C_{r2}'(w_{r2}) < p_2^* \) for one region. In this case, one region has an initial marginal cost in period 2 that is below the first-best outcome. Here, we want to move permits out of this region, lower emissions, and raise hence marginal abatement costs. But this is impossible without linking in the second period that can move permits from one region to the other; banking can only increase second-period permit supply and raise emissions in both regions. So the asymmetric delinking outcome has higher costs than the first-best outcome.

Case 2. \( C_{r2}'(w_{r2}) > p_2^* \) for both regions, but \( C_{H2}'(w_{H2} + w_{H1}) > p_2^* \). Here, we want to move permits into both regions in period 2 to lower marginal abatement costs, which is generally doable with banking. However, permit demand in the second period for region \( H \) is such that, even when all of the region \( H \) permits from both periods are used in period 2, marginal costs are still above the first-best equilibrium. In order to achieve the first-best, some region \( L \) permits need to be usable for compliance in second-period region \( H \). This cannot be done directly, because markets are delinked in period 2. It cannot be done indirectly, because of the maximum link constraint. It follows that asymmetric delinking cannot obtain the first-best outcome and hence has higher costs than first-best.

Case 3. \( C_{r2}'(w_{r2}) > p_2^* \) for both regions, and \( C_{H2}'(w_{H2} + w_{H1}) < p_2^* \). Again, we want to move permits into both regions in period 2 to lower marginal abatement costs. In this case, marginal costs can be sufficiently lowered solely through banking, and so the asymmetric delinking outcome has the same costs as the first-best outcome.

Among these three cases, case 2 highlights an interesting outcome in the asymmetric delinking system. First, unquenchable demand for \( H \) permits in period 2 drives up their price and leads all of them to be banked; compliance in both regions in period 1 is achieved using \( L \) permits that establish the marginal cost in both regions. That is, prices diverge in period 1, but marginal costs do not. (In the first-order conditions for the asymmetric
delinking problem, if \( \gamma^{a*} \) is positive, then the first period prices are not the same.) Second, the problem (in terms of increased costs) arises from the need to bank more permits into one of the regions than the rules allow. This contrasts with Case 1, where the higher costs associated with asymmetric delinking arise from the inability to borrow permits from a second-period region with low marginal costs.\(^{12}\)

These observations highlight the potential fear of delinking: current prices may diverge and costs rise. For policymakers, divergent prices may create problems as holders of different permits experience opposing gains and losses, while the high-price region experiences a significant influx of foreign permits. This points to the question: Are there alternatives to asymmetric delinking that would avoid price divergence and possibly lower costs? This leads directly to our idea for symmetric delinking.

### 3.3 Symmetric Delinking

Instead of each region’s banked permits reverting to their region of origin, suppose they are instead treated symmetrically. We specify that region \( H \) gets \( \pi (B_H + B_L) \) permits and region \( L \) gets \( (1 - \pi)(B_H + B_L) \) permits, where \( 0 \leq \pi \leq 1 \). It does not matter whether permits are banked in region \( L \) or \( H \); they are treated symmetrically. And, by specifying that the total emissions available in period two equals \( \pi (B_H + B_L) + (1 - \pi)(B_H + B_L) = B_H + B_L \), we preserve the aggregate emission level across time and region. By similar logic as in the case of no delinking, we can assume, without loss of generality, that the maximum link constraint will always be satisfied under symmetric delinking.\(^{13}\)

\(^{12}\)It is also useful to point out that most emission trading programs to date have encouraged considerable banking, making Case 1 seem less likely. The EU ETS, for example, currently holds a bank equal to roughly one year’s worth of compliance needs.

\(^{13}\)Suppose that the optimal solution to the symmetric delinking problem called for \( \Delta_1 \) to be greater than \( e_{H1} \). Then let \( k = \Delta_1 - e_{H1} \). Decrease \( \Delta_1 \) by \( k \), decrease \( B_H \) by \( k \), increase \( B_L \) by \( k \). Then we have a new feasible solution that satisfies the maximum link constraint, and in addition, has the exact same values for \( e_{rt} \) as the optimal solution. So it is optimal as well.
Under these provisions, the market constraints become

\[ e_{H1} - \Delta_1 + B_H = w_{H1} \]
\[ e_{L1} + \Delta_1 + B_L = w_{L1} \]

\[ e_{H2} = w_{H2} + \pi(B_H + B_L) \]
\[ e_{L2} = w_{L2} + (1 - \pi)(B_H + B_L) \]

\[ B_r \geq 0. \]

The corresponding first-order conditions are

\[ \text{FOC for } e_{rt}: \quad -C_r' = p_{rt}^* \quad \text{(MAC equals price)} \] (6)

\[ \text{FOC for } \Delta_1: \quad p_{H1}^{s*} = p_{L1}^{s*} = p_1^{s*} \quad \text{(Linking equates } H \text{ and } L \text{ prices in period 1)} \]

\[ \text{FOC for } B_r: \quad \lambda_r^{s*} = p_{r1}^{s*} - \pi p_{H2}^{s*} - (1 - \pi)p_{L2}^{s*} = \lambda^{s*} \quad \text{(Banking under symmetric delinking)} \]

where superscript \( s^* \) indicates the solution to the symmetric delinking problem. The first feature of these first-order conditions to note is that first-period prices are equated. Thus symmetric delinking addresses one concern with asymmetric delinking.

Also notice that the banking equation is slightly different from the first-best banking equation. But \( \pi \) is a free parameter, and so by picking its value correctly, we may be able to obtain the the first-best outcome in some cases. Moreover, even if we can’t obtain the first-best, perhaps we can select \( \pi \) such that symmetric delinking leads to costs that are at least as low as asymmetric delinking. These conjectures are indeed correct, as shown in the following proposition:

\[ \text{Proposition 1. Suppose that } C_{r2}'(w_{r2}) > p_2^{f*} \text{ for both regions and let } \pi = \frac{e_{H2}^* - w_{H2}}{e_{H2}^* - w_{H2} + e_{L2}^* - w_{L2}}. \]

Then the solution under symmetric delinking achieves the first-best outcome. Now suppose that \( C_{r2}'(w_{r2}) < p_2^{f*} \) for one region and let \( \pi = \frac{B_H^{s*}}{B_L^{s*}}. \) Then the solution under symmetric delinking has costs no higher than asymmetric delinking.

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The intuition for this result is as follows. Under case 2 and case 3, relative to the initial endowment, the first-best solution requires permits to flow into both regions in period 2. Because the maximum link constraint is always satisfied with symmetric delinking, we can mimic the first-best solution with the symmetric delinking solution. Under case 1, however, the first-best solution requires permits to flow out of one region in period 2. This cannot be done with symmetric delinking. But we can, however, mimic the outcome of asymmetric delinking in this case.

In summary, we see that asymmetric delinking obtains the first-best outcome only in case 3, but symmetric delinking obtains the first-best outcome in cases 2 and 3. In case 1, symmetric delinking is at least as good as asymmetric delinking.

3.4 Uncertainty About Delinking

Up to now, we have assumed that market participants know whether or not the markets will be linked in the second period and act accordingly. It is more realistic to assume that there may be uncertainty about the second period link. The results in this case can be obtained by building on our previous results without uncertainty, as explained in the following proposition:

**Proposition 2.** Let $\phi$ be the subjective probability, as viewed by the market in period 1, that markets are delinked in the second period. If the market assumes asymmetric delinking will occur, the first-order conditions in period one are given by those in (4) with $p_{rv2}^a$ replaced by $E[p_{rv2}^a]$. If the market assumes symmetric delinking will occur, the first-order conditions in period one are given by those in (6) with $p_{rv2}^s$ replaced by $E[p_{rv2}^s]$.

These first-order conditions reflect the simple intuition that uncertainty or speculation about delinking will lead to a solution where expected second-period prices replace the otherwise certain delinked second-period price, but otherwise keep the same form. Importantly, mere speculation can cause divergence from the first-best, no delinking outcome, and the
form of divergence will also depend on the assumed policy regarding banked allowances; that is, whether they are treated asymmetrically or symmetrically. An assumed policy of asymmetric delinking will lead first-period prices to diverge from one another, as well as from the first-best level, in Cases 1 and 2. In contrast, an assumed policy of symmetric delinking will always maintain parity between the prices of each region’s permits. This leads us to our final proposition, really a corollary of Proposition 1:

**Corollary 1.** Let $\phi$ be the subjective probability, as viewed by the market in period 1, that markets are delinked in the second period. If (a) the market assumes symmetric delinking will occur, (b) $C_{r2}'(w_{r2}) > p_{r2}^*$ for both regions, and (c) the market assumes $\pi = \frac{e_{H2}^{r*} - w_{H2}}{e_{H2}^{r*} - w_{H2} + e_{L2}^{r*} - w_{L2}}$, then first-period prices are invariant to $\phi$.

Thus we see that it is possible that speculation about delinking will not affect first period prices at all, thereby preserving the first-best outcome. This occurs if markets believe that banked allowances will be treated symmetrically and that, in designing the symmetric treatment, governments will seek to maintain prices. Notice for this to happen we must have $C_{r2}'(w_{r2}) > p_{r2}^*$ for both regions, which implies banking must be occurring in both regions (which is typically the case for most observed programs).

### 3.5 Numerical Example: Australia and the European Union

To illustrate our results we present a simple numerical example based on the proposed linking of the Australian and European Union trading systems. Imagine we are in the year 2018 and the systems are fully linked. Some divergence of views emerge—over offsets, allocation, or some other design issue—and government officials are contemplating delinking the programs in 2020.

We parameterize the model so that period 1 can be thought of as the years 2019, when the programs are clearly linked, and period 2 can be thought of as 2020-2027, a later and much larger (8x) period where they might or might not be linked. In a world where the systems
remain linked, we imagine current and future allowance prices equalling $15 per ton. At $15 per ton, equilibrium period 1 demand for allowances is 300 million tons in Australia ($e^*_{AU,1}$) and 1,900 million tons in the EU ($e^*_{EU,1}$); period 2 demand is 8 times period 1 demand (2,400 and 15,200 million tons, respectively, for $e^*_{AU,2}$ and $e^*_{EU,2}$).

We assume that an allocation that is generally 100 million tons short for Australia and long for the EU each year, so that there is a net permit flow from the EU to Australia, as 2013 prices suggest. On top of this, we also assume period 1 is 1,000 million tons long—all in the EU—and period 2 is 1,000 million tons short, assuring active banking from period 1 to period 2. This amounts to $w_{AU,1} = 200, w_{EU,1} = 3,000, w_{AU,2} = 1,600$, and $w_{EU,2} = 15,000$.

This arrangement—a small country linking with a large country, the small country being the net buyer, and the large partner having a large excess allocation in period 1—are sufficient ingredients for “case 2” in our taxonomy of asymmetric delinking cases, where the maximum link constraint is binding, and symmetric delinking can reduce costs. To see this, note that in the linked equilibrium Australia needs 800 million permits in period 2, compared to the EU who needs 200 million tons. There are 1000 banked allowances flowing in to meet demand—but no more than 200 million Australian permits, because that is the extent of Australia’s endowment in period 1. This is all fine when the programs are linked in period 2, but becomes a constraint with asymmetric delinking.

Note that this parameterization fits the two scenarios we have seen to date: Both Quebec and Australia are small and have higher prices than their larger linking partner. In the case of the EU, there is also a large initial bank equal to over a billion tons. While it is too early to know whether there will be a similarly bank in California, previous US programs—such as the acid rain trading program from SO$_2$—did.

In order to see what happens with asymmetric and symmetric delinking, we first assume abatement supply schedule in each region is defined by a constant elasticity of supply around the linked equilibrium described above,
\[-C'_r = \$15(e_{rt}/e_{rt}^*)^{\frac{1}{\varepsilon_r}},\]

where \(\varepsilon_r\) is the elasticity in region \(r\). We choose \(\varepsilon_r\) such that the EU price equals \$10 and the Australian price equals \$30 when the 100 million trade balance each year is unsatisfied \((e_{AU,1} = (1/8)e_{AU,2} = 200 \text{ and } e_{EU,1} = (1/8)e_{EU,2} = 2000)\). This leads to \(\varepsilon_{AU} = 0.585\) and \(\varepsilon_{EU} = 0.127\).

We can now simulate delinking. Table 1 shows the results for asymmetric delinking and the results for symmetric delinking for various values of \(\pi\). As we would expect with case 2, the asymmetric delinking solution results in high period 2 prices in Australia. Also notice the price dispersion in the first period. All of the first period Australian permits are banked for use in the second period and first period compliance in both regions is completely covered by EU permits.

Moving to symmetric delinking, we see that for any value of \(\pi\) the first period price dispersion is eliminated; all allowances are treated and priced the same until the systems are delinked. Also notice that the variability over time and regions is reduced compared to the asymmetric case, most clearly so long as \(\pi > 0.2\). For these reasons, symmetric delinking can lower cost. For the value of \(\pi = 0.8\), the symmetric delinking solution attains first-best as shown in Proposition 1.

Next consider the effects of uncertainty about delinking. For symmetric delinking, with \(\pi = 0.8\), Corollary 1 implies that the first period price is invariant with respect to the probability of delinking. This is not the case, of course, for asymmetric delinking, as shown in Table 2. If the probability of delinking is zero, then we get the first-best solution. As the probability of delinking increases, the total amount of banked EU permits decreases. For Australia, the maximum link constraint binds for any non-zero probability. As the probability of delinking approaches zero, the first period prices approach the first-best solution. The first price dispersion increases in the probability of delinking.
Table 1: Australia and the EU: Symmetric and Asymmetric Delinking

<table>
<thead>
<tr>
<th>Asymmetric</th>
<th>$\pi$</th>
<th>$p_{AU,1}$</th>
<th>$p_{EU,1}$</th>
<th>$p_{AU,2}$</th>
<th>$p_{EU,2}$</th>
<th>$B_{AU}$</th>
<th>$B_{EU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asymmetric</td>
<td></td>
<td>24.53</td>
<td>11.67</td>
<td>24.53</td>
<td>11.67</td>
<td>200</td>
<td>691</td>
</tr>
<tr>
<td>Symmetric</td>
<td>1</td>
<td>13.48</td>
<td>13.48</td>
<td>13.48</td>
<td>16.65</td>
<td>955</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>15.74</td>
<td>15.74</td>
<td>17.25</td>
<td>13.47</td>
<td>1020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>15.46</td>
<td>15.46</td>
<td>20.40</td>
<td>12.17</td>
<td>1012</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>13.89</td>
<td>13.89</td>
<td>24.69</td>
<td>11.19</td>
<td>967</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>10.75</td>
<td>10.75</td>
<td>30.00</td>
<td>10.75</td>
<td>853</td>
<td></td>
</tr>
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</table>

Table 2: Asymmetric Delinking: Uncertainty about Delinking

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$p_{EU,1}$</th>
<th>$p_{AU,1}$</th>
<th>$B_{AU}$</th>
<th>$B_{EU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.67</td>
<td>24.53</td>
<td>200</td>
<td>691</td>
</tr>
<tr>
<td>0.8</td>
<td>12.30</td>
<td>22.70</td>
<td>200</td>
<td>714</td>
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<tr>
<td>0.6</td>
<td>12.95</td>
<td>20.83</td>
<td>200</td>
<td>737</td>
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<td>0.4</td>
<td>13.62</td>
<td>18.92</td>
<td>200</td>
<td>759</td>
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<tr>
<td>0.2</td>
<td>14.30</td>
<td>17.00</td>
<td>200</td>
<td>780</td>
</tr>
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<td>0</td>
<td>15</td>
<td>15</td>
<td>0</td>
<td>1000</td>
</tr>
</tbody>
</table>
4 Discussion and extensions

So far, the discussion has focused almost exclusively on cost-effectiveness. However, a decision to delink to trading systems, by its nature, is a decision against cost-effectiveness. For that reason, it may be unwise to focus exclusively on the cost savings associated with different delinking policies. This brings us back to our initial discussion of some of the other issues on the minds of policymakers making a decision to link or delink. Most of the reasons for unlinked systems relate to a desire for different features, particularly different prices or levels of ambition, that are incompatible practically and/or politically within a linked system. If two linked regions evolve to a position where they really want different prices, it makes no sense to emphasize a delinking policy that lower costs by equalizing future prices.

As we have seen, however, the decision to delink (or speculation about delinking) could lead current permit prices to diverge across regions before delinking occurs—perhaps long before. This creates additional and unnecessary transaction costs as firms with current compliance obligations try to acquire cheaper permits while the more expensive permits have to migrate to those firms with the resources to invest and hold onto them until period 2. While the divergence of prices across regions (and implicitly higher costs) after delinking may be necessary for the political reasons discussed above, along with some price shift over time, the transaction costs associated with divergent regional permit prices before delinking are not.

This suggests it may be useful to design a delinking policy that avoids price divergence prior to delinking but allows prices to diverge in a flexible way after delinking. To do this, we could issue specific banking rights into each region rather than attaching them entirely to the current permits themselves. To date, the design of banking provisions have assumed that the holder of any current permit is free to hold that permit indefinitely for future use rather than using it for compliance right now. Our previous suggestion for symmetric treatment merely specified how that future use might be redefined.

Instead of allowing any permit holder to freely decide whether to use or bank their
permit, suppose we issued $M_H$ distinct rights to bank into region $H$ and $M_L$ rights to bank into region $L$—rights distinct from the emission permits themselves. To bank into region $r$, a firm is required to hold one of these banking rights for region $r$ along with a period one permit from either region. The market constraints for this situation are:

\[
\begin{align*}
    e_{H1} - \Delta_1 + B_{H1} &= w_{H1} \\
    e_{L1} + \Delta_1 + B_{L1} &= w_{L1} \\
    B_{H2} + B_{L2} &= B_{H1} + B_{L1} \\
    B_{H2} &\leq M_H \\
    B_{L2} &\leq M_L \\
    e_{H2} &= w_{H2} + B_{H2} \\
    e_{L2} &= w_{L2} + B_{L2} \\
    B_{r1} &\geq 0
\end{align*}
\]

with corresponding first-order conditions:

\[
\begin{align*}
    \text{FOC for } e_{rt}: & \quad -C'_r = p_{rt}^{m*} \quad \text{(MAC equals price)} \\
    \text{FOC for } \Delta_1: & \quad p_{H1}^{m*} = p_{L1}^{m*} = p_1^{m*} \quad \text{(Linking equates period 1 prices)} \\
    \text{FOC for } B_{r1}: & \quad \lambda_r^{m*} = \lambda^{m*} = p_1^{m*} - p_{2,\text{base}}^{m*} \quad \text{(Rubin-Schennach banking equation)} \\
    \text{FOC for } B_{r2}: & \quad p_{r2}^{m*} = p_r^{m*} + p_{2,\text{base}}^{m*} \quad \text{(Flexible period 2 prices)}
\end{align*}
\]

where the superscript $m^*$ indicates the solution to the problem with the $M_r$ constraints, $p_{2,\text{base}}$ is the Lagrange multiplier on the constraint equating total (across region) banking from period 1 with total banking into period 2, and $\mu_r^{m*}$ is the Lagrange multiplier on the new banking limits established by $M_r$.

Like the symmetric delink policy, there is no distinction between regional permits in period 1: They are both usable for compliance in each region and identically bankable (with
the same FOC for $\Delta_1$). In symmetric delink case, however, the fractional split of banked allowance into region $H$ and $L$ determines one of the three prices ($p^{*s}_1$, $p^{*s}_{2H}$, $p^{*s}_{2L}$) given the other two (based on the last FOC in (10)) when banking is active.

In this case, $M_L$ and $M_H$ can be used to precisely control emissions and prices in both regions in period 2. The Lagrange multipliers on the two associated constraints allow $p^{*m*}_{H2}$ and $p^{*m*}_{L2}$ to be the same or different from $p^{*m*}_1$ in a flexible way (so long as $p^{*m*}_{r2} \geq p^{*m*}_1$). Higher period two prices in either region can be achieved by reducing the relevant $M_r$. Lower period two prices in either region can be achieve by raising the relevant $M_r$ (up to the point where prices are equalized with period one). At that point, the constraint will become slack and it is impossible to shift any more permits into that region.

This delink policy is more complex than the previous ones and requires additional decisions about how to allocate banking rights. However, an initial announcement that all permits would be treated the same, regardless of origin, would be consistent with this eventual outcome should it arise.

A final issue is whether and how addressing the issue of delinking encourages speculation that it will occur (that is, the perception of $\phi$ rises with any delinking provision). This is analogous to the perception that a prenuptial agreement prior to marriage signals uncertainty about marriage.\(^{14}\) There are, of course, differences: The consequences of uncertainty in marriage rest almost entirely with the decision-makers; the consequences of uncertainty in linking rest significantly with market participants. Perhaps more importantly, the motivation in a prenuptial agreement tends to be differentiation of assets; the suggested delinking provision for trading programs would emphasize equal treatment. In fact, such a provision might not even mention delinking explicitly, and instead refer to “any future policy changes.” In many ways, this creates a situation that would be quite similar to a permit system that did not distinguish the region of origin at all.\(^{15}\)

\(^{14}\)See Mahar (2003).
\(^{15}\)We did not explicitly discuss a scenario where period one permits were indistinguishable. This might be the case if Iceland decided to separate its emission regulation from the EU ETS. Iceland is not a member of the EU but gave the European Commission the authority to regulate its domestic emissions under the EU
5 Conclusions

Policymakers must decide whether, when, and how to address the possibility of delinking when they decide to create a compliance link between two emission trading programs. Absent any announcement or provision, we believe market participants would likely assume a decision to delink would lead to banked allowances that are valid only in their region of origin (at least in linked systems with distinct registries and tracking systems). This assumption, in turn, would lead to a divergence in prices if speculation about delinking emerged. An easy solution to this dilemma is to announce that any policy changes would treat circulating permits in both programs in the same way, and to do so before speculation emerges. This might or might not prevent current prices from changing from what they would be absent speculation, but it would certainly prevent divergence between prices of regional permit already in circulation.

One motivation for such a policy would be to reduce costs. A delinking policy that treats all circulating permits in the same way could be designed to maintain the original, fully-linked outcome in most cases. This would minimize the total costs in a delinked system. However, it well may be that costs alone are not the only criteria when delinking is pursued. A quite flexible array of alternative outcomes could be managed while continuing to treat circulating permits in the same way. The main advantage of announcing such treatment may be that it prevents the market turmoil associated with a divergence in prices among circulating permits, were speculation about delinking to arise before the issue could be addressed.

It may be interesting to analyze a policymaker’s decision to address delinking in a signaling model. Does reference to delinking, explicitly or implicitly, create additional uncertainty about the link? This would lead to a necessary trade off between the benefit of reduced consequences from delinking speculation on the one hand, and the cost of increased speculation on the other. In any case, more open discussion and analysis of these issues is likely warranted.

ETS. In this scenario, the option for asymmetric delinking is ruled out.
Appendix

Proof of Proposition 1.

For the first part of the proposition, we are assuming that $C_{r_2}(w_{r_2}) > p_2^*$ for both regions. We construct a feasible solution to the symmetric delinking problem from the first-best solution $e^*_{r_1}, \Delta^*_t$, and $B^*_{r}$ as follows. Let

$$B_r = e^*_{r_2} - w_{r_2}. \quad (7)$$

Each $B_r$ is strictly positive (by assumption, $C_{r_2}(w_{r_2}) > p_2^*$ and so in the first-best solution, emissions in both regions in period 2 are greater than the permit endowment). Also let

$$\Delta_1 = e^*_{H_1} - (w_{H_1} - B)_H. \quad (8)$$

With these definitions in hand, we verify the market constraints for the symmetric delinking problem are satisfied.

Start with the second period. We have

$$e_{H_2} = w_{H_2} + \pi(B_H + B_L) = w_{H_2} + e^*_{H_2} - w_{H_2} = e^*_{H_2}$$

where the second equal sign follows from (7) and the definition of $\pi$ in the proposition statement. By a similar logic we have

$$e_{L_2} = w_{L_2} + (1 - \pi)(B_H + B_L) = e^*_{L_2}.$$

Thus the second period emissions are the same as the first-best emissions.

Turning to the first period, we have

$$e_{H_1} = w_{H_1} + \Delta_1 - B_H = w_{H_1} + e^*_{H_1} - (w_{H_1} - B_H) - B_H = e^*_{H_1}.$$
where the second equality follows from (8). Likewise, we have

$$e_{L1} = w_{L1} - \Delta_1 - B_L = w_{L1} - (e_{H1}^* - (w_{H1} - B_H)) - B_L = w_{L1} + w_{H1} + w_{H2} + w_{L2} - e_{H1}^* - e_{H2}^* - e_{L2}^*.$$ 

Now, summing the four first-best equality market constraints at the optimal solution gives

$$w_{L1} + w_{H1} + w_{H2} + w_{L2} - e_{H1}^* - e_{L1}^* - e_{H2}^* - e_{L2}^*.$$ 

It follows from the previous two equations that

$$e_{L1} = e_{L1}^*.$$ 

Thus we have described a feasible symmetric delinking solution, and this solution gives the same emissions, and therefore costs, as the first-best solution.

For the second part of the proposition, we are assuming $C_{r2}^r(w r_2) < p_{L1}^*$ for one region. Start with the optimal asymmetric delinking solution $e_{rt}^{a*}, \Delta_1^{a*}$ and $B_r^{a*}$. Now let

$$e_{rt} = e_{rt}^{a*}, \Delta_1 = \Delta_1^{a*}, \text{ and } B_r = B_r^{a*}.$$ 

By construction, when we substitute the values for these variables into the market constraints for the symmetric delinking problem (with $\pi$ defined as in the statement of the proposition) we get the market constraints for the asymmetric delinking solution, evaluated at the optimal asymmetric delinking solution. For example, consider a market constraint for the symmetric delinking problem:

$$e_{H2} = w_{H2} + \pi (B_H + B_L).$$ 

Using (9) gives

$$e_{H2}^{a*} = w_{H2} + \frac{B_H^{a*}}{B_H^{a*} + B_L^{a*}} (B_H^{a*} + B_L^{a*}) = w_{H2} + B_H^{a*}.$$ 

This equation holds by definition of the asymmetric delinking problem, so it follows that
values for the variables specified in (9) satisfy the symmetric delinking market constraint. So the optimal solution for the asymmetric delinking problems is feasible for the symmetric delinking problem. Therefore, the optimal symmetric delink solution will have costs at or below costs with asymmetric delinking.

Proof of Proposition 2.

With uncertainty about delinking resolved in the second period, we can no longer solve the optimization problem (1) all at once; we have to use dynamic programming and solve the model backwards from the second period. In the second period, given their banked permits, facilities in the two regions decide on second period emissions and buy and sell permits in the appropriate market. In the first period, facilities in the two regions decide on first period emissions, buy permits, and decide on the amount of permits to bank for the second period. When making these first period decisions, they take into account the probability that the systems will be linked in the second period and the resulting expected second period costs.

We characterize the second-period solutions by defining a value function for each possible outcome in terms of the banking level in each region. In particular, for the event that the systems remained linked we define,

\[
V^*_2(B_H + B_L) = \min_{e_H, e_L, \Delta_2} C_H(e_H) + C_L(e_L)
\]

such that

\[
e_H - \Delta_2 = w_H + B_H
\]

\[
e_L + \Delta_2 = w_L + B_L
\]

For the event that the systems face an asymmetric delink we define,
\[ V^a_2(B_H, B_L) = \min_{e_{H2},e_{L2}} C_{H2}(e_{H2}) + C_{L2}(e_{L2}) \]

such that

\[ e_{H2} = w_{H2} + B_H \]
\[ e_{L2} = w_{L2} + B_L. \]

And, for the event that the systems face a symmetric delink we define,

\[ V^s_2(B_H, B_L) = \min_{e_{H2},e_{L2}} C_{H2}(e_{H2}) + C_{L2}(e_{L2}) \]

such that

\[ e_{H2} = w_{H2} + \pi(B_H + B_L) \]
\[ e_{L2} = w_{L2} + (1 - \pi)(B_H + B_L). \]

We define \( E[V(B_L, B_H)] = \phi V^d(B_L, B_H) + (1 - \phi) V^t(B_L, B_H) \) and \( d \in \{a, s\} \) depending on whether asymmetric or symmetric delinking is assumed. Note that

\[
\begin{align*}
\frac{\partial E[V]}{\partial B_r} &= \phi \frac{\partial V^d}{\partial B_r} + (1 - \phi) \frac{\partial V^t}{\partial B_r} \\
\frac{\partial V^a}{\partial B_r} &= p_r^a \\
\frac{\partial V^s}{\partial B_r} &= \pi p^s_{H2} + (1 - \pi) p^s_{L2} \\
\frac{\partial V^t}{\partial B_r} &= p_r^t = p^t_2 = \pi p_2 + (1 - \pi) p_2
\end{align*}
\]

where, as before, \( p_{r2} \) are the Lagrange multipliers on the permit market constraints in period 2 with superscript indicating the linking or delinking policy. The last result also makes use
of the first-order condition that $p^{f}_{H2} = p^{f}_{L2}$ arising from $\Delta_2$.

From these, we can see that

$$\frac{\partial E[V]}{\partial B_r} = \phi p^o_{r2} + (1 - \phi)p^f_{r2} = E[p_{r2}]$$

when asymmetric delinking is assumed and

$$\frac{\partial E[V]}{\partial B_r} = \pi E[p_{H2}] + (1 - \pi)E[p_{L2}]$$

when symmetric delinking is assumed.

We then recast the first-period problem as

$$\min_{e_{r1,B_r}} C_{H1}(e_{H1}) + C_{L1}(e_{L1}) + E[V(B_L, B_H)]$$

such that

$$e_{H1} - \Delta_1 + B_H = w_{H1}$$
$$e_{L1} + \Delta_1 + B_L = w_{L1}$$

$$B_r \geq 0$$

$$\Delta_1 \leq e_{H1}$$

where, and where the last constraint can be ignored without loss of generality with symmetric delinking.

With asymmetric delinking, the first-order conditions are then

- **FOC for $e_{L1}$**:
  $$-C'_{L1} = p^{o*}_{L1}$$
  (MAC equals price except maybe $H1$)

- **FOC for $e_{H1}$**:
  $$-C'_{H1} = p^{o*}_{H1} - \gamma^{o*}$$

- **FOC for $\Delta_1$**:
  $$p^{o*}_{H1} - \gamma^{o*} = p^{o*}_{L1}$$
  (Linking equates H and L MAC in period 1 but perhaps not prices)
FOC for $B_r$: $\lambda^*_{r} = p_{r_1}^* - E[p_{r_2}^*]$ (Rubin-Schennach banking equation)

And, with symmetric delinking, the first-order conditions are then

FOC for $e_{r1}$: $-C'_{r_1} = p_{r_1}^*$ (MAC equals price)

FOC for $\Delta_1$: $p_{H_1}^* = p_{L_1}^* = p_1^*$ (Linking equates $H$ and $L$ prices in period 1)

FOC for $B_r$: $\lambda_{r}^* = p_{r_1}^* - \pi E[p_{H_2}^*] - (1 - \pi)E[p_{L_2}^*] = \lambda^*$ (Banking under symmetric delinking)

Proof of Corollary 1.

Start with the first-best emissions $e_{r_1}^*$. Under the conditions in the corollary, we can construct a feasible solution to the dynamic problem with uncertainty in which first period emissions are $e_{r_1}^*$ and second period emissions are $e_{r_2}^*$ whether or not markets are linked by letting $\Delta_2 = 0$ and letting $B_H, B_L$ and $\Delta_1$ be defined as in (7) and (8). This solution must be in fact optimal, for if it was not, then there would exist a solution that gave lower costs than first-best when markets were either linked or delinked, or in both cases.


