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## **Some Informal Remarks on Devising a “Fair” Decision-Making Rule for Representative Assemblies<sup>1</sup>**

In his presentation Moshé Machover distinguished between two kinds of representative assemblies, each of which can be elected by using either a deterministic or a probabilistic voting procedure:

- 1) An assembly which is a microcosm of the entire electorate and where every member represents an ideologically homogeneous but geographically dispersed constituency. Machover argues – and I agree – that the only way to obtain such an assembly by using a deterministic voting procedure is to use the list system procedure.
- 2) An assembly where every member represents (is an agent of) an ideologically diverse but geographically contiguous constituency, of which s/he is in some sense the “best” representative.

However, Machover decided not to address an important related issue, i.e., how to devise a “fair” decision-making rule to be used by the assembly. I would like to dwell on this issue.

As far as I know, it is a universal practice in democracies that the decision rule used by both types of assembly – regardless of whether they are elected by a deterministic or by a probabilistic procedure – as well as the decision rule used in popular referenda, is majoritarian.<sup>2</sup> The fact that democracies employ decision-making rules within representative assemblies that are based solely on the majority principle could lead to the conclusion that a decision based on this principle can always be reconciled with democratic principles. In fact, “there is nothing inherent in democracy that requires majority rule” (Guinier, 1994, p. 17). Moreover, the majority rule principle implies that the minority of representatives in an assembly are unable to affect reality – even if their number is exactly proportional to the proportion of the entire electorate who supported them in the election.

So the universal use of a majoritarian decision rule within representative assemblies, as well as in popular referenda, causes the minority to become totally impotent in shaping public policies. Such impotence is especially serious if the

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<sup>2</sup> A majoritarian decision rule requires that in order to change the status quo slightly more than half the voters and fewer than all the voters must support this change. Note that since a requirement of unanimity enables every voter to veto a change of the status quo, unanimity is not considered a majoritarian voting rule.

minority group is relatively large and it is a permanent one, i.e., it always consists of the same (type of) voters who may belong to the same ethnic group, or the same ideological party, or the same geographic region.

In contemplating which decision rule(s) ought to be used by a representative assembly when it engages in tasks involving the selection of one out of several possible alternatives, let us first consider three alternative political/philosophical principles/goals:

(i) **Majority rule:** to guarantee that the alternative preferred by the majority of voters (or representatives) will be selected.

(ii) **Equiprobability of success:** to let every voter (or representative) have the same probability that his/her preferred alternative is the one selected.

(iii) **Equal opportunity to avoid the worst:** to provide every voter (or representative, or alliance) with an ability to prevent that his/her/its least preferred alternative is the one selected.

To achieve the first goal one must use a deterministic voting procedure and select the alternative supported by the majority of voters/representatives.

To achieve the second goal one must use a probabilistic voting procedure which assigns to every voter/representative the same chance of being selected - and the selected voter will then state which alternative s/he prefers.<sup>3</sup>

To achieve the third goal one must enable every voter, or group of voters of some minimal size, to veto one of the alternatives under consideration, thus guaranteeing that every voter or group of voters has some minimal effect on the selected outcome.

Achieving the first goal can never make, by definition, the majority of voters very miserable, but it may make the minority of voters very miserable. On the other hand, achieving the second goal may make either the majority or the minority very miserable if a member of the other group is selected to choose the alternative to be adopted. (Of course there is a higher probability that the selected person will belong to the majority than to the minority group).

It seems natural that we prefer the possibility that the minority may be miserable over the possibility that the majority may be miserable - and hence prefer to realize the first goal over the realization of the second goal. Moreover, since the realization of the second goal involves the

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<sup>3</sup> In an assembly made up of party-blocs, the achievement of this goal would require the probability of success of each bloc to be proportional to its weight.

employment of a probabilistic voting procedure, this procedure may cause additional problems such as instability and/or inconsistency in public policies and decision-making.<sup>4</sup>

The realization of the third goal has two advantages over the realization of the other two goals: a) It leads to the selection of an alternative that is stable regardless of whether the social preference ordering among the available alternatives contains cycles.<sup>5</sup> b) If voters behave rationally, the selected alternative is not only Pareto-optimal – that is, no other alternative is preferred by all voters over the selected alternative – but it also does not constitute any voter's least preferred alternative.

It is quite easy to realize the third goal when the number of representatives ( $n$ ) is smaller than the number of policy alternatives ( $m$ ) of which one or more must be selected. In this case every representative, in his/her turn, can veto one or more alternative(s) (depending on his weight) – and the alternative(s) that was/were not vetoed are selected.<sup>6</sup> However, implementing this goal is difficult when the number of representatives in the assembly is larger than the number of available alternatives. Implementing this goal in this case – which is common in actual representative assemblies – implies that more than one representative is needed to veto any given alternative, and the formation of the needed coalition(s) may become complicated. Moreover, a good theory as to how to analyze such cases is still lacking.

So how, if at all, is it possible to adopt one or more decision rules in representative assemblies which will provide the minority of voters with both a priori as well as actual voting power proportional to their weight?

I think the answer to this question is twofold:

a) First, act according to a fourth political/philosophical principle/goal designed to achieve **proportionality of a priori voting power – which means equiprobability of being critical-to weight**. Given that the number of seats controlled by the various parties or districts in a representative assembly is proportional to the number of relevant voters in the

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<sup>4</sup> Instability may be caused by the losers' demand following any given division to conduct another round of voting, whereas inconsistency may be a result of adopting contradictory policies that were selected interchangeably by voters belonging to the majority and minority groups.

<sup>5</sup> A stable alternative is an alternative that cannot profitably be objected to by any voter or coalition of voters. Note that, in contrast, no alternative is stable if one uses a majoritarian decision rule and the social preference ordering among the available alternatives contains a top cycle.

<sup>6</sup> This procedure is known as Sequential Voting by Veto (SVV). It was proposed originally by Mueller (1978) who presented an algorithm for determining the winning alternative under SVV, given the order in which the representatives cast their vetoes. Moulin (1981, 1983: 138–140) extended Mueller's idea to any situation in which  $n$  voters have to select one out of  $n + 1$  alternatives and they have complete information on all other voters' preference orderings among the alternatives. Felsenthal and Machover (1992) generalized the Mueller-Moulin result to a situation in which  $n$  voters must select  $s$  out of  $m$  alternatives ( $s > 0$ ;  $m > n \geq 2$ ). For laboratory experiments with small groups operating under SVV see Yuval (2002) and Yuval and Herne (2005).

electorate,<sup>7</sup> one looks for a decision rule to be used by the assembly such that the **a priori relative voting power** of every representative (as measured by Banzhaf's index of relative voting power) will be as close as possible to each representative's relative weight, i.e., the proportion of voters s/he represents.<sup>8</sup>

However, the realization of this goal too is not problem-free. To understand just one of the problems associated with it consider the following simple example.

Suppose a 99-seat assembly with three parties, each controlling 33 seats because each received an equal proportion of the votes in an election. The **relative a priori** voting power of each party will be  $1/3$  - which is proportional to each party's weight - regardless of whether the quota ( $q$ ) needed to pass resolutions is  $34 \leq q \leq 66$  (simple or qualified majority) or  $67 \leq q \leq 99$  (unanimity). However, the **absolute a priori** voting power of every party (as measured by the Penrose measure) would be  $1/2$  if only simple/qualified majority is required to pass a resolution but only  $1/4$  if unanimity is needed to pass resolutions. So which quota would be fairer in this case? It would seem that here unanimity would be fairer than simple/qualified majority, because under simple/qualified majority any single party may become a de facto dummy if the other two parties form a relatively long-term binding coalition (alliance). Of course the price to be paid for granting in this case veto power to each of the three parties, is not only significant loss of (a priori) absolute voting power by each of the parties, but also the possibility of total paralysis of the legislature inasmuch as any of the parties is unable to agree on the passage of any bill.

b) So perhaps some milder form of de facto (proportional) voting power would be preferable than awarding each of the three parties in the above example veto power regarding all proposed bills. It should be possible to institute arrangements, at least with respect to certain kinds of decisions, e.g., budgetary decisions, or decisions regarding certain regions or policy areas, which will enhance the a posteriori (actual) voting power of representatives belonging to the minorities in legislatures. For example, if the Red and Blue parties control 40 and 60 percent of the seats, respectively, in a representative assembly, then one can institute an arrangement where the Blue party would be given the prerogative of determining the total size of the annual budget, as well as dividing it into two parts - one containing

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<sup>7</sup> The relevant electorate is either the number of voters who supported each party represented in the assembly, or the number of voters belonging to each constituency (district) represented in the assembly, or the size of financial contribution of each member-country to the common fund, e.g., the financial contribution of each member of the International Monetary Fund to its fund.

<sup>8</sup> For the various measurements of a priori voting power see Felsenthal and Machover (1998).

60 percent of the total planned expenditure and the other containing 40 percent of the total expenditure – and let the Red party have the sole prerogative to decide how the 40-percent part of the budget would be allocated.

Of course such power-sharing arrangements – in decision making bodies in general and legislatures in particular – are not problem-free, and their details are crucial. Yet it seems to me that the discussion and development of such proportional power-sharing arrangements should constitute a major new area for social choice theory to be engaged in.

So far the relevant disciplines (social choice, political science, economics, mathematics, law, philosophy) focused mainly on how to elect a representative assembly, and to a lesser degree on how to measure the a priori voting power of representatives in an assembly. In my opinion the time has come for them to shift their focus to how to devise a fair and practical decision rule(s) for a representative assembly so that **all** its members will have **actual** voting power which is as close as possible to their relative weight.

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