

Recent Developments in Carbon Finance

A stochastic equilibrium model for the CO_2 permit price in discrete time

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CO₂ permit price formation

CO₂ marketable permits can be considered as a *pseudo*-commodity whose price is, as any standard commodity, a function of demand and supply.

Supply The supply side is determined by the amount of emission permits (EUAs) and carbon credits available in the market, i.e. by:

- ▶ the initial allocation of EUAs via NAP;
- ▶ the supply and conversion of CDM credits (CERs) into EUAs;
- ▶ the possibility of borrowing and banking EUAs.

Demand The demand side is determined primarily by the projected (or actual) pollution emissions of the installations covered by the EU ETS. Most of the literature identifies the following macroeconomic factors as main drivers of projected (or actual) emissions:

- ▶ pollution abatement options at disposal;
- ▶ economic growth and energy-commodities prices;
- ▶ weather;

CO₂ permit price modeling

While the rules behind the CO₂ permit price formation mechanism and the price dynamics require further investigations, we can identify three main different approaches for modeling CO₂ price:

1. Macroeconomic models (economic growth, energy-related commodities, weather, financial risk factors);
2. Pure econometric investigation of CO₂-related time-series (unconditional and conditional analysis of univariate - and multivariate - time series);
3. Stochastic equilibrium models for the analysis of the spot and futures price dynamics.

Several authors support the argument that permit price responds to macroeconomic fundamentals and try to identify statistically which factors affect permits demand and/or supply.

Modeling CO₂

Chesney and Taschini
(2008)

The Model

Numerical Results

Extensions

References

Weather Mansanet-Bataller et al. [2007] identifies extreme weather events as CO₂ price drivers.

Energy Convery and Redmond [2007] and Alberola et al. [2008] highlight the importance of energy-commodities (oil, gas, coal) due to the presence of fuel-switching options.

Securities Chevallier [2009] examines the empirical relationship with financial risk-factors and identifies the existence of a weak link.

CO₂ permit price formation

Other authors investigate the behavior of emission permits based on the pure univariate CO₂ time-series

Discrete The analysis of the conditional variance of the price return is performed by Paoletta and T. (2008) which propose and operationalize the use of a mixed-normal GARCH model. Benz and Trück [2009] suggests a Markov-switching model, and a standard AR(1)-GARCH(1,1) model.

Continuous Daskalakis et al. [2009] use a jump diffusion model to approximate the random behavior of the emission permit spot price.

CO₂ permit price formation

Finally, some authors develop stochastic equilibrium models in order to analyze the dynamic behavior of the permit price.

Discrete Chesney and T. (2008) develop an equilibrium model in the short term that accounts for trading interactions in the market for permits in the presence of asymmetric information.

Fehr and Hinz (2007) develop an equilibrium model considering utilities (in aggregate) that decide how much to abate relying on fuel-switching technology

Continuous Seifert et al. [2008] develop an equilibrium model considering a representative agent that decides whether or not to spend money on lowering emission levels.

Chesney and T. (2008) in a nutshell

- ▶ In Chesney and T. (2008) we model the price dynamics of marketable permits under asymmetric information, allowing banking and borrowing.
- ▶ The basic setup is a permit market lasting a finite T number of periods. There are a finite number of firms and each firm's emission follows an exogenously given stochastic process.
- ▶ The initial allocation of permits in each period to these firms is pre-determined and publicly known. In each period, a firm knows its own accumulated pollution level and those of the other firms up to the previous period.
- ▶ At the end of the period T , firms reconcile their permit holding with the accumulated emissions: if a firm fails having enough permits, it has to pay a penalty for each permit in shortage at a price P .

The model, the approach and results

Aim: The aim is to take into account the most important features of the EU ETS and provide a simple conceptual framework for the price of the emission permits.

Approach: A firm's strategy is to choose the optimal number of permits to buy and to sell in each period up to $T - 1$. Given the firms' trading decisions, the market clearing condition in each period determines the equilibrium permit price.

Results:

- ▶ The paper gives insights into the dynamics of the CO₂ permit price for a finite time horizon in the presence of asymmetric information.
- ▶ The model captures most of the features of the EU ETS and the equilibrium price for emission reflects the scarcity or excess of permits in the system;
- ▶ The optimal traded quantities have an impact on the evolution of permit prices.

Two-firm & Multi-period model: the ingredients

- ▶ Two companies, $i = \{1, 2\}$. At each time t , company i -th can buy (sell) a quantity $X_{i,t} > 0$ ($X_{i,t} < 0$) of emission allowances;
- ▶ The sum of each firm initial endowment, $N_{i,0}$, add-up to N , the total number of permits in the market is $N_{1,0} + N_{2,0} = N$
- ▶ Let us define $\delta_{i,t} := N_{i,0} + \sum_{s=0}^t X_{i,s}$ the net amount of permits for company i -th. Since the total number of permits is fixed, the market clearing condition is:

$$\delta_{1,t} + \delta_{2,t} = N \quad \Rightarrow \quad X_{1,t} = -X_{2,t} \quad \forall \quad t = 0, 1, \dots, T-1;$$

- ▶ Each company is characterized by an exogenous and instantaneous stochastic pollution process:

$$\frac{dQ_{i,t}}{Q_{i,t}} = \mu_i dt + \sigma_i dW_{i,t}.$$

where μ_i and σ_i are respectively the constant drift term and the constant volatility of the pollution process;

The boundary conditions

We assume that at time T the emission price is either zero or P (penalty).

$$S_T = \begin{cases} 0 & \text{if both want to sell} \\ P & \text{if at least one wants to buy} \end{cases}$$

Moreover, if company i -th is in allowance-excess at time T , it tries to sell a number of permits equal to Γ at the price P per unit where

$$\Gamma := \min \left(\left(\delta_{1,T-\Delta t} - \int_0^T Q_{1,s} ds \right)^+, \left(\int_0^T Q_{2,s} ds - \delta_{2,T-\Delta t} \right)^+ \right)$$

On the other hand, if i -th is in allowance-shortage, it has to "cover" (buying permits or paying the penalty) all emissions at a price P per unit of permits.

Hence, the final cash-outflow (inflow) of company i -th at time T is:

$$f(X_{1,T-\Delta t}) = \left(\int_0^T Q_{1,s} ds - \delta_{1,T-\Delta t} \right)^+ \cdot P - \Gamma \cdot P$$

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References

The equilibrium price at time $T - k\Delta t$

Starting at time $T - \Delta t$, we solve backward the problem H for company i at each instant $T - k\Delta t$ where $k \in [1, 2, \dots, T/\Delta t]$:

$$H \equiv \min_{\{X_{i,T-k\Delta t}\}} S_{T-k\Delta t} \cdot X_{i,T-k\Delta t} + e^{-\eta\Delta t} \mathbb{E}_{\mathbb{P}} \left[\sum_{h=1}^k e^{-\eta(h-1)\Delta t} \bar{S}_{T-(k-h)\Delta t} \cdot \bar{X}_{i,T-(k-h)\Delta t} | \mathcal{F}_{T-k\Delta t} \right].$$

We account for **asymmetric information** imposing a one-time lag effect on the known amount of the net emission permits of the other company. In other words, at time $t \in [0, T]$, company i knows

$$\int_0^t Q_{i,s} ds - \delta_{i,t-1}, \quad \int_0^{t-1} Q_{j,s} ds - \delta_{j,t-1};$$

and company j at time $t \in [0, T]$ knows

$$\int_0^t Q_{j,s} ds - \delta_{j,t-1}, \quad \int_0^{t-1} Q_{i,s} ds - \delta_{i,t-1}.$$

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Extensions

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The equilibrium price at time $T - k\Delta t$

Solving the problem, we obtain

$$\bar{S}_{T-k\Delta t} = e^{-\eta\Delta t} \mathbb{E}_{\mathbb{P}} \left[\bar{S}_{T-(k-1)\Delta t} | \mathcal{F}_{T-k\Delta t} \right].$$

Substituting, we can express the equilibrium price as

$$\bar{S}_{T-k\Delta t} = e^{-\eta k\Delta t} \cdot P \cdot \left\{ 1 - \mathbb{E}_{\mathbb{P}} \left[\Phi(-d_{i,T-\Delta t}) \cdot \Phi(-d_{j,T-\Delta t}^{\text{lag}}) | \mathcal{F}_{T-k\Delta t} \right] \right\},$$

where

$$d_{i,T-\Delta t} = \frac{\ln \left(\frac{Q_{i,T-\Delta t} \cdot \Delta t}{N_{i,T-2\Delta t} + X_{i,T-\Delta t} - \int_0^{T-\Delta t} Q_{i,s} ds} \right) + \left(\mu_i - \frac{\sigma_i^2}{2} \right) \cdot \Delta t}{\sigma_i \cdot \sqrt{\Delta t}},$$

$$d_{j,T-\Delta t}^{\text{lag}} = \frac{\ln \left(\frac{Q_{j,T-2\Delta t} \cdot 2\Delta t}{N_{j,T-2\Delta t} + X_{j,T-\Delta t} - \int_0^{T-2\Delta t} Q_{j,s} ds} \right) + \left(\mu_j - \frac{\sigma_j^2}{2} \right) \cdot 2\Delta t}{\sigma_j \cdot \sqrt{2\Delta t}}.$$

The equilibrium price at time $T - k\Delta t$

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Therefore, at each time step $T - k\Delta t$ where $k \in [1, 2, \dots, T/\Delta t]$ we obtain a pair ($i \neq j$) of equations for the emission price:

$$\bar{S}_{T-k\Delta t} = e^{-\eta k\Delta t} \cdot P \cdot \{1 - \mathbb{E}_{\mathbb{P}} [\mathcal{P}_i | \mathcal{F}_{T-k\Delta t}]\}, \quad (1)$$

where $\mathcal{P}_i =: \Phi(-d_{i,T-\Delta t}) \cdot \Phi(-d_{j,T-\Delta t}^{\text{lag}})$.

With these two equations and the market clearing condition, at each time step we determine the equilibrium permit price by numerically evaluating the quantity of permits that satisfies the following equality:

$$\mathbb{E}_{\mathbb{P}} [\mathcal{P}_i | \mathcal{F}_{T-k\Delta t}] = \mathbb{E}_{\mathbb{P}} [\mathcal{P}_j | \mathcal{F}_{T-k\Delta t}], \quad (2)$$

for a given set of parameters ($\{\mu, \sigma, \mathbf{Q}_0, \mathbf{N}_0\} \in \mathbb{R}^2$) that characterizes the two pollution processes.

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The Model

Numerical Results

Extensions

References

Multi-firm and Multi-periods trading

Along similar lines and splitting the set \mathcal{I} into two parts ($\mathcal{I} = I^- \cup i$), we generalize the model to \mathcal{I} companies in a multi-period setting. The equilibrium permit price result from the solution of a system of \mathcal{I} equations.

Proposition Given the exogenous pollution processes

$$\{Q_{i,t}\}_{t=0}^T \quad \text{for } i = 1, 2, \dots, \mathcal{I}$$

the process $\bar{S} = \{\bar{S}_t\}_{t=0}^T$ is called equilibrium permit price process, if there exists

$$\{\bar{X}_{i,t}\}_{t=0}^{T-\Delta t} \quad \text{for } i = 1, 2, \dots, \mathcal{I}$$

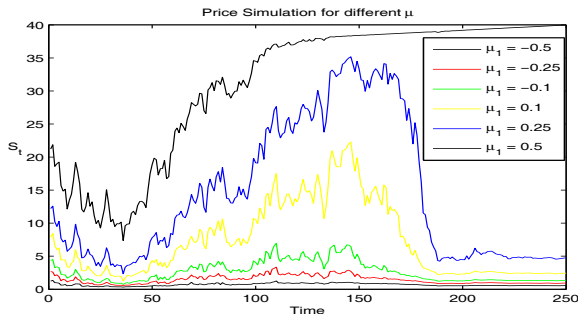
such that for all $i = 1, 2, \dots, \mathcal{I}$ and $t = 0, \dots, T - \Delta t$

$$\mathbb{E}_{\mathbb{P}}[\mathcal{P}_i | \mathcal{F}_t] = \mathbb{E}_{\mathbb{P}}[\mathcal{P}_i | \mathcal{F}_t],$$

and the market clearing condition is satisfied $\sum_{i=1}^{\mathcal{I}} \bar{X}_{i,t} = 0$ for all $t = 0, \dots, T - \Delta t$.

Numerical results: Sensitivity

The common starting pollution parameters are $\mu_2 = 0.001$, $\sigma = [0.10; 0.10]$, $\mathbf{Q}_0 = [50; 25]$, $\mathbf{N}_0 = [52; 25]$ - $N_0 \approx Q_0 \cdot \int_0^T e^{\mu T} dt$ where $T = 1$.

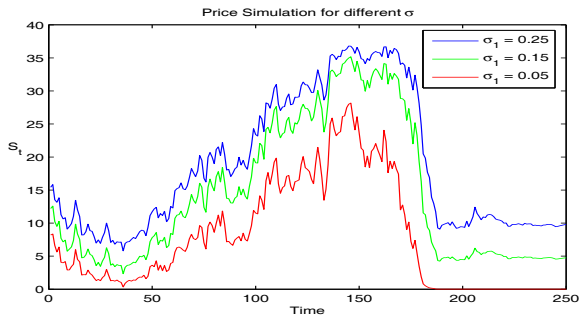


The larger μ_1 the higher the probability of being in shortage by the end of the period, i.e. the higher the permit price.

Remark: For each drift but $\mu_1 = 0.50$, as time goes by and uncertainty is resolved, the permit amount value is sufficiently large to reverse such a trend.

Numerical results: Sensitivity

The common starting pollution parameters are $\mu = [-0.15; 0.001]$, $\sigma_2 = 0.10$, $\mathbf{Q}_0 = [50; 25]$, $\mathbf{N}_0 = [52; 25]$.

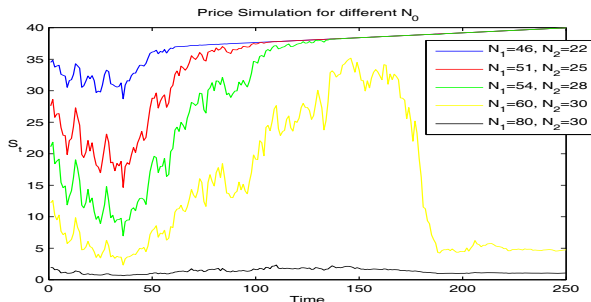


The larger σ_1 the higher the uncertainty about the net permit position before the compliance date, and consequently about the probability of future shortage situations for both companies.

Remark: When more information about the accumulated pollution volumes is collected, the confidence about the permit amount value takes precedence over the overall uncertainty level and leads to a price decrease.

Numerical results: Sensitivity

The common starting pollution parameters are $\mu = [-0.15; 0.001]$,
 $\sigma = [0.10; 0.10]$, $\mathbf{Q}_0 = [50; 25]$.



The more generous the initial allocation, i.e. \mathbf{N}_0 , the lower the permit price.

Remark: The shortage status becomes obvious in the first three cases (and the permit price is simply the discounted penalty level) only after some time. Uncertainty about future pollution (production) and about the permit net-positions of the market players play an important role.

Potential implications and extensions

Pricing: The obtained dynamics can be used to price any project whose value *derives* from the future CO₂ spot permit price. Important examples are:

- ▶ Project-based investments, i.e. those investments committed under the so called CDM and JI mechanisms, that at regular intervals return emission reduction certificates yielding a payoff that depends on the CO₂ permit market price.
- ▶ Technological abatement investments or production process modifications that can be valued in terms of costs saved from purchasing emission permits or revenue from the sales of extra unused permits.

Policy: Introducing endogenous pollution processes would allow to test whether the modification of the production process - according to the accumulated pollution level - have an impact on the corresponding permit price evolution. In other words, test if markets for permits do indeed spur innovation.

- E. Alberola, J. Chevallier, and B. Chéze. Price drivers and structural breaks in the european carbon prices 2005-2007. *Energy Policy*, 36(2):787–797, 2008.
- E. Benz and S. Trück. Modeling the price dynamics of CO₂ emission allowances. *Energy Economics*, 31(1):4–15, 2009.
- M. Chesney and L. Taschini. The endogenous price dynamics of emission allowances and an application to CO₂ option pricing. Swiss Banking Institute, University of Zürich, Switzerland, 2008.
- J. Chevallier. Carbon futures and macroeconomic risk factors: A view from the eu ets. *Energy Economics - forthcoming*, 2009.
- F. J. Convery and L. Redmond. Market and price developments in the European Union Emissions Trading Scheme. *Review of Environmental Economics and Policy*, 1(1):88–111, 2007.

- G. Daskalakis, D. Psychoyios, and R. Markellos. Modeling CO₂ emission allowance prices and derivatives: Evidence from the european trading scheme. *Journal of Banking & Finance*, page forthcoming, 2009.
- M. Mansanet-Bataller, A. Pardo, and E. Valor. CO₂ prices, energy and weather. *The Energy Journal*, 28(3):67–86, 2007.
- M. S. Paoletta and L. Taschini. An econometric analysis of emission–allowances prices. *Journal of Banking and Finance*, 32(10):2022–2032, 2008.
- J. Seifert, M. Uhrig-Homburg, and M. Wagner. Dynamic behavior of CO₂ spot prices. *Journal of Environmental Economics and Managements*, 56:180–194, 2008.