Manipulability, Decisiveness, and Responsiveness in Voting Rules

Voting Power in Practice Workshop
Sponsored by the Leverhulme Trust
Château du Baffy, Normandy
July 30 - August 2, 2010

William S Zwicker

Abstract One method for comparing voting rules is to measure the probability (under some distribution) that a profile is subject to a "voting paradox":

- a failure of *monotonicity*, or
- a *no-show paradox*, or
- a failure of *one-way monotonicity*, or
- a susceptibility to *strategic voting*.

We argue that a comparison based on the severity of these paradoxes is incomplete and potentially misleading, unless it also takes two other aspects into account:

- *decisiveness* (infrequency of ties), and
- *positive responsiveness*.

When a single voter switches her ballot, we do not wish the election outcome to change *inappropriately*, as it does with the paradoxes. But there are two alternatives to inappropriate change: no change whatsoever, and appropriate change. The *omninominator* rule (wherein an alternative wins if it is top-ranked by at least one voter) is almost completely immune to the above paradoxes. Yet for most voting applications, it would never be considered – it has far too many ties. Indeed, omninominator is resistant to strategic manipulation precisely because a change in one ballot almost never affects the outcome.

It seems that one way for a voting rule to dodge inappropriate outcome changes is to resist change in general, and one way to do that is through an elevated frequency of ties. With three alternatives, for example, the Copeland rule is more resistant than is the Borda count to manipulation. Yet Copeland has many more ties than Borda. What should we conclude?

There are further subtleties in terms of distinguishing appropriate from inappropriate change.
Suppose voter v switches her ranking from σ to τ, and the election winner changes from x to y. If we assume σ to be v's sincere vote and if y is above x in σ, then we deem the move a manipulation by v. But suppose y is above x in τ, and we instead assume τ to have been the sincere vote. We might now consider the change in winner to be appropriate; it is a *positive response* in that it encourages a sincere vote. One might argue, then, that certain appropriate responses are paid for with inappropriate ones. Indeed, for *one-way monotonic* rules (such as Borda) *each* manipulation can be seen as positive response, turned on its head. Not every rule can offer such a mitigating interpretation, though.

We have, as yet, no recommended method for weighting the relative importance of avoiding paradox, achieving positive response, and choosing a unique winner. There does seem to be evidence, though, that such a method is needed. These factors can only be reasonably understood as a package.