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On non-marginal benefit-cost analysis

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Abstract

Conventional benefit-cost analysis incorporates the normally reasonable assumption that the policy or project under examination is marginal. In particular, it is assumed that the policy or project does not change the underlying growth rate of the economy. However, this assumption may be inappropriate in some important circumstances, notably responding to climate change. One example is the benefit-cost analysis of global targets for carbon emissions, while another might be a large renewable energy project in a small economy, such as a hydropower dam. This paper develops some theory on the evaluation of non-marginal policies and projects, with simple empirical applications to climate change. We examine the conditions under which evaluation of a non-marginal project using marginal methods may be wrong, and in our empirical examples we show that both qualitative and large quantitative errors are plausible.

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1. Introduction

Benefit-cost analysis (BCA) of major policies, programmes and projects is becoming more widely used to inform and improve decisions (Hahn and Tetlock, 2008). In the United States and the United Kingdom, for instance, there is now a legislative requirement to conduct BCA of significant new policies and policy reforms, while other countries and regional organisations such as the European Commission have made steps in the same direction (Pearce et al., 2006). In addition, there is a long tradition of BCA of major projects by the World Bank and other multilateral financial institutions.

Conventional BCA incorporates the normally reasonable assumption that the project[1] under examination is marginal in the sense that it will not significantly change relative prices. While many projects clearly satisfy this condition, not all of them do, and indeed it is arguable that some of the most worthwhile projects are unlikely to be small in this sense (Hammond, 1990). Indeed, many projects are designed precisely to change relative prices in a non-marginal way.

A rare but important category of project not only changes relative prices, but is also large enough to shift the underlying growth rate of the relevant economy. Most notably, proposals to spend several per cent of global GDP on the deployment of "low-carbon" technologies, such as renewable energy, smart electricity grids and transport infrastructure, are explicitly intended to shift the global growth path by avoiding climate change. As part of this global infrastructure investment programme, there is likely to be a renewed impetus for large development projects in small economies, for example to generate renewable electricity, while adaptation to climate change will require similarly large projects to, for example, store freshwater and protect against coastal flooding. Such projects may also change the growth rate of the small economies in which they are developed.

In their classic text on project appraisal, Dasgupta et al. (1972) largely focus on marginal, rather than non-marginal, projects. Nevertheless they do note that different considerations may apply to large projects:

[1] Henceforth we will use the word "project" to denote any change in "business as usual", whether arising from a private-sector or government policy, programme or project.

> we tacitly assumed that...the proposed project is "small", i.e. the "range" of the net benefits of the project is small compared with the size of aggregate consumption. [Where this assumption is untrue], it might seem plain that the EPV rule will not suffice then. One would like to know what rule should replace it. One would also like to know whether the evaluator would make serious errors if he stuck to the EPV rule in such cases. (p111)

While Dasgupta et al. (1972) briefly examine whether errors might occur, they do so with a simple back-of-the-envelope calculation involving a highly specific utility function ($u(c) = -10000/c$) and a project that results in a once-off cash flow. Surprisingly – and compared with the literature on BCA in the presence only of changes in *relative* prices – it does not appear that a wider literature has developed to address their questions, even though it is not particularly difficult to think of examples where the project undertaken might have been large enough to shift the growth path of the economy (Dasgupta et al. (1972) give the Aswan dam in Egypt as a possible example of the time). This is particularly surprising given the problem of applying standard BCA in the presence of large changes was widely recognised at the time the basic theory was set out: Harberger (1971) does so in his classic paper, although like Dasgupta et al. (1972) his consideration of the issue is brief. Hammond (1990) only makes limited reference to non-marginal projects, and only considers the impact of changes to relative prices rather than the economic growth rate, while there is no treatment of non-marginal projects in recent texts in public economics, such as Myles (1995), or project appraisal, such as Mishan and Quah (2007).

In the case of global carbon emissions abatement, many analyses have ignored the possibility that investing in abatement could be non-marginal, at least in terms of how they conducted BCA. For example, Tol's (2005) review of the empirical literature shows that, of the 103 estimates of the shadow value of emissions abatement he considered, 62 ignored the possibility of a shift in the growth path, because they took a partial-equilibrium approach in which the consumption discount rate (which depends on estimated future growth) was set irrespective of the size of future cash flows and their effect on the growth rate. That is to say, these 62 cases carried out marginal analysis. Other analyses, such as those of Nordhaus (1994, 2008) and Stern (2007), did use a non-marginal approach, evaluating the project in a general-equilibrium framework in which the consumption discount rate was endogenous. However, there is currently no way of knowing whether the move from marginal to non-marginal (i.e. partial to general equilibrium) matters empirically. For instance, does it matter as much as celebrated controversies in the literature, notably over the parameters of the social discount rate? Furthermore, does it matter to project appraisal more widely, for instance as practised by multilateral institutions such as the World Bank? Standard procedure in this area is to apply a marginal analysis with an exogenous discount rate, irrespective of the size of a project's net benefits. Climate change will require an increase in the rate of energy infrastructure investment worldwide (IEA, 2009), as well as increased investment in various forms of climate resilience such as freshwater storage and flood defence (Agrawala and Fankhauser, 2008), including in small economies. We can expect project appraisal to play an important role in the design and implementation of such projects on the ground.

Hence this paper attempts to address the question of whether "serious errors" could be made by evaluating non-marginal projects with conventional BCA, which uses discounted cash flow (DCF) analysis to determine net present value (NPV). By the term "non-marginal", we mean sufficiently large for the first-order Taylor approximation of the utility function of aggregate consumption per capita not to hold (see proposition 1 below). In defining non-marginal projects this way, we are interested not in the effects of small projects on relative prices, which have been much more comprehensively explored, but on the effects of large projects on aggregate consumption. Section 2 reviews the relevant public economic theory and presents the result that if a project is evaluated to have positive NPV, then it is also welfare-improving, provided that the project is marginal. It follows that if the project is non-marginal, the result may not hold. A Taylor-series expansion provides an expression of the error involved in evaluating non-marginal projects with DCF analysis, and comparative statics, including the impact of growing population, are examined. These provide intuition for the circumstances in which DCF analysis may produce an error, especially errors of large size. Section 3 then applies this theory to climate-change mitigation, using two simple empirical examples. The first employs a well-known integrated assessment model of climate change to estimate the value of a project to reduce global carbon emissions. The second uses data from the World Bank to evaluate a large renewable energy project in a small economy, namely the "Nam Theun II" hydroelectric power project in Laos. Armed with these examples, we are able to examine numerically the sign and size of the potential error caused by evaluation of a non-marginal project using marginal analysis. We find that it is possible for marginal BCA to provide both qualitatively and quantitatively incorrect guidance, by ignoring the impacts of projects on the underlying economic growth path. Section 4 concludes.

2. Theory

2.1. Marginal BCA of a non-marginal project

A core proposition of BCA is that if DCF analysis shows that a project has positive NPV, then the project is welfare-improving (see proposition 1). Define Δ_t as the cash flows at time t from the project, and denote the consumption discount rate by ρ_t. Conventional *marginal BCA* is conducted by DCF analysis, which examines whether the sum of the discounted cash flows $\sum_{t=0}^{\tau} \Delta_t(1+\rho_t)^{-t}$ exceeds zero. In contrast, *full BCA* calculates the true NPV by examining whether discounted utility with the project exceeds discounted utility without it. Define c_t as real, aggregate business-as-usual consumption, which provides utility $u(c_t)$, with $u'(c_t) > 0$ and $u''(c_t) \leq 0$, and with corresponding utility discount rate δ_t. By focusing on real, aggregate consumption, we abstract from issues raised by price changes, which would require the use of a price index in order to generate a money metric (see e.g. Aronsson et al., 2004; Weitzman, 2001). The utility discount rate and the consumption discount rate are connected by the Euler identity $(1+\rho_t)^{-t} = \frac{u'(c_t)}{u'(c_0)}(1+\delta_t)^{-t}$ in discrete time. In continuous time with isoelastic utility this is equivalent to $\rho_t = \delta_t + \eta g_t$, where η is the elasticity of marginal utility and g_t is the growth rate in consumption (Ramsey, 1928).

Provided the project under examination is reasonably small, and the curvature of the utility function is not too large, marginal BCA is a reasonable approximation for full BCA. Proposition 1 sets out the core justification for the use of marginal BCA by DCF analysis in project appraisal.

PROPOSITION 1. *If $u(c_t + \Delta_t) = u(c_t) + u'(c_t)\Delta_t$, then*

$$\sum_{t=0}^{\tau} \Delta_t(1+\rho_t)^{-t} > 0 \Longrightarrow \sum_{t=0}^{\tau} [u(c_t + \Delta_t) - u(c_t)](1+\delta_t)^{-t} > 0 \quad (1)$$

Proof. Apply the Euler equation to substitute for the consumption discount factor $(1+\rho_t)^{-t}$ on the left-hand side of Eq. (1). Further, provided the first-order Taylor approximation of the utility function around c_t is exact, so that $u(c_t + \Delta_t) = u(c_t) + u'(c_t)\Delta_t$, it follows that:

$$\sum_{t=0}^{\tau} \Delta_t (1+\rho_t)^{-t} = \sum_{t=0}^{\tau} \Delta_t \frac{u'(c_t)}{u'(c_0)} (1+\delta_t)^{-t} = \frac{1}{u'(c_0)} \sum_{t=0}^{\tau} [u(c_t + \Delta_t) - u(c_t)](1+\delta_t)^{-t} \tag{2}$$

As $u'(c_0) > 0$, it follows from Eq. (2) that the implication in Eq. (1) holds. ■

Proposition 1 states that for marginal projects (where the first-order Taylor approximation holds), if a project has positive NPV, it is also welfare-increasing (Little and Mirrlees, 1974). What if the first-order approximation does not hold? The full Taylor series expansion of utility around consumption level c_t is:

$$u(c_t + \Delta_t) = u(c_t) + u'(c_t)\Delta_t + \Omega \tag{3}$$

where Ω is the error in the first-order approximation, which may be given by the expression for Cauchy's remainder:

$$\Omega = \sum_{j=2}^{\infty} u^j(c_t) \frac{\Delta_t^j}{j!} \tag{4}$$

For an isoelastic utility function, $u(c_t) = c_t^{1-\eta}/(1-\eta)$, with elasticity of marginal utility η, this error is:

$$\Omega = \sum_{j=2}^{\infty} \left[\prod_{i=2}^{j} (\eta + i - 2) \right] (-1)^{j+1} c_t^{1-\eta-j} \frac{\Delta_t^j}{j!} \tag{5}$$

For linear utility, $\eta = 0$, the error $\Omega = 0$ and the first-order Taylor expansion is exact. At the other extreme, as $\eta \to \infty$, it is also true that $\Omega = 0$ provided $c_t > 1$. In other words, when the elasticity of marginal utility, η, takes on values at the extreme ends of the range $[0, \infty)$, the error in using conventional BCA is likely to be limited, even for a non-marginal project.

However, when η has an intermediate value, the error involved in evaluating a non-marginal project could be substantial. Unfortunately, reasonable values of η *are* intermediate values; η is generally taken to be in $[0.5, 10]$ (Stern, 1977), and often values of $[1, 4]$ are seen as being appropriate (Atkinson, 1970; Johansson-Stenman et al., 2002). For instance, it is often convenient and not unreasonable to assume logarithmic utility, with $\eta = 1$, in public economic analysis. The review of climate-change economics by Stern (2007) did just that. Following the *Stern Review*, several economists (Weitzman, 2007; Dasgupta, 2007) argued that more suitable values of η were in the range $[2, 4]$. On the other hand, Atkinson and Brandolini (2008) point to evidence from the literature on inequality, which supports values in the range $[0.125, 2]$, and Layard et al. (2008), in analysing data on subjective happiness, put η at just over unity.

In part, the range of estimates arises because η simultaneously represents preferences for intertemporal substitution, aversion to risk, and aversion to (spatial) inequality with a utilitarian social welfare function (Atkinson et. al., 2009). Nevertheless, very few economists would argue that a central estimate for η is much below 0.5 or much above 5.

With logarithmic utility, the error in applying marginal DCF to a non-marginal project is:

$$\Omega = -\frac{1}{2}\left(\frac{\Delta_t}{c_t}\right)^2 + \frac{1}{3}\left(\frac{\Delta_t}{c_t}\right)^3 - \frac{1}{4}\left(\frac{\Delta_t}{c_t}\right)^4 + ... = \sum_{j=2}^{\infty} -\frac{\Delta_t^j}{j(-c_t)^j} \tag{6}$$

How significant could this error be? Consider a once-off, non-marginal positive cash flow at time t of Δ_t. The true increase in utility derived from this cash flow is $\log(c_t + \Delta_t) - \log(c_t)$. The first-order approximation (see Eq. (3)) is Δ_t/c_t, and the error in that approximation is given by Eq. (6). Suppose the cash flow Δ_t from the project is positive but much smaller than business-as-usual consumption, so that $0 < (\Delta_t/c_t) << 1$, but is nevertheless large enough to be non-marginal. Then the error can itself be approximated by the Lagrange remainder, which here is the same as the second-order term in Eq. (6), namely: $-\frac{1}{2}(\Delta_t/c_t)^2$. The increase in utility is therefore roughly overestimated by the fraction:

$$\frac{\frac{1}{2}(\Delta_t/c_t)^2}{\log(c_t + \Delta_t) - \log(c_t)} \approx \frac{\frac{1}{2}(\Delta_t/c_t)^2}{\Delta_t/c_t} = \frac{1}{2}\left(\frac{\Delta_t}{c_t}\right) \tag{7}$$

For instance, if a project delivers a once-off benefit (Δ_t/c_t) of 10% of current consumption, then conventional DCF analysis will overestimate the actual increase in utility by approximately 5%, simply because the marginal evaluation ignores curvature in the utility function. A 5% overestimate of benefits could make some welfare-reducing projects appear welfare-enhancing, and vice versa for a 5% underestimate. Of course, there are not many projects that involve increasing business-as-usual consumption by 10% in one year. However, for projects with moderately high cash flows over several decades or more, even annual errors of just a percentage point or two might add up to a significant overall error, and potentially an incorrect policy prescription.

2.2. Non-marginal projects and population growth

The foregoing analysis assumed constant population. Yet many large projects are conducted in economies with (sometimes rapid) population growth. As we will see, allowing for population growth with non-marginal projects can generate some counterintuitive results. Denote population at time t as n_t, and the population growth rate as g_t per period so that $n_t = n_0(1+g_t)^t$. Define a "population-augmented discount factor" $\beta_t = (1+g_t)^t(1+\delta_t)^{-t}$, where $\beta_0 = 1$, to reflect utility discounting and population growth combined. Finally, assume that individuals have identical utility functions, $u(c_t)$. The utilitarian welfare increase, denoted ΔV, generated by a project with per capita cash flows of Δ_t is given by:

$$\Delta V = n_0 \sum_{t=0}^{\tau} \beta_t \left[u(c_t + \Delta_t) - u(c_t) \right] \tag{8}$$

Let π denote the NPV per capita (in terms of consumption) corresponding to welfare increase ΔV, so that π is implicitly defined by the equation:

$$\Delta V = n_0 \left[u(c_0 + \pi) - u(c_0) \right] \tag{9}$$

Combining Eqs. (8) and (9) and incrementing the summation index implicitly defines π as follows:

$$u(c_0 + \pi) + \sum_{t=1}^{\tau} \beta_t u(c_t) = \sum_{t=0}^{\tau} \beta_t u(c_t + \Delta_t) \tag{10}$$

Assume that costs are incurred before benefits are accrued. To fix ideas, suppose there are two periods, $t = 0, 1$, where the project is represented by $\Delta_0 < 0 < \Delta_1$. In this case, π is implicitly defined by the equation

$$u(c_0 + \pi) + \beta_1 u(c_1) = u(c_0 + \Delta_0) + \beta_1 u(c_1 + \Delta_1) \tag{11}$$

For a marginal project in a growing economy (so $c_1 > c_0$), an increase in the concavity of the utility function — an increase in η for an isoelastic utility function — reduces π. This is because increasing η reduces the marginal utility of consumption in the period with high consumption ($t = 1$) relative to the period with low consumption ($t = 0$), and the benefits Δ_1 are realised in the period of high consumption. That is to say, in a simple two-period model, provided there is positive consumption growth g, the consumption discount factor falls with η, so future benefits are discounted more heavily and π is lower.[2] Conversely, if the sequence of project benefits and costs is reversed, so that $\Delta_0 > 0 > \Delta_1$,

[2] Note that we have abstracted here from questions of equity weighting and inequality aversion, discussed by Dasgupta (2007), by assuming that individuals are identical.

then an increase in η increases π. While the latter profile of benefits and costs might be unusual (being a disinvestment rather than the more standard investment project), in a multi-period setting it might not be as difficult to find projects which reduce intertemporal fluctuations in consumption, at least over a subset of time-periods.

However, the interesting result in the present case is that, if the project is non-marginal, and population is increasing, it is possible that increasing η from zero can *increase* the project's π, even in a simple two-period model with growth and where costs are incurred before benefits accrue. We observe this later in our empirical modeling, as seen in the plot of 'True NPV' in Figure 3. Proposition 2 sets out two necessary conditions for this result in a two-period investment in a growing economy.

PROPOSITION 2. *Suppose that* $c_0 \leq c_1$ *and* $\Delta_0 \leq 0 \leq \Delta_1$. *Necessary conditions for* $\partial\pi/\partial\eta > 0$ *at* $\eta = 0$ *are*

$$\begin{aligned} \beta &> 1 \text{ and} && (12) \\ \Delta_1 &> \frac{c_1 - c_0 - \Delta_0}{\beta_1 - 1} && (13) \end{aligned}$$

Proof. Implicitly differentiating Eq. (11) with respect to η, using the chain rule, and setting $\eta = 0$ yields

$$\left.\frac{\partial\pi}{\partial\eta}\right|_{\eta=0} = (c_0 + \pi)\left[f(c_0 + \pi) - f(c_0 + \Delta_0) + \beta_1\{f(c_1) - f(c_1 + \Delta_1)\}\right] \tag{14}$$

where $f(x) \equiv x \ln x$. As $(c_0 + \pi) > 0$ for any plausible project, and as $\pi = \Delta_0 + \beta_1\Delta_1$ at $\eta = 0$, $\partial\pi/\partial\eta > 0$ at $\eta = 0$ requires

$$\frac{f(c_0 + \Delta_0 + \beta_1\Delta_1) - f(c_0 + \Delta_0)}{\beta_1\Delta_1} > \frac{f(c_1 + \Delta_1) - f(c_1)}{\Delta_1} \tag{15}$$

Denote $m(x, d)$ as the gradient of the chord from $(x, f(x))$ to $(x + d, f(x + d))$, so that $m(x, d) = [f(x + d) - f(x)]/d$. We can reexpress the inequality in Eq. (15) as

$$m(c_0 + \Delta_0, \beta_1\Delta_1) > m(c_1, \Delta_1) \tag{16}$$

To derive the necessary condition in Eq. (12), note that $m_x(x, d) > 0$ and $m_d(x, d) > 0$, as $f'' > 0$, and because $m_d(x, d) > 0$, increasing β_1 increases $m(c_0 + \Delta_0, \beta_1\Delta_1)$. Note that if $\beta_1 = 1$, then for the inequality in Eq. (16) to hold true would require $m(c_0 + \Delta_0, \Delta_1) > m(c_1, \Delta_1)$, which would require $c_0 + \Delta_0 > c_1$ because $m_x(x, d) > 0$. However, $c_0 + \Delta_0 < c_1$ as by assumption the economy is growing

($c_1 > c_0$) and the project is costly ($\Delta_0 < 0$). As $\beta_1 = 1$ is inadequate and as $m_d(x,d) > 0$, it follows that $\beta_1 > 1$ is a necessary condition for Eq. (16) to hold and hence for $\partial\pi/\partial\eta > 0$ at $\eta = 0$.

To derive the necessary condition in Eq. (13), compare a chord joining the point $(x_1, f(x_1))$ to $(s, f(s))$ with a chord joining the point $(x_2, f(x_2))$ to $(s, f(s))$ where $s > x_2 > x_1$. Note that $m(x_1, s - x_1) < m(x_2, s - x_2)$ because $f'' > 0$. Applying this, now suppose $s = c_0 + \Delta_0 + \beta_1\Delta_1 = c_1 + \Delta_1$, $x_2 = c_1$ and $x_1 = c_0 + \Delta_0$. Then for the inequality in Eq. (16) to hold would require $c_0 + \Delta_0 > c_1$, because $m(x_1, s - x_1) < m(x_2, s - x_2)$. But as noted above, the opposite is true. Hence, because $m_x(x,d) > 0$, $c_0 + \Delta_0 + \beta_1\Delta_1 > c_1 + \Delta_1$ is required, which implies $\Delta_1 > \frac{c_1 - c_0 - \Delta_0}{\beta_1 - 1}$ is a necessary condition for Eq. (16) to hold and hence for $\partial\pi/\partial\eta > 0$ at $\eta = 0$. ■

The intuition behind this result may be seen in three parts. First, when population growth is fast enough that $\beta_1 > 1$, project NPV per capita, π, can be extremely high. Second, in Eq. (11) the utility with the project Δ_t (on the right-hand side) is by definition equal to the utility without the project, but where initial consumption is increased by the project NPV per capita π (on the left-hand side). If π must be large for this equality to hold, then obviously $c_0 + \pi$ is also large. Third, introducing concavity in the utility function (by increasing η from 0) leads to a greater reduction in marginal utility in periods of relatively high consumption. When $\beta_1 > 1$ and $c_0 + \pi$ is large, it is possible that, for the equality in Eq. (11) to hold, π must *increase* to offset the relative reduction in the marginal utility of $c_0 + \pi$ brought about by increasing η above zero. The relationship is not monotonic, however, and increases in η above a certain level will have the expected effect of reducing π, because of reductions in the marginal utility of the project benefits.

In other words, it follows from proposition 2 that an otherwise unexpected relationship between project NPV and the elasticity of marginal utility, can emerge when population growth is fast enough that $\beta > 1$, and when the project is sufficiently non-marginal in the sense that the benefits are large enough that $\Delta_1 > \frac{c_1 - c_0 - \Delta_0}{\beta_1 - 1}$. These are not highly unusual conditions — in small developing economies it is not implausible that the population growth rate, g_t, might exceed the utility discount rate, δ_t, so that $\beta_t > 1$. It is also far from impossible that a project could be large enough that the second condition is satisfied. Precisely this increasing relationship between NPV and the elasticity of marginal utility is also observed in some scenarios of BCA of global carbon emissions reductions, as discussed in the following section, and as observed in the plot of 'True NPV' in Figure 3.

3. Application to Climate Change

3.1. Global Emissions Abatement

Consider a globally-coordinated investment project, with cash flows Δ_t, which reduces emissions of carbon dioxide (CO_2) on a large scale over many decades. Let c_t^b represent business-as-usual global consumption per capita when carbon emissions are uncontrolled, and suppose that it results in climate change that has a non-marginal cost, both through the cost of adapting to it (e.g. raising coastal defences) and through its residual impacts (e.g. coastal flooding). Let $c_t^b + \Delta_t$ represent consumption along a path where carbon emissions are controlled by project Δ_t, which involves net costs from t_0 to t^* and net benefits from t^* to the terminal period, τ. The project cash flows are structured in this way, because physical inertia in the climate system causes the externality to respond slowly to costly abatement efforts. It is not inconceivable that the abatement costs associated with Δ_t are themselves non-marginal. It may also be that climate change is initially beneficial, which is another reason for net costs from t_0 to t^*. Let c_t^u represent consumption under a 'utopian' counterfactual, in which CO_2 emissions are uncontrolled but there are no damages from climate change. This is often referred to as the 'baseline', and is in effect an extrapolation of past trends in consumption growth, which have neither been affected by the cost of anthropogenic climate change nor by the cost of emissions reductions. While fictitious, many previous BCAs of climate change have calibrated the consumption discount rate on the path c_t^u. These three consumption pathways are represented in Figure 1.

[Insert Figure 1 (three consumption pathways in theory) about here.]

We want to examine the circumstances in which DCF analysis may give a misleading evaluation of the welfare consequences of the project to control carbon emissions. Suppose the project is indeed non-marginal, such that the stream of cash flows Δ_t is large (as in Figure 1). Then the difference between business-as-usual consumption and consumption if the project is undertaken will be large (and it follows that the difference between both of these paths and the 'utopian' counterfactual will also be large). Welfare analysis based on proposition 1 (the first-order Taylor approximation) may be unreliable because it applies to a set of consumption discount factors along a particular path (be it c_t^b, $c_t^b + \Delta_t$, or c_t^u), even though the project itself shifts the path.

Instead we must go back to the underlying welfare model and measure the difference between social welfare on the path corresponding to the investment in emissions reductions $c_t^b + \Delta_t$ and social welfare on the business-as-usual path c_t^b. Eq. (9) provided an obvious measure of the true welfare increase of the project ΔV, which can be rearranged for π and also $\Pi = n_0 \pi$, which denotes the true aggregate net present (consumption) benefit from the project, given by:[3]

$$\Pi = n_0 \left[u^{-1} \left(\frac{\Delta V}{n_0} + u(c_0) \right) - c_0 \right] \tag{17}$$

Eq. (4) set out the error in project *utility* which arises from using the marginal method. It follows that an equivalent expression for the error in terms of the net present *consumption* of the project (i.e. a money measure), denoted Ω_Π, is the difference between Π and the sum of discounted cash flows:

$$\Omega_\Pi = \Pi - \sum_{t=0}^{\tau} \Delta_t (1 + \rho_t)^{-t} \tag{18}$$

To explore whether "serious errors" might be made in the application of BCA to global carbon emissions abatement, we use a so-called 'integrated assessment model' (IAM) of the linkages between economy and climate. Such models have been used quite extensively over the last two decades, with the ultimate aim of evaluating the welfare effects of planned reductions in carbon emissions. Perhaps the best known IAM is William Nordhaus' DICE (Nordhaus, 1994; 2008; Nordhaus and Boyer, 2000), and we use it here.

The structure and parameterisation of DICE is described in full in Nordhaus (2008). Unless explicitly noted, we make no changes either to the model structure or to its parameter values. In brief, DICE couples a standard Ramsey-Cass-Koopmans model of economic growth to a simple model of the climate system. Output of a composite good is produced using aggregate capital and labour inputs, augmented

[3] Note that the operation of taking inverse utility requires that $\Delta V / n_0 + u(c_0) < (>) 0$ when $\eta > (<) 1$.

by exogenous total factor productivity in a Cobb-Douglas production function. Production is associated with the emission of CO_2, resulting in radiative forcing of the atmosphere and an increase in global mean temperature. The climate model couples back to the economy by means of a so-called "damage function", which is a reduced-form polynomial equation associating a change in temperature, as an index of changes in a range of climatic variables, with a loss in utility, expressed in terms of equivalent output. The damage function in DICE implicitly takes account of adaptation to climate change, which reduces the amount of output lost for a given increase in global mean temperature, so that the representative agent is left just to choose how much to invest in abating CO_2 emissions from production.[4] The model is globally aggregated (i.e. there is a single, representative global agent), so we can simplify the analysis and bound it more tightly with the comparative statics in section 2 by abstracting from questions of the spatial distribution of consumption. This would be a worthwhile future extension, however, as would the incorporation of consumption risk.

The abatement project we consider reduces emissions of CO_2 with the aim of stabilising its atmospheric stock at 1.5 times its preindustrial level, 420 parts per million (ppm). Stabilising the stock of CO_2 at 1.5 times its preindustrial level has been a focus for international political and scientific discussions on climate change, featuring prominently in, for instance, the Fourth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC, 2007). It constitutes one of the most aggressive emissions abatement proposals currently on the table (Stern, 2007), with high costs of abatement (Nordhaus, 2008) and potentially large avoided climate damages accordingly. Here, we use DICE to estimate the NPV of CO_2 stabilisation at 420 ppm over the 200 years from 2005 to 2205.[5]

Figure 2 plots estimates from DICE of the NPV of the global mitigation project as a function of the curvature of the isoelastic utility function, η. Four sets of estimates are shown, including three estimates

[4] As a neoclassical growth model, DICE can of course be used to optimise investment in CO_2 emissions abatement, versus investment in the composite capital good for future consumption. In line with the aims of this paper, however, we consider exogenous policy settings, and for the sake of tractability we also specify an exogenous savings rate.

[5] DICE is capable of running out as far as 2595, yet model predictions are especially speculative so far into the future, and in any case Nordhaus (2008) sets up the model such that business-as-usual emissions abatement reaches 100% in the 23rd century (due to the availability of zero-carbon energy technologies at no incremental cost), so that the effect of an explicit abatement project is most pronounced over the next 200 years or so.

of the DCF of the project, with the consumption discount rate in each set calibrated on DICE's estimate of growth along one of the three paths outlined above (i.e. c_t^b, $c_t^b + \Delta_t$ and c_t^u). The fourth set is our estimate of the true NPV of the project, which is the change in social welfare due to the project, normalised to present consumption (Π in Eq. (17)).

The Figure reports results where the utility discount rate $\delta = 1$. As we would expect, there is no difference between DCF and true NPV — no error — when $\eta = 0$. As η increases from zero, all four estimates of NPV fall, since per-capita consumption in the standard version of DICE is projected to grow over the period 2005-2205, irrespective of climate change (Nordhaus, 2008). However, for our purposes the important result is that the error Ω_Π (see Eq. (18)) increases, because the three DCF estimates fall faster than true NPV does. Thus DCF analysis generates a fairly substantial quantitative error for small positive values of η, although it does give the correct signal qualitatively. When $\eta = 0.5$, for example, $\Omega_\Pi =$ \$US 25 trillion, or in relative terms roughly 30% of the true value of the project.

For larger values of η, Ω_Π is smaller in absolute terms. When $\eta > 1.5$, DCF analysis continues to return estimates of project value that are below its true value, but it is qualitatively correct in estimating that the project reduces social welfare. The interesting case is when $\eta = 1$, a very common setting. Here, the error is small in absolute terms, but DCF analysis on the commonly used path c_t^u, as well as on the path $c_t^b + \Delta_t$, is qualitatively wrong. That is to say, true NPV continues to be positive when $\eta = 1$, and yet marginal analysis on these two paths estimates a negative discounted cash flow.

[Insert Figure 2 (NPV as a function of η) for $\delta = 1$ about here.]

Figure 3 reports results where $\delta = 0$. In this case, the true NPV of the project increases rapidly in the range $0 < \eta < 1$, while the three DCF estimates fall rapidly, thereby generating a very large quantitative error. As η increases beyond unity, the true NPV of the project falls rapidly, reducing the error brought about by estimating DCF. By the time $\eta \approx 1.5$, Ω_Π is small in absolute terms. We find that the peak in NPV is caused by population growth, as explained in section 2.2. In this case, the discount rate on utility is sufficiently small that, when allied with population growth, the population-augmented discount factor β_t is strictly greater than unity for all t. This may be compared with the first case, when $\delta = 1\%$, where we find that β_t is strictly less than unity for all t.

[Insert Figure 3 (NPV as a function of η) for $\delta = 0$ about here.]

As a final permutation of the emissions-abatement example, we make a small but significant modification to the standard DICE model, in order to increase the difference in consumption between c_t^b, $c_t^b + \Delta_t$ and c_t^u. In particular, we increase the important 'climate-sensitivity' parameter in the model. This is the change in the global mean temperature (in equilibrium) for a doubling of the atmospheric concentration of CO_2, and it thus plays an important role in governing how much the planet is assumed to warm up for a given pulse of emissions, and therefore how much damage will result. Standard DICE assumes that this parameter takes the value of 3°C, but IPCC (2007) estimates that the subjective probability of the climate sensitivity being greater than 4.5°C is as much as 17%. Indeed, estimates in excess of 10°C have been made (IPCC, 2007; Weitzman, 2009), although these should be considered extreme outliers. Here, we simply double the value of the climate-sensitivity parameter to 6°C and re-analyse the error in DCF analysis (Figure 4). We also return δ to 1%.

[Insert Figure 4 (NPV as a function of η) high climate sensitivity and $\delta = 1$.]

Figure 4 shows that it is possible for the true NPV of the project to initially increase in η with $\delta = 1\%$ and the population-augmented discount factor is less than unity. The reason is that in the simple two-period model examined in Proposition 2 above, it was necessarily true that an investment made by the (poorer) present for the benefit of the (richer) future increased inequality over time. In contrast, for a complex multi-period investment such as that considered in the DICE model, it is possible that investment over (several) earlier periods yields benefits over (many) later periods which may reduce inequality. Since η serves as a measure of aversion to intertemporal inequality, as η increases from low levels it is possible that such a multi-period investment is assessed to have a higher NPV because of its impact on inequality.

We conclude from these examples that marginal analysis of global carbon emissions abatement can result in "serious errors", both qualitative and quantitative. Many modifications of the standard DICE model are possible, including a more pessimistic damage function (Weitzman, 2010). Often, these permutations would tend to make the abatement project still larger, and would presumably reinforce our results in terms of the possible errors in estimating NPV by DCF analysis. However, the three figures above are sufficient to make our point.

3.2. A Large Energy Project in a Small Economy

Large infrastructure projects in small economies could also be non-marginal. Mitigation and adaptation of climate change will require considerable infrastructure investment, including in small economies, and it follows that economic evaluation of non-marginal projects could be an important issue. In our second example, we estimate the error committed in carrying out DCF analysis of such a project, using as our particular case the "Nam Theun II" hydroelectric power project in Laos, construction on which commenced in 2005 and is due to finish shortly. According to the BCA of the World Bank (World Bank and MIGA, 2005), which has provided loans and guarantees for the project, the net benefits of the dam range from around -$US 240 million during the construction phase to $US 250 million during its operation. To put these figures in context, current consumption in Laos is around $US 2.5 billion, so the construction costs alone are in the region of 10% of national consumption.

As in our first example, let c_t^b represent Laos' consumption per capita along a business-as-usual path, which in this case is a simple projection of growth in the absence of the dam. For purposes of this analysis, we define business-as-usual by projecting growth in Laos' aggregate consumption, total population and consumption per capita by a simple extrapolation of the average growth rate of GDP and population over the period 1984-2008 as estimated in the World Bank's World Development Indicators database (World Bank, 2010).[6] We can then use the estimated annual cash flows in World Bank and MIGA (2005) to calculate the error in welfare evaluation. Let $c_t^b + \Delta_t$ denote consumption per capita if the dam is constructed, being initially lower than c_t^b, due to the costs of construction, but being subsequently higher, due to the benefits of power generation. [7]

[6] Data on consumption and saving are unavailable.

[7] The path c_t^u is not relevant in this case. Note also that one can conceive of several additional costs and benefits, which are not included in the World Bank's formal BCA, including environmental costs and the social costs of community dislocation, as well as the potential benefits of the project to Laos' long-run growth (i.e. the World Bank BCA only looks

Figure 5 plots estimates of the NPV of the dam project, again as a function of the elasticity of marginal utility, η. Three sets of estimates are shown. These correspond to our estimate of true NPV, which is analogous to Π in Eq. (17), and the DCF of the project, as estimated along the paths c_t^b and $c_t^b + \Delta_t$. The utility discount rate $\delta = 1$. It can be seen that there is again no error when $\eta = 0$, and that, when η is increased, all three estimates of NPV decrease, due to the effect of increasing the concavity of the utility function in a growing economy.[8] But in a similar fashion to global carbon emissions abatement, our estimate of true NPV falls more slowly than the two estimates of DCF, so that when $0 < \eta \leq 1.5$ a relatively large quantitative error arises, albeit the conclusion from marginal analysis is qualitatively correct. Furthermore, when $3 \leq \eta \leq 3.75$ DCF analysis is qualitatively wrong on either c_t^b or $c_t^b + \Delta_t$, respectively estimating positive discounted cash flows when in fact the project decreases social welfare, and *vice versa*.

[Insert Figure 5 (NPV of Nam Theun II) about here.]

4. Conclusion

This paper has examined the theory of non-marginal BCA, and made two simple empirical applications to climate change. After defining non-marginality in terms of the inappropriateness of applying a first-order Taylor approximation, theoretical expressions for the error in welfare analysis (in utility and consumption terms) were developed. The curvature in the utility function (the elasticity of marginal utility, η) is the source of the error, so the errors are very small for extremely low η, or extremely high η. However, extreme values of η are not well supported by the empirical evidence, and more serious errors are theoretically possible for non-marginal projects evaluated with intermediate η, for which there is good evidence. Further, non-marginality creates the possibility of some unusual counterintuitive results when projects are evaluated in the context of a growing population. This paper found the conditions under which an

at the 'levels' effect of the project). A full analysis would incorporate these factors.

[8] Interestingly, we find that even when $\delta = 1$, the population-augmented discount factor $\beta_t > 1$ for all t. Intuitively, this is due to Laos' relatively rapid rate of population growth. That NPV is nevertheless decreasing in η initially, despite the fact that $\beta_t > 1$ for all t, implies that the project is insufficiently large, as defined in the necessary condition in Eq. 13.

increase in η can increase project NPV, in a setting without risk or distributional considerations.

The empirical part of the paper explored two climate-change mitigation projects at different scales, in order to investigate whether conventional BCA could yield significant quantitative errors, or, perhaps even worse, suggest outcomes which were qualitatively wrong. The first 'project' reduces global carbon emissions to a low level, and was explored using the DICE integrated assessment model developed by Nordhaus (2008). Both qualitative and large quantitative errors were found to be plausible outcomes from the DICE model results, depending on η, the utility discount rate δ, and, as an example of dependence on other model parameters, the climate sensitivity. The second project was the 'Nam Theun II' hydroelectric power plant in Laos. Using data from the World Bank, we again found both qualitative and large quantitative errors were possible for reasonable values of η and δ.

Following Dasgupta et al. (1972), we conclude that if there is cause to suspect a project under evaluation is not "small", in the sense that the range of net benefits might be a significant share of aggregate consumption, then the NPV rule will not suffice. Instead, analysts must fall back on a general-equilibrium model, which is capable of evaluating the underlying change in social welfare brought about by the project. This has important implications for the evaluation of climate-change projects from the global to the national level.

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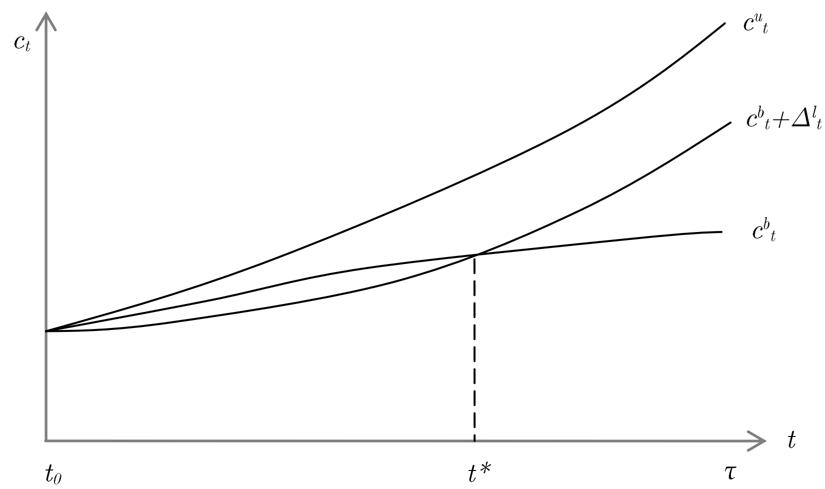

Figure 1: Three theoretical consumption pathways.

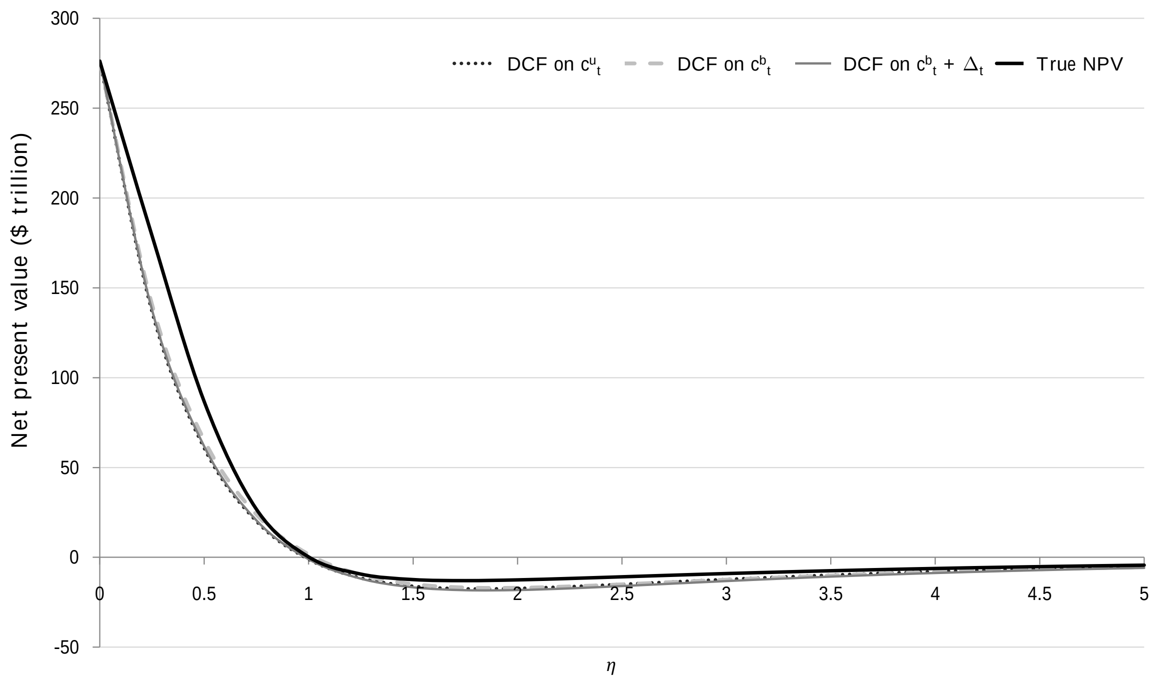

Figure 2: NPV of global carbon emissions abatement as a function of η for $\delta = 1$.

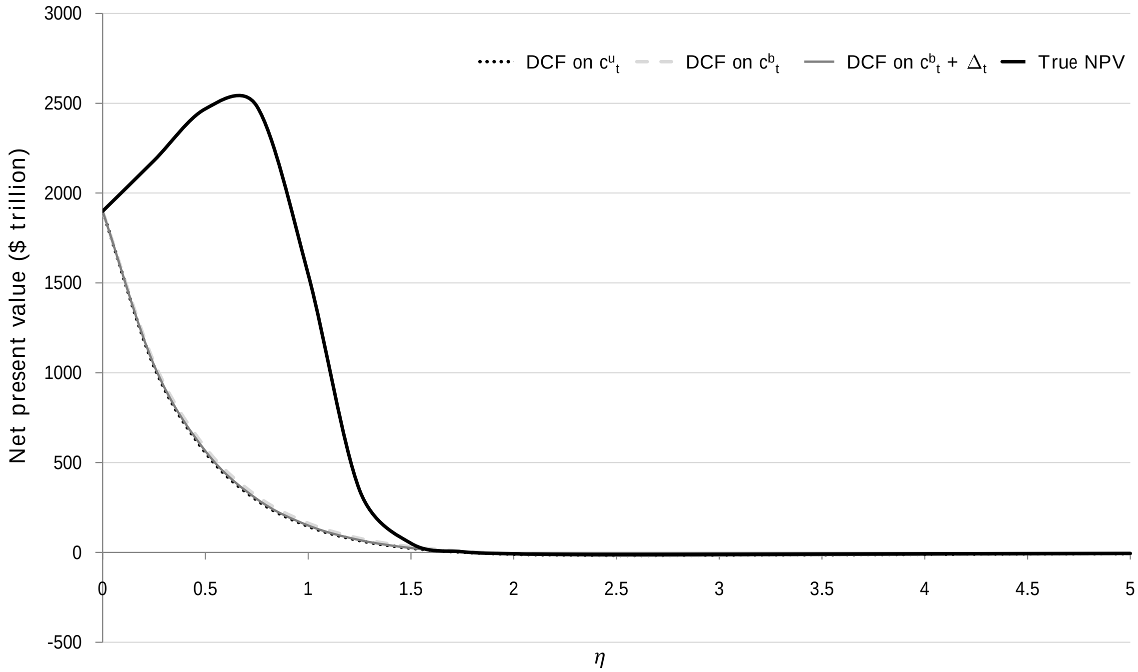

Figure 3: NPV of global carbon emissions abatement as a function of η for $\delta = 0$.

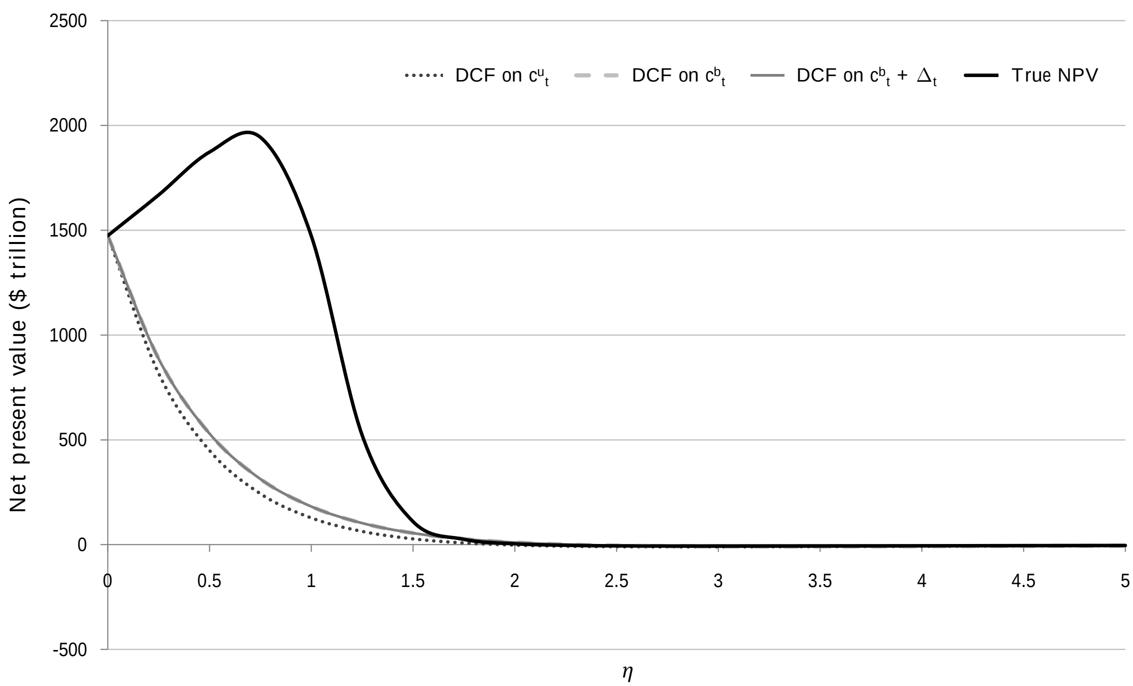

Figure 4: NPV of global carbon emissions abatement for $\delta = 1$ with high climate sensitivity.

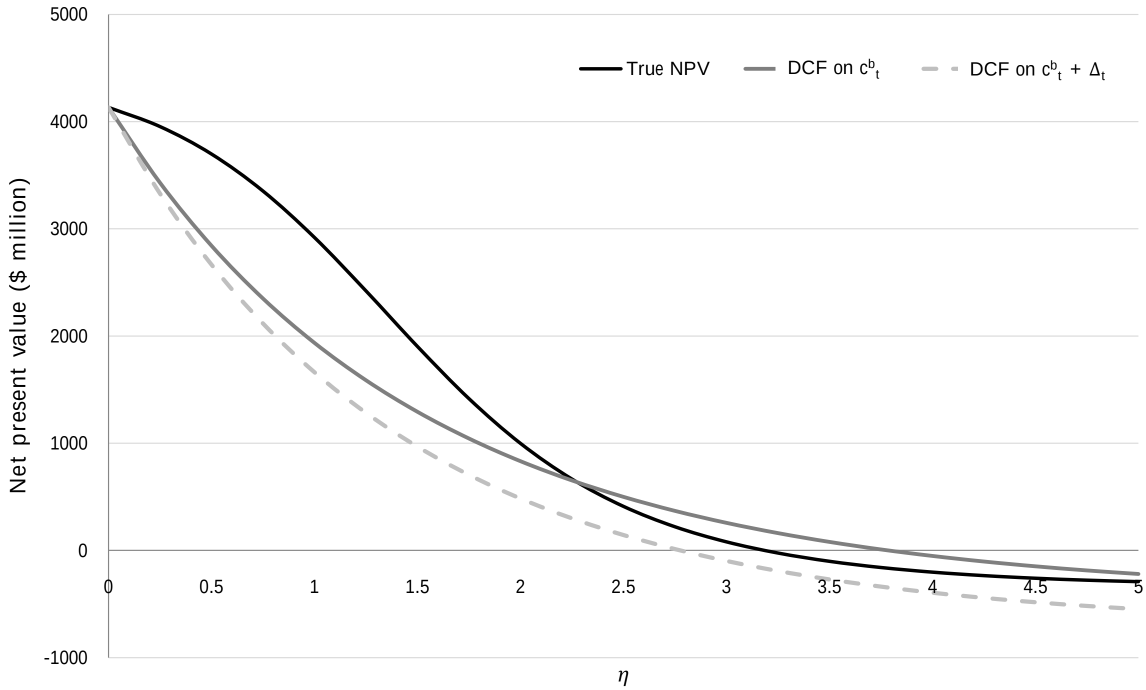

Figure 5: NPV of Nam Theun II as a function of η for $\delta = 1$.