Grantham Research Institute on
Climate Change and
the Environment

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August 2010

Grantham Research Institute on Climate Change and the Environment

Working Paper No. 22

Auctioning Conservation Contracts in the Presence of Externalities

Exploring joint bidding in the uniform-price auction[*]

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Abstract

Current models of conservation auctions do not permit for the presence of environmental externalities and synergies between bidders. Yet, conservation auctions are usually set up for the very purpose of addressing problems associated with environmental externalities. Clearly, our models do not tell the whole story, and they consequently fail to identify waste and inefficiency in these auctions. This paper shows how externalities between bidders can be incorporated into our models of conservation auctions, and uses this framework to investigate the cost-efficiency of the uniform-price auction when neighbours can bid jointly. Allowing neighbours to bid jointly allows them to internalise these externalities, but also reduces the competitiveness of the auction. The net effect on cost-efficiency is ambiguous, so we show how simulation can be used to determine in what circumstances joint bidding can be expected to reduce the payments needed to secure a given amount of ecosystem services.

Keywords: Externalities, Joint bidding, Multi-unit auctions, Payments for Ecosystem Services
JEL: D44, H23, Q19, Q57

[*]This paper is based on the author's M.Sc. thesis. I would like to thank my supervisor Malcolm Pemberton. I am also grateful for comments from Sam Fankhauser, Christian Johansson, Noel Lobo, Anthony Millner, and Flavio Toxvaerd, as well as the participants at Envecon 2010 and the 4th World Congress of Environmental and Resource Economists. I would like to acknowledge the financial support of the Jan Wallander and Tom Hedelius Foundation.

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1 Introduction

Billions of dollars in payments for ecosystem services (PES) are allocated through auctions every year. PES schemes like the Conservation Reserve Program in the US, the Bush Tender in Australia, and the Country Stewardship Scheme in the UK all make use of auctions (Latacz-Lohmann and Schilizzi, 2005).

Such schemes are set up to encourage activities that produce more (less) positive (negative) environmental externalities, and thus deliver ecosystem services. Because bidders in these auctions are usually engaged in environmentally sensitive production, chiefly agriculture, and are located in relatively cohesive geographical areas, it is difficult to imagine that the conservation measures taken would not also have external effects on others bidding in the same auction. There may be *site synergies* (also referred to as co-benefits and agglomeration benefits), such that there are added conservation benefits if neighbouring parcels of land commit to some conservation measure (Banerjee et al., 2009; Saïd and Thoyer, 2007). There may be *cost synergies*, which arise if there are decreasing marginal costs of conservation, so that the cost of committing to some conservation measure is lower if neighbours also agree to conserve. But even if neither of these are relevant, the *environmental externalities* for which the scheme is set up, and other externalities associated with the same activities, are by construction an important part of PES schemes.

Yet, our models of conservation auctions begin by assuming that there are no such externalities between bidders. They assume that individuals have *independent* valuations of winning.[1] Thus, on the one hand, we are setting up auctions to correct for environmental externalities, and, on the other hand, relying on models that assume there are no such externalities to design and evaluate these auctions. This is clearly not an ideal situation. In the presence of externalities, your neighbours' valuations of winning conservation contracts will, in part, depend on whether or not you win a conservation

[1] A standard reference is Latacz-Lohmann and van der Hamsvoort (1997).

contract. Valuations are, in a word, *inter*dependent.

If bidders fail to internalise externalities, the auctioneer may end up making excessively large payments to secure a given amount of ecosystem services. If neighbours are permitted to submit bids jointly, they will internalise the externalities, but this comes at the expense of competitive pressure in the auction, again resulting in excessive payments. Because current models assume there are no externalities, they fail to identify such waste and inefficiency.

Several reports have asked whether conservation auctions should use individual or joint bidding. Chan et al. (2003), writing for the Australian Productivity Commission, devote a section to the "possibility of joint bidding", but are forced to conclude that analysis of joint bidding is virtually absent from the literature. In a report to the Scottish Executive Environment and Rural Affairs Department, Latacz-Lohmann and Schilizzi (2005) echo this conclusion, writing that "the efficiency and payment properties of joint bidding are barely explored in the literature and detailed auction rules are yet to be developed" (p. 31). This paper attempts to fill this gap. It is important to emphasize that, while current models do not consider externalities and joint bidding, joint bidding may well already be going on in individual and joint bidding situations alike. Some of the cost-efficiency gains sometimes associated with joint bidding maybe is already being captured in practice. However, analysts are clearly lacking a framework that allows them to identify these gains. By introducing and reinterpreting some important results from auction theory in the context of PES auctions, the present study hopes to enable such analysis. Gaining control of this design dimension—individual or joint bidding—may then help policy makers implement more cost-efficient auctions.

Section 2 introduces a general framework that allows us to incorporate externalities between bidders into models of conservation auctions. Building on this framework, section 3 explores the properties of the uniform-price auction—one of the most commonly used conservation auction formats—with individual and joint bidding. The main conclusion is that joint bidding is more cost-efficient when externalities are 'sufficiently large'. We show, by means of a simple simulated example, how the policy maker can determine precisely what 'sufficiently large' means for a particular PES scheme, and hence which bidding structure would be more cost-efficient. Section 4 concludes.

2 A 'Neighbourhood World'

Imagine farmers dumping their waste into a river. All the farmers affect the water quality in the town located downstream, but the upstream farmers will also affect water quality for the downstream farmers, who use the river for irrigation. Figure 1 draws an illustrative map, where some farmers affect their nearest downstream neighbour. Each farmer is represented by a vertex, and each edge indicates the presence of externalities between a pair of farmers. The assumption of independent valuations corresponds visually to farmers just being isolated dots on a plane. Introducing externalities into the analysis of conservation auctions corresponds to introducing edges into the graph.

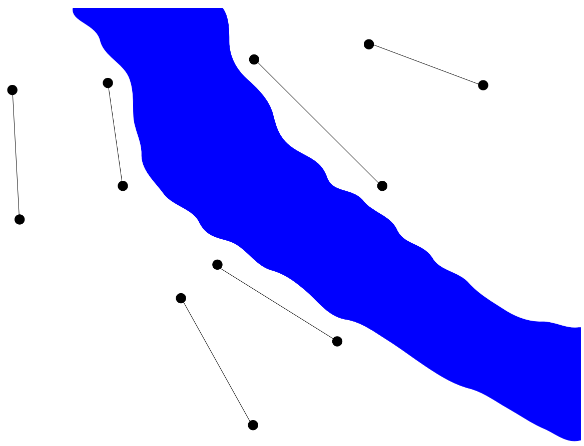

Figure 1: An illustrative map

Each farmer i (where $i = 1, \ldots, I$) chooses his own production activities. Unconstrained in his choice, i's profit is π^i. The PES scheme is in place to encourage him to take some conservation measure y^i. The conservation measure could be something like reducing the use of a pesticide, a fertiliser, or setting aside land for some alternate use. If i is awarded a conservation contract, he commits to doing y^i, which is associated with the lower profit $\pi^{i\prime}$ (i.e. conservation is costly). i's *individual valuation* of this event, not counting the payment he receives, is $v^i = \pi^{i\prime} - \pi^i \leq 0$.

However, because i produces an externality if he conserves, his individual valuation will differ from his neighbourhood's valuation of this event. To see this, let us consider formally what we mean by a 'neighbourhood'. A neighbourhood, denoted n (where $n = 1, \ldots, N$), is a disjoint complete subgraph, meaning that if $i \in n$, then $i \notin m$ for all $m \neq n$, and that if $i, j \in n$, then i and j are connected by an edge (i.e. they are 'neighbours'). Each neighbourhood has L members. Taking the twelve farmers in figure 1 as an example, figure 2 draws illustrative graphs for different neighbourhood configurations, where neighbourhood size L varies between 2 and 4. We obtain a complete graph when $L = I$.[2] When $L = 1$, we have a totally disjoint graph, which corresponds to the traditional assumption of independent valuations. We call the world of $L \geq 2$ the 'neighbourhood world', to contrast it with the 'neighbour-less world' of independence.

Each edge connects two neighbours, i and j, and signifies that i's production choices affect j, and/or vice versa. For conservation measures y^i and y^j, an edge is associated with two externality flows, e^{ij} and e^{ji}. When i wins a conservation contract, he alters his production such that he produces an externality e^{ij} that impacts j's payoff. Conversely, e^{ji} is the externality produced by j and experienced by i. Naturally, we permit $e^{ij} \neq e^{ji}$, because i's actions may have a different effect on j's payoff than the converse, as we would expect in our above example with upstream and downstream farmers. This asymmetry also allows us to consider synergies, which we could represent by letting the externality be zero for the first neighbour to win, but positive for subsequent neighbours.

[2]Notice that every possible graph with I vertices is nested in the complete graph obtained for $L = I$, and can be obtained simply by letting some edges be associated with externalities of zero magnitude. Thus, this basic structure permits analysis of any graph the researcher may find of interest.

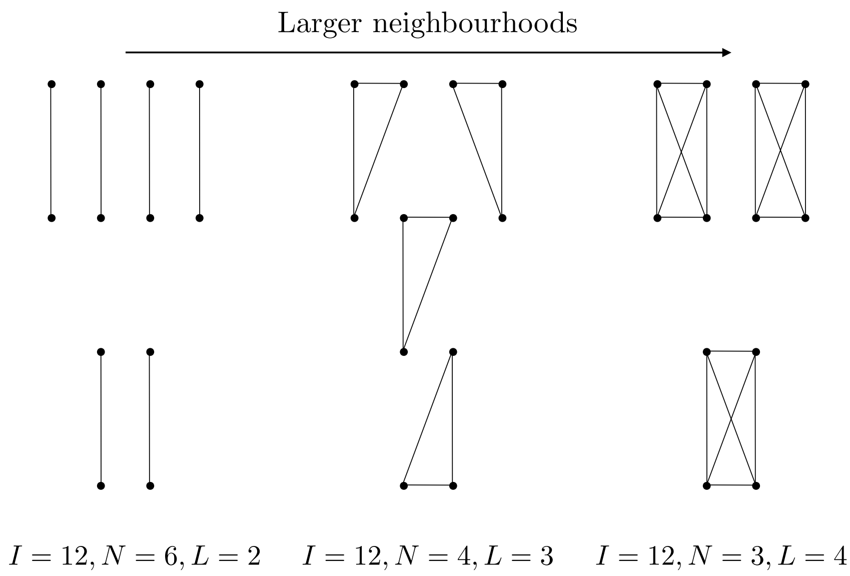

Figure 2: Graphs of the 'neighbourhood world'

We follow the convention of modeling this as an additive externality, such that i's payoff is $\pi^i + e^{ji}$ if π^i is i's profit and only individual j from among his neighbours wins a contract (Krishna and Rosenthal, 1996; das Varma, 2002). This implies no loss of generality, since it is possible to let the size of j's impact on i depend also on the scale of i's production, such that $e^{ji}(y^j, \pi^i, \pi^{i\prime})$.

The simplest 'neighbourhood world' will suffice to illustrate the effects of introducing externalities into the analysis. Therefore, we shall henceforth focus our attention on the case where $L = 2$ (as in figure 1 and the leftmost panel of figure 2). In this case, we can drop the double superscript for externalities, so that $e^{ij} = e^i$ and $e^{ji} = e^j$.

Without any conservation contracts, as is the status quo, n's neighbourhood payoff is $\pi^n = \pi^i + \pi^j$. If only i is awarded a contract, the neighbourhood payoff is $\pi^{in} = \pi^{i\prime} + \pi^j + e^i$, not including any payment received. The *neighbourhood valuation* of this event is therefore $v^{in} = \pi^{in} - \pi^n = v^i + e^i$. Table 1 completes this notation.

We impose the restriction that $v^{in}, v^{jn} \leq 0$, which simply says that conservation com-

Table 1: Neighbourhood Payoffs and Valuations

Outcome	Payoff	Valuation
i and j lose	$\pi^n = \pi^i + \pi^j$	0
i wins, j loses	$\pi^{in} = \pi^{i\prime} + \pi^j + e^i$	$v^{in} = v^i + e^i$
j wins, i loses	$\pi^{jn} = \pi^i + \pi^{j\prime} + e^j$	$v^{jn} = v^j + e^j$
i and j win	$\pi^{2n} = \pi^{i\prime} + \pi^{j\prime} + e^i + e^i$	$v^{2n} = v^i + v^j + e^i + e^j$

mitments are costly, so they would not be undertaken voluntarily even when accounting for intra-neighbourhood externalities. Alternatively, one can think of it as excluding from the auction those who would conserve voluntarily.

For expositional convenience, let us define $v_1^n = \max(v^{in}, v^{jn})$ and $v_2^n = \min(v^{in}, v^{jn})$, which are simply the first and second order statistics of neighbourhood n's valuations.[3] We shall refer to v_1^n as n's valuation of winning the 'first' contract, and v_2^n as its valuation of winning a 'second' contract. Let π_1^n and π_2^n denote the payoffs associated with winning the first and second contract respectively.

Individual valuations ignore externalities, while neighbourhood valuations take them into account. In the next section, we use this representational framework to model and simulate the uniform-price auction in the 'neighbourhood world'.

3 The Uniform-Price Auction in a 'Neighbourhood World'

The framework introduced in the previous section allows us to incorporate externalities between bidders into models of conservation auctions. In this section, we illustrate the

[3] If the size of externalities depends on scale of production, then

$$e^i(y^i, \pi^j, \pi^{j\prime}) = \begin{cases} \alpha^i(y^i, \pi^j) & \text{if } v^i + \alpha^i(y^i, \pi^j) > v^j + \alpha^j(y^i, \pi^i) \\ \beta^i(y^i, \pi^{j\prime}) & \text{otherwise} \end{cases}$$

must satisfy $v^i + \alpha^i(y^i, \pi^j) > v^j + \beta^j(y^j, \pi^{i\prime})$ whenever $v^i + \beta^i(y^i, \pi^{j\prime}) > v^j + \alpha^j(y^j, \pi^i)$. Otherwise, we have both $v^{in} > v^{jn}$ and $v^{in} < v^{jn}$. This makes it very difficult to make any reasonable *a priori* guess about which neighbour in n will submit the highest bid (although we can analyse both possibilities), unless we have more information about the bargaining problem. Of course, the neighbourhood may be fully able to resolve the question whether or not we know enough to predict the outcome.

use of this framework by analysing the properties of the uniform-price sealed bid auction in the 'neighbourhood world'. To highlight the relevance to practical policy decisions, we specifically address the question of whether the policy maker should opt for an individual or joint bidding structure. Which bidding regime results in the policy maker having to make lower expected payments to achieve his conservation target?

We can think of the policy maker's problem as figuring out what action he should take in the first stage of a three-stage game. First, he chooses whether individuals should be forced to bid individually, or whether they should be allowed to form bidding coalitions with their neighbours. Second, coalitions are formed, as permitted, and individuals/neighbourhoods formulate their optimal bidding strategies. Third, bids are submitted, and contracts and payments are awarded according to the rules of the uniform-price sealed bid auction. As a policy maker, we are interested in what our choice in the first stage should be: individual or joint bidding? Solving by backward induction, let us describe these three stages in reverse order. We continue to focus on the case where $L = 2$, for expositional ease.

Stage 3: The Auction

There are $K < I$ identical and indivisible conservation contracts for sale. Each contract mandates a conservation measure i must take if he is awarded the contract, for each i. Let the vector of conservation measures be denoted simply by y. Given y, individual valuations and externalities are independent realisations of two random variables with known distributions with full support on the intervals $[\underline{\nu}, \overline{\nu}]$ and $[\underline{e}, \overline{e}]$, respectively. Individual variation in valuations and externalities can be thought to result from individual differences in production technologies, location, and the potentially different conservation measures they would need to undertake.[4] Neighbourhood valuations are

[4] In practice, of course, a particular individual will be associated with a given production technology and located in a particular environmental context, so that his individual valuation and externality is completely determined. However, the policy maker's problem is one of asymmetric information, and

then independent realisations of the sum $v^i + e^i$, which, given y, has a distribution G with full support on $[\underline{v}, \overline{v}]$. Our earlier assumption that conservation is costly, even for neighbourhoods, implies that $\overline{v} \leq 0$.

In the individual bidding scenario, individual i submits a sealed bid b^i, which specifies what he is prepared to pay to commit to taking conservation measure y^i. An individual bids only for one contract. In the joint bidding scenario, each neighbourhood submits a sealed bid profile $\mathbf{b}^n = (b_1^n, b_2^n)$, which specifies what it is prepared to pay to commit to the conservation measure specified for each additional contract it may win. Because conservation is costly, bids are negative.[5]

Following Krishna (2002), let us construct a K-vector of competing bids facing individual i, $\mathbf{c}^{-i} = (c_1^{-i}, \ldots, c_K^{-i})$, by arranging the bids submitted by all other individuals in descending order and selecting the first K of these. c_k^{-i} is the kth order statistic of bids not submitted by i (where $k = 1, \ldots, K$). If individual i bids alone, submitting a bid b^i, then

$$i \text{ wins } \kappa = \begin{cases} 1 & \text{contract if } b^i > c_K^{-i} \\ 0 & \text{contracts otherwise} \end{cases}$$

and receives a payment of $(-p)$ if he wins, where p is the value of the highest rejected bid, given by

$$p = \begin{cases} c_K^{-i} & \text{if } b^i > c_K^{-i} \\ \max(b^i, c_{K+1}^{-i}) & \text{otherwise} \end{cases}$$

Suppose individual i forms a coalition with his neighbour j. We can construct $\mathbf{c}^{-n}$ for neighbourhood n in the same way as before. n now submits a bid profile $\mathbf{b}^n = (b_1^n, b_2^n)$,

this set up allows us to analyse the problem when we only know about the distributions of individual valuations and externalities in the population.

[5] This set up describes an auction where conservation measures are fixed and bidders compete in prices, and we do not consider the auctioneer as having any budget constraint. By changing the the fixed and random variables, however, the model can be adapted to consider a situation where the prices are fixed and bidders compete in conservation measures. In that model, the auctioneer's budget constraint could be trivially satisfied.

and

$$
n \text{ wins } \kappa = \begin{cases} 2 & \text{contracts if } b_2^n > c_{K-1}^{-n} \\ 1 & \text{contract if } b_1^n > c_K^{-n} \text{ and } b_2^n < c_{K-1}^{-n} \\ 0 & \text{contracts otherwise} \end{cases}
$$

and all winners are awarded the same per contract payment $(-p)$, were p is given by

$$
p = \begin{cases} c_{K-1}^{-n} & \text{if } b_2^n > c_{K-1}^{-n} \\ \max(b_2^n, c_K^{-n}) & \text{if } b_1^n > c_K^{-n} \text{ and } b_2^n < c_{K-1}^{-n} \\ \max(b_1^n, c_{K+1}^n) & \text{otherwise} \end{cases}
$$

A neighbourhood or individual winning κ contracts thus receives a total payment of $\kappa \times (-p)$. The uniform-price auction can be straightforwardly extended to bidding coalitions of any size (Krishna, 2002). The total cost to the auctioneer of achieving the conservation target is $K \times (-p)$.

Stage 2: Coalition Formation and Optimal Bidding Strategies

How coalitions are formed, and hence, what coalitions end up bidding in the auction, will affect optimal bidding strategies and the outcome of the auction. This is a point of particular interest, because this is really where the outcome of the auction is being determined. The standard models used to describe PES auctions consider only the possibility that all participants are bidding individually. The polar opposite would be if each neighbourhood maximised the joint expected payoff of all of its neighbours. For simplicity, our analysis will contrast these two scenarios. We shall simply assume that, if coalitions are prohibited, every individual bids to maximise his individual expected payoff. This is the individual bidding scenario normally modeled, and therefore serves as an interesting benchmark. In the presence of externalities, even this benchmark is inefficient. If coalitions are permitted, we assume that every neighbourhood forms a separate coalition, becomes aware of externalities, and bids to maximise its joint expected payoff.[6] Although neighbourhoods cooperate in our model by assumption, there does exist a set of potential side payments that could sustain neighbourhood cooperation (this is indicated by the fact that, as we shall see momentarily, the optimal individual and joint bidding strategies differ). Restricting our attention to these two scenarios will serve to illustrate the impact of introducing externalities and joint bidding into our models of PES auctions.

Individual bidding strategy

There are I bidders in the individual bidding scenario, each bidding for a single contract. Individual i formulates his bid based on his individual valuation, as privately optimising behaviour dictates. In this scenario, the uniform-price auction is strategically equivalent to the Vickrey auction, which means that it will be a weakly dominant strategy for each bidder to submit a bid equal to his individual valuation. In equilibrium, therefore, each bidder i submits a bid $b^i = v^i$.[7]

Joint bidding strategy

There are N bidders in the joint bidding scenario, each submitting two bids, one for each neighbour. Individuals still select production individually, but they now formulate a joint bidding strategy based on neighbourhood valuations. Thus, externalities still exist in the joint bidding scenario, but individuals are now able to use the joint bidding mechanism to account for their presence.[8]

[6] In practice, PES auctions are likely to occur somewhere along the intermediate range between our individual and joint bidding schenarios. The choice of a individual or joint bidding regime is probably more akin to pushing us in one direction or another along this range, and bidding strategies and auction outcomes will depend in very complicated ways on this choice within this range. The question of coalition formation is very interesting and highly pertinent, and should prove a fruitful area for experimental examination.

[7] See Krishna (2002) for a proof.

[8] If neighbours merged their production activities, neighbourhoods could be treated as 'individuals', and there would be no externalities to start with. Notice that if we tried to achieve this artificially by designing the auction so that each neighbourhood could submit only a single bid covering the conservation activities of the entire neighbourhood, we would be forced to either award contracts to every member

We define the joint bidding strategy as a function $B^n(v_1^n, v_2^n) = (B_1^n(v_1^n, v_2^n), B_2^n(v_1^n, v_2^n)) : [\underline{v}, 0]^2 \to \mathbf{R}^2_{--}$ returning two bids from two valuations. For easy comparison, the optimal individual bidding strategy could be written as the identity function, $B^i(v^i) = v^i$.

Define $F_k(x)|B$ as the probability that, if all neighbourhoods except n use bidding strategy B, at least k bids made by those neighbourhoods are less than or equal to x. In a symmetric equilibrium all bidders pursue the same bidding strategy B^*. Let us then write $F_k(x)|B^* = F_k^*(x)$, to simplify notation, and $f_k^*(x)$ as the corresponding density. Then, n's expected payoff is

$$\begin{aligned}\mathbf{E}(\pi^n) &= \int_{\underline{v}}^{b_2^n} (\pi^{2n} - 2c_{K-1}^{-n}) f_{K-1}^*(c_{K-1}^{-n}) dc_{K-1}^{-n} + \int_{b_2^n}^{b_1^n} (\pi_1^n - c_K^{-n}) f_K^*(c_K^{-n}) dc_K^{-n} \\ &\quad + (\pi_1^n - b_2^n)(F_K^*(b_2^n) - F_{K-1}^*(b_2^n)) + (\pi^n)(1 - F_K^*(b_1^n)) \end{aligned} \tag{1}$$

The first term is n's payoff from winning two contracts, weighted by the probability of that event. The second term is the probability-weighted payoff from winning a single contract and paying c_K^n, the third is for winning a single contract but paying b_2^n, and the final term is for the event of not winning any contracts at all. The expected payoff is maximised with respect to b_1^n when

$$b_1^n = \pi_1^n - \pi^n = v_1^n \tag{2}$$

The second equality is obtained from the definitions in section 2. Thus, in equilibrium, each neighbourhood's first bid will equal its neighbourhood valuation of the first contract, $B_1^*(v_1^n) = v_1^n$. This looks similar to the individual bidding scenario, except that it now accounts for externalities. The expected payoff is maximised with respect to b_2^n when

$$b_2^n = (\pi^{2n} - \pi_1^n) - \frac{F_K^*(b_2^n) - F_{K-1}^*(b_2^n)}{f_K^*(b_2^n)} = v_2^n - \frac{F_K^*(b_2^n) - F_{K-1}^*(b_2^n)}{f_K^*(b_2^n)} \tag{3}$$

The second equality, again, comes from definitions in section 2. The numerator of the second term on the right hand side is the probability that b_2^n is the highest rejected

of a neighbourhood, or none. This would be inefficient because we could only award contracts on the basis of 'aggregate neighbourhood bids', rather than the 'marginal bids' of each individual within a neighbourhood.

bid, which is strictly positive. Since the denominator is positive as well[9], we can see that the neighbourhood has an incentive to shade its bid for the second contract, i.e. $b_2^n \leq v_2^n$. Put another way, there is an incentive to demand excessive payment for the second contract. The intuition is as follows. If b_2^n were the highest rejected bid, n would receive a payment of $(-b_2^n)$. The more negative b_2^n is, the larger would be the payment in this event. Because the probability of this event is positive, the neighbourhood has an incentive to shade its bid for the second contract. On the other hand, lowering the second bid reduces the chances of n actually winning a second contract and receiving a total payment of $(-2c_{K-1}^{-n})$. This tempers the neighbourhood's incentive to shade its bid, but the equilibrium strategy is still to bid below its valuation of the second contract. Clearly, this complication does not arise for the first bid because a neighbourhood's own first bid never affects how much it is paid if it wins. Completely solving the neighbourhood's optimisation problem yields the following equilibrium joint bidding strategy:

Theorem 1. *A symmetric undominated Bayes-Nash equilibrium in the uniform-price sealed bid auction with joint bidding is given by a set of continuous monotonic bidding strategies $B^*(v_1^n, v_2^n) = (B_1^*(v_1^n), B_2^*(v_2^n))$, one for each neighbourhood n, such that*

$$B_1^*(v_1^n) = v_1^n \tag{4}$$

and

$$B_2^*(v_2^n) = \begin{cases} \underline{v} & \text{if } v_2^n < T(\underline{v}) \\ v_2^n - T(v_2^n) & \text{otherwise} \end{cases} \tag{5}$$

where

$$T(b_2^n) = \frac{\sum_{X,Y;2X+Y=2N-K-1} R(b_2^n)}{\sum_{t=2N-K}^{2N-2} \sum_{X,Y;2X+Y=t} \left(\frac{\partial R}{\partial b_2^n} + \frac{\partial R}{\partial B_2^{-1*}} \frac{\partial B_2^{-1*}}{\partial b_2^n} \right)} \tag{6}$$

and

$$R(b_2^n) = \frac{(N-1)!}{X!Y!(N-X-Y-1)!} \left(\int_{\underline{v}}^{B_2^{-1*}(b_2^n)} \int_{\underline{v}}^{b_2^n} h(v_1^m, v_2^m) dv_1^m dv_2^m \right)^X \tag{7}$$

[9] A proof that $f_K^*(b_2^n) > 0$ can be found in Noussair (1995).

$$\left(\int_{\underline{v}}^{B_2^{-1*}(b_2^n)} \int_{b_2^n}^{0} h(v_1^m, v_2^m) dv_1^m dv_2^m\right)^Y \left(\int_{B_2^{-1*}(b_2^n)}^{0} \int_{b_2^n}^{0} h(v_1^m, v_2^m) dv_1^m dv_2^m\right)^{N-X-Y-1}$$

and where

$$B_2^*(0) = 0 \quad \textit{if } N < K$$

$$B_2^*(\underline{v}) = \underline{v} \quad \textit{otherwise}$$

See Noussair (1995) for a proof of theorem 1.[10] B_2^{-1*} in equation 6 is just the inverse of the function B_2^* (we have dropped the n-superscript here to simplify notation, and because all neighbourhoods follow the same strategy in equilibrium). $R(b_2^n)$, defined in equation 7, is the probability in equilibrium that X bidders, not including n, submit two bids lower than or equal to b_2^n, that Y bidders other than n submit exactly one bid greater than b_2^n, and that $N - X - Y - 1$ bidders other than n submit two bids above b_2^n. $h(v_1^m, v_2^m)$ in equation 7 is the joint density of the order statistics of neighbourhood valuations of a randomly selected neighbourhood m, where $m \neq n$.

Stage 1: Individual or Joint bidding?

Comparing the individual bidding strategy with the joint bidding strategy, we find that the individual and the neighbourhood alike will submit a first bid equal to the valuation of winning only one contract. However, the presence of externalities means that the actual bids of a given individual will differ between the two scenarios. If the externality associated with that bidder is positive, so that $v^i + e^i > v^i$, the neighbourhood's honest bid will exceed the individual's. The neighbourhood will also take account of the externality relevant to the second bid, so that we have $v^j + e^j > v^j$ for a positive externality.

[10]Noussair (1995), considers a slightly different formulation of the underlying problem than presented here. Firstly, what we call neighbourhoods, Noussair imagines as inseparable entities, which means that valuations do not have the structure of being sums of random variables. Secondly, he considers a standard auction (i.e. valuations are positive), while we are considering a reverse auction (i.e. valuations are negative). Thirdly, there is a zero-payoff from loosing in Noussair's auction, while in our problem individuals still make a profit if they loose. Noussair's result is presented here with these alterations, but the form of the proof is unchanged.

As noted above, in the joint bidding scenario a neighbourhood's second bid might end up the highest rejected bid, and therefore determines how much it is paid for the first contract it wins. This creates an incentive to make excessive demands payment for the second contract. The neighbourhood has to balance this against the consequent reduced chances of winning the second contract. The result of this balancing act is that $v^j + e^j \geq v^j + e^j - T(v^j + e^j)$. The net effect of accounting for externalities and shading the bid for the second contract, $e^j - T(v^j + e^j)$, can be either positive or negative, corresponding to an increase or reduction in the second bid compared with j's individually optimal bid.

Switching from individual to joint bidding thus has two effects on bidding behaviour. Firstly, it induces individuals to consider the externalities they produce if they win. Secondly, it prompts neighbourhoods to strategically exaggerate their second bids. If all externalities are zero or negative, joint bidding will result in the auctioneer having to make higher expected payments to secure a given amount of ecosystem services. But as long as some portion of externalities are positive, it is unclear whether joint bidding increases or decreases the payment per contract. If externalities are sufficiently large, they will outweigh the loss in competitive pressure, and permitting joint bidding would then reduce the expected cost of achieving the conservation target. Such cost-savings may be substantial, since, even if the payment per contract is only a little lower, the cost-saving is multiplied by every sold conservation contract (recall that the total cost to the auctioneer is $K \times (-p)$).

Whether such cost-savings exist, and how large they are, depends on the size and distribution of externalities, the distribution of individual valuations, the number of individuals that participate, and the number of contracts that are auctioned (these terms appear in theorem 1). If the policy maker has some idea of these, he can calculate the expected cost of using individual and joint bidding, and compare them. To demonstrate how to conduct such a calculation, let us simulate a simple example of the uniform-price auction.

A simple simulation

We fix the number of individuals ($I = 4$) and contracts ($K = 3$), and assume that, given some y, individual valuations are independently and uniformly distributed on $[\underline{\nu}, \underline{\nu} + 1]$, and externalities are independently and uniformly distributed on $[\underline{e}, \underline{e} + 1]$. To satisfy our assumptions that individual and neighbourhood valuations are non-positive, we let $\underline{\nu} \leq -1$ and $\underline{\nu} + \underline{e} + 2 = 0$. We also assume that the size of externalities are independent of scale of production. Since the sum of two independently and uniformly distributed random variables follows a triangular distribution, neighbourhood valuations follow a triangular distribution on $[-2, 0]$, with mode -1. It is then possible to compute the joint distribution of the order statistics of neighbourhood valuations, which is needed to specify the joint bidding strategy.

We simulate the individual bidding scenario by independently drawing four values from the uniform distribution on $[\underline{\nu}, \underline{\nu} + 1]$. Since bids are equal to individual valuations, the three bidders with the highest valuations will win one contract each, and each winner will each receive a payment equal to negative of the highest rejected bid.

We simulate the joint bidding scenario by independently drawing two pairs of values from the triangular distribution on $[-2, 0]$ with mode -1. For each pair of neighbourhood valuations, we obtain the bids using the equilibrium bidding strategy $B^*(v_1^n, v_2^n) = (B_1^*(v_1^n), B_2^*(v_2^n))$, where

$$B_1^*(v_1^n) = v_1^n \tag{8}$$

and

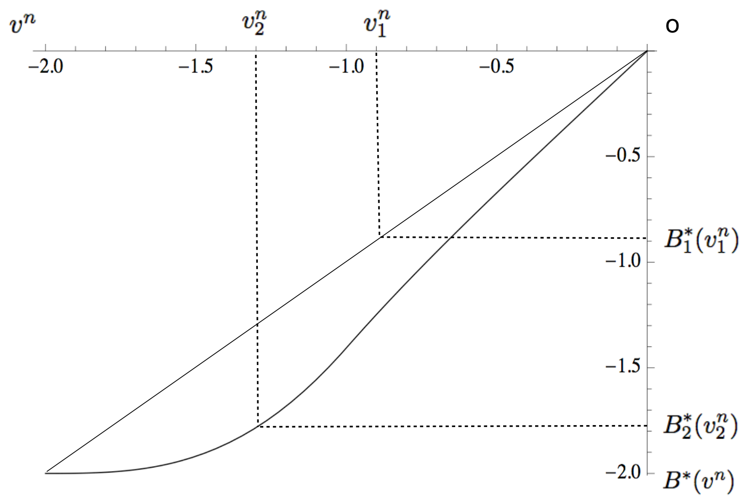

Figure 3: Plot of $B_2^*(v_2^n)$ in equation 9

$$B_2^*(v_2^n) = \begin{cases} v_2^n - \frac{1}{16}\left((v_2^n)^2 + 4v_2^n + 2\right)^2 \\ \quad \left(-\frac{4(v_2^n+2)}{(v_2^n)^2+4v_2^n+2} - \sqrt{2}\log\left(-v_2^n + \sqrt{2} - 2\right) + \sqrt{2}\log\left(v_2^n + \sqrt{2} + 2\right)\right) & \text{if } -2 \leq v_2^n \leq -1 \\ \\ v_2^n - \frac{1}{48}v_2^n\left((v_2^n)^3\left(4 + 3\sqrt{2}\log\left(\sqrt{2}-1\right) - 3\sqrt{2}\log\left(1+\sqrt{2}\right)\right) + 16\right) & \text{if } -1 < v_2^n \leq 0 \end{cases} \tag{9}$$

Equation 8 is just the same as equation 4, and equation 9 is the special case of equation 5 for this particular auction. Equation 9 is derived in the appendix.

To help us visualise joint bidding strategy, figure 3 plots equations 8 and 9 over the interval $[-2, 0]$, and reads off the first and second bids (vertical axis) corresponding to some neighbourhood valuations (horizontal axis). The neighbourhood bids honestly for the first bid, as reflected by the 45° line. The neighbourhood shades its second bid, as is apparent from the fact that the curve falls below the 45° line. Thus, as drawn in the figure, although the neighbourhood only requires a payment of 1.3 to be indifferent to conservation, it will demand nearly 1.8 in equilibrium (exaggerating by nearly 40%).

The three individuals with the highest bids will be awarded one contract each and receive a payment equal to the negative value of the highest rejected bid.

For a given realisation of individual valuations and externalities, we can calculate how much the auctioneer will end up paying per contract with individual and joint bidding regimes. However, since the policy maker knows only the distribution of individual valuations and externalities, it will only be able to figure out how much it will need to pay in expectation. For repeated realisations of individual valuations and externalities, we can approximate the distribution of payments. Figure 4 plots such approximations of the probability density functions (pdfs) of p, obtained by repeating the auction one million times.[11] Panel *(a)* plots the pdf in the joint bidding scenario. The same distribution is obtained for any pair of intervals $[\underline{\nu}, \underline{\nu}+1]$ and $[\underline{e}, \underline{e}+1]$ that satisfy the boundary restrictions. Panel *(b)* plots a family of pdfs for the individual bidding scenario, as the interval over which individual valuations are distributed $[\underline{\nu}, \underline{\nu}+1]$ is shifted from $[-1, 0$ down to $[-3, -2]$ in half-unit steps. Notice that because the distribution of neighbourhood valuations is fixed, the interval over which externalities are distributed, $[\underline{e}, \underline{e}+1]$, must simultaneously shift up from $[-1, 0]$ to $[1, 2]$. The downward shift of the distribution of p in panel *(b)* can therefore be interpreted as a consequence of increasing the proportion of neighbourhood valuations accounted for by externalities. As the size of externalities increases, the expected payments obtained through joint bidding begins to look more favourable by comparison.

It is a well-known result that the order statistics of independent and uniformly distributed random variables are Beta random variables. Using this fact, the expectation of p would be $\mathbf{E}(p) = \frac{I-K}{I} = \frac{1}{4}$, were individual valuations to be distributed on $[0, 1]$. Since taking the expectation is a linear operation, the expectation of p is given by

$$\mathbf{E}(p) = \frac{1}{4} + \underline{\nu} \tag{10}$$

when individual valuations are distributed on the interval $[\underline{\nu}, \underline{\nu}+1]$. It is reassuring that the curve given by equation 10 also emerges in our simulation as the envelope of

[11] Details of simulations are available from the author on request.

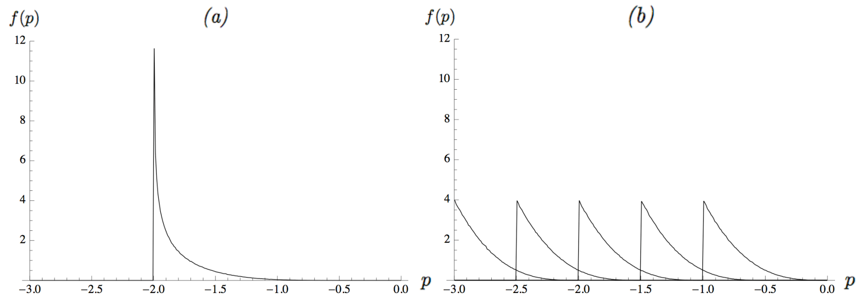

Figure 4: Approximate distributions of the per contract payment. Panel *(a)* shows the approximate pdf of the per contract payment in the uniform-price auction with joint bidding. Panel *(b)* shows a corresponding family of pdfs of the per contract payment obtained with individual bidding. The pdfs in panel *(b)* approximate the pdf of a beta random variable.

the expected transfers as $\underline{\nu}$ changes (see figure 5). The expectation of p in the joint bidding scenario (panel *(a)* in figure 4) can be numerically approximated to -1.806. It is then a matter of simple arithmetic to show that joint and individual bidding yield approximately equal payments per contract in expectation when $\underline{\nu} = -2.056$. The point $(-1.806, -2.056)$ is located where the curve intersects the vertical axis in figure 5. At this point, individual valuations are uniformly distributed on the interval $[-2.056, -1.056]$, and externalities on $[0.056, 1.056]$. Thus, individual and joint bidding result in the same payments in expectation when the average size of externalities is approximately 30% of the expected size of payments. When externalities are smaller (in the right quadrant of figure 5), individual bidding results in lower payments in expectation. When externalities are larger (to the right), joint bidding results in lower payments. Notice also that, since the total cost of achieving the conservation target is $K \times (-p)$, the expected cost-saving of using a joint bidding structure as we move to the right increases at 3 times the rate of decline of the expected per contract payment. Thus, even if externalities are only slightly larger than the critical level, there are potentially large cost-savings from allowing joint bidding.

Suppose, then, that a policy maker was organising a uniform-price auction for four

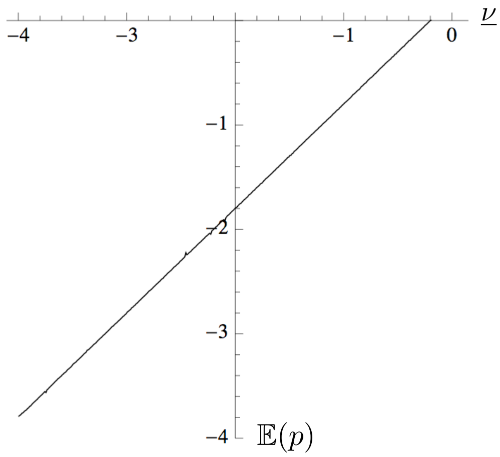

Figure 5: Envelope of approximate expected per contract payment

individuals, and y is chosen such that individual valuations and externalities are uniformly distributed on unit intervals. Having some prior belief about the size of externality flows, this simulation will tell him whether he should opt for an individual or joint bidding structure.

In our simulated special case, joint bidding will only be preferable when all externalities are positive. However, in principle joint bidding may reduce payments even when some proportion of externalities are negative. This kind of simulation can be conducted for any auction scenario, and will locate the point at which the balance tips in favour of joint bidding.

Simulations can also locate tipping points with respect to the number of bidders, and the number of contracts for sale, and with respect to changes in the characteristics of the underlying distributions of individual valuations and externalities. Given information about some of these variables, it is then possible to optimise the auction design with respect to the remaining unknowns.[12] This can help the policy maker select a more cost-efficient auction format for his PES scheme.

[12]Further information about simulations are available from the author on request.

4 Conclusion

Previous models of conservation auctions have looked at the behaviour of isolated vertices on a plane—neighbour-less individuals—but it is both inappropriate and unnecessary to ignore externalities between bidders. We can now connect these dots to form a crude blueprint, and although we have not connected all the dots yet, we have suggested a toolkit for doing so. In real life PES auctions, individuals may already be deviating from isolated optmisation, and it is important that our theoretical framework be similarly expanded to include joint bidding. This paper has introduced auction theory results not normally considered in the context of PES auctions, and provided a neighbourhood structure that allows us to reinterpret and make use of these theoretical results. We have demonstrated that this toolkit can be operationalised, analysing the properties of the uniform-price auction. The properties of the uniform-price auction in our simple example hold for the 'neighbourhood world' in general, both existence of an equilibrium (Reny, 1999; Bresky, 1999) and the systematic exaggeration of bids (Krishna, 2002). This offers a stronger starting position for the study of conservation auctions than the traditional models of a 'neighbour-less world'.

We have focused on the potential for using joint bidding to reduce costs, but the underlying machinery allows us to ask further questions. How allocatively inefficient will individual and joint bidding be in the presence of externalities, and what does allocative inefficiency depend on? Bresky (2009) suggests that the policy maker can use the reservation price to reduce allocative inefficiency under a joint bidding regime. Thus, a joint bidding regime with an appropriately set reservation price could reduce both the public and private cost of conservation.

Additional questions have been raised with regard to the properties of auctions in the presence of synergies. We can represent synergies in our framework by letting the externality flow associated with the first winner in a neighbourhood to be zero, and subsequent externality flows reflect the synergies with previous winners in the neighbourhood. If these synergies accrue to the policy maker instead of individuals, we can still use this framework, but we would now need to condition payments on these added benefits (see Banerjee et al. (2009) for a discussion of how to design an auction that conditions payments in this way, and Saïd and Thoyer (2007) for an experimental study). The framework presented here is flexible enough to include all of these possibilities.

Externalities are recognised to be important in conservation auctions. In fact, despite the apparent lack of a theoretical framework to evaluate joint bidding, in 2004-5, the Australian Auction for Landscape Recovery actually allowed joint bidding. In the future, such pilot projects can be more systematically evaluated with the aid of the framework presented here.

An important objective for future research will be to investigate bidding behaviour under alternative conditions, and to extend the simulations presented here. In particular, this will involve extending this analysis to other auction formats, such as the pay-your-bid auction (Chakraborty, 2006), and to incorporate more sophisticated models of coalition formation. These endeavours must be complemented with measurements of externality flows taken in the field and through interviews, to guide theoretical work towards empirically interesting scenarios, and with experimental work, to evaluate new predictions about conservation auctions.

Appendix: Deriving equation 9

The Bapat-Beg theorem (Bapat and Beg, 1989) gives the joint cumulative distribution of order statistics. The joint cumulative distribution function (cdf) of the order statistics of two i.i.d random variables is given by

$$\begin{aligned} H(v_1^n, v_2^n) &= \text{Prob}(\max(v^{in}, v^{jn}) \leq v_1^n, \min(v^{in}, v^{jn}) \leq v_2^n) \\ &= 2G(v_1^n)G(v_2^n) - (G(v_2^n))^2 \end{aligned}$$

where $v_1^n > v_2^n$ (otherwise the density h of H is zero). G is the cdf of neighbourhood valuations. The probability that, for a randomly selected neighbourhood m (where $m \neq n$), both bids are below or equal to b_2^n is

$$\begin{aligned}
\text{Prob}(b_1^m \leq b_2^n, b_2^m \leq b_2^n) &= \int_{\underline{v}}^{B_2^{-1*}(b_2^n)} \int_{\underline{v}}^{b_2^n} h(v_1^m, v_2^m) dv_1^m dv_2^m \\
&= \int_{-2}^{b_2^n} \int_{-2}^{b_2^n} h(v_1^m, v_2^m) dv_1^m dv_2^m \\
&= H(b_2^n, b_2^n) \\
&= 2G(b_2^n)G(b_2^n) - (G(b_2^n))^2 \\
&= (G(b_2^n))^2
\end{aligned}$$

Neighbourhood valuations are distributed on $[-2, 0]^2$, so it is straightforward to replace $\underline{v}$ with -2 above. Moreover, $h = 0$ always where $B_2^{-1*}(b_2^n) > b_2^n$, so we can replace $B_2^{-1*}(b_2^n)$ with b_2^n without altering the value of the integral, thus obtaining the second equality.

By a similar argument, the probability that both of bids are greater than b_2^n is

$$\begin{aligned}
\text{Prob}(b_1^m > b_2^n, b_2^m > b_2^n) &= \int_{B_2^{-1*}(b_2^n)}^{0} \int_{b_2^n}^{0} h(v_1^m, v_2^m) dv_1^m dv_2^m \\
&= \int_{B_2^{-1*}(b_2^n)}^{0} \int_{B_2^{-1*}(b_2^n)}^{0} h(v_1^m, v_2^m) dv_1^m dv_2^m \\
&= H(0,0) - H(0, B_2^{-1*}(b_2^n)) - H(B_2^{-1*}(b_2^n), 0) + H(B_2^{-1*}(b_2^n), B_2^{-1*}(b_2^n)) \\
&= H(0,0) - H(0, B_2^{-1*}(b_2^n)) - H(B_2^{-1*}(b_2^n), B_2^{-1*}(b_2^n)) + H(B_2^{-1*}(b_2^n), B_2^{-1*}(b_2^n)) \\
&= H(0,0) - H(0, B_2^{-1*}(b_2^n)) \\
&= 1 - H(0, B_2^{-1*}(b_2^n)) \\
&= 1 - 2G(0)G(B_2^{-1*}(b_2^n)) - (G(B_2^{-1*}(b_2^n)))^2 \\
&= (1 - G(B_2^{-1*}(b_2^n)))^2
\end{aligned}$$

The probability that m submits exactly one bid that is greater than b_2^n is

$$\text{Prob}(b_1^m > b_2^n, b_2^m \leq b_2^n) = \int_{\underline{v}}^{B_2^{-1*}(b_2^n)} \int_{b_2^n}^{0} h(v_1^m, v_2^m) dv_1^m dv_2^m$$

$$
\begin{aligned}
&= H(0, B_2^{-1*}(b_2^n)) - H(b_2^n, V_2^*(b_2^n)) \\
&= H(0, V_2^{-1*}(b_2^n)) - H(b_2^n, b_2^n) \\
&= 2G(B_2^{-1*}(b_2^n)) - (G(B_2^{-1*}(b_2^n)))^2 - (G(b_2^n))^2
\end{aligned}
$$

Then, the probability that X bidders other than n submit two bids lower than or equal to b_2^n, that Y bidders other than n submit exactly one bid greater than b_2^n, and that $N - X - Y - 1$ bidders other than n submit two bids above b_2^n is

$$
\begin{aligned}
R(b_2^n) = \quad & \frac{(N-1)!}{X!Y!(N-X-Y-1)!} \left((G(b_2^n))^2\right)^X \left(2G(B_2^{-1*}(b_2^n)) - (G(B_2^{-1*}(b_2^n)))^2 - (G(b_2^n))^2\right)^Y \\
& \left((1 - G(B_2^{-1*}(b_2^n))^2\right)^{N-X-Y-1}
\end{aligned}
$$

Assuming that $N = 2$, the multinomial coefficient equals 1. We then want to calculate the numerator and denominator of the second term on the right hand side of the equation

$$
B_2^*(v_2^n) = v_2^n - \frac{\sum_{X,Y;2X+Y=2N-K-1} R(b_2^n)}{\sum_{t=2N-K}^{2N-2} \sum_{X,Y;2X+Y=t} \left(\frac{\partial R}{\partial b_2^n} + \frac{\partial R}{\partial B_2^{-1*}} \frac{\partial B_2^{-1*}}{\partial b_2^n}\right)}
$$

Let us first consider the numerator. Assuming that $K = 3$ and $N = 2$, there is only one possible combination of values for X and Y, namely $(X, Y) = (0, 0)$. We can then write

$$
\sum_{X,Y;2X+Y=2N-K-1} R(b_2^n) = (1 - G(B_2^{-1*}(b_2^n)))^2
$$

In the denominator, t goes from 1 to 2, so only $(X, Y) = (0, 1)$ or $(1, 0)$ are possible. We can therefore write the denominator as

$$
\sum_{t=2N-K}^{2N-2} \sum_{X,Y;2X+Y=t} \left(\frac{\partial R}{\partial b_2^n} + \frac{\partial R}{\partial B_2^{-1*}} \frac{\partial B_2^{-1*}}{\partial b_2^n}\right) = \frac{\partial}{\partial b_2^n} \left(2G(B_2^{-1*}(b_2^n)) - (G(B_2^{-1*}(b_2^n)))^2\right)
$$

Putting these expressions together and simplifying gives

$$
B_2^*(v_2^n) = v_2^n - (1 - G(v_2^n))^2 \int_{-2}^{v_2^n} \frac{1}{(1 - G(x))^2} dx \tag{11}
$$

with initial condition $B_2^*(0) = 0$, since $N < K$. In our case, G is the cdf of the triangular distribution on the interval $[-2, 0]$ with mode -1. We can write it in the form

$$
G(x) = \begin{cases} x - \operatorname{sign}(x+1)\frac{(x+1)^2}{2} + \frac{3}{2} & \text{if } -2 \leq x \leq 0 \\ 0 & \text{otherwise} \end{cases}
$$

Since the integrand on the right hand side of equation 11 is a piecewise rational function, it is possible to retrieve the anti-derivative. It is then a matter of arithmetic manipulation to obtain equation 9, restated below

$$B_2^*(v_2^n) = \begin{cases} v_2^n - \frac{1}{16}\left((v_2^n)^2 + 4v_2^n + 2\right)^2 & \text{if } -2 \le v_2^n \le -1 \\ \quad \left(-\frac{4(v_2^n+2)}{(v_2^n)^2+4v_2^n+2} - \sqrt{2}\log\left(-v_2^n + \sqrt{2} - 2\right) + \sqrt{2}\log\left(v_2^n + \sqrt{2} + 2\right)\right) & \\ & \\ v_2^n - \frac{1}{48}v_2^n\left((v_2^n)^3\left(4 + 3\sqrt{2}\log\left(\sqrt{2}-1\right) - 3\sqrt{2}\log\left(1+\sqrt{2}\right)\right) + 16\right) & \text{if } -1 < v_2^n \le 0 \end{cases}$$

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