Voting power as a probability when votes are not independent

by Serguei Kaniovski

WIFO

Austrian Institute of Economic Research

The Weberian Definition of Power

"Power is the probability that one actor within a social relationship will be in position to carry out his own will despite resistance, regardless of the basis of on which this probability rests"

Voting Power - The Power of the Vote

- Voting power is the probability of casting a decisive vote
- Voting power depends on circumstances created by others casting their votes so that the voter has opportunities to be decisive
- Power, influence, or authority exerted by means other than casting one's vote is not voting power, but it may augment or diminish voting power indirectly

An “Absurd” Simple Majority Game

Coalitions			Prob.
1	1	1	0.5
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0.5

- Voters may have power, but they have no voting power

- Thinking why a situation like this may occur takes us beyond Simple Voting Games and into richer empirical contexts

Straffin's (1977) Prescription for Empirical Work

- Assessment of the empirical distribution of *a priori* voting power

- Independence Assumption: The p_i's are selected independently from the uniform distribution on $[0, 1]$ – Principle of Insufficient Reason

- Homogeneity Assumption: A number p is selected from the uniform distribution on $[0, 1]$, and $p_i = p$ for all i – Common Standards

- Independence Assumption – Binomial model – Bz measure

Bz Conditional (Swing) Probability

- All votes correlate

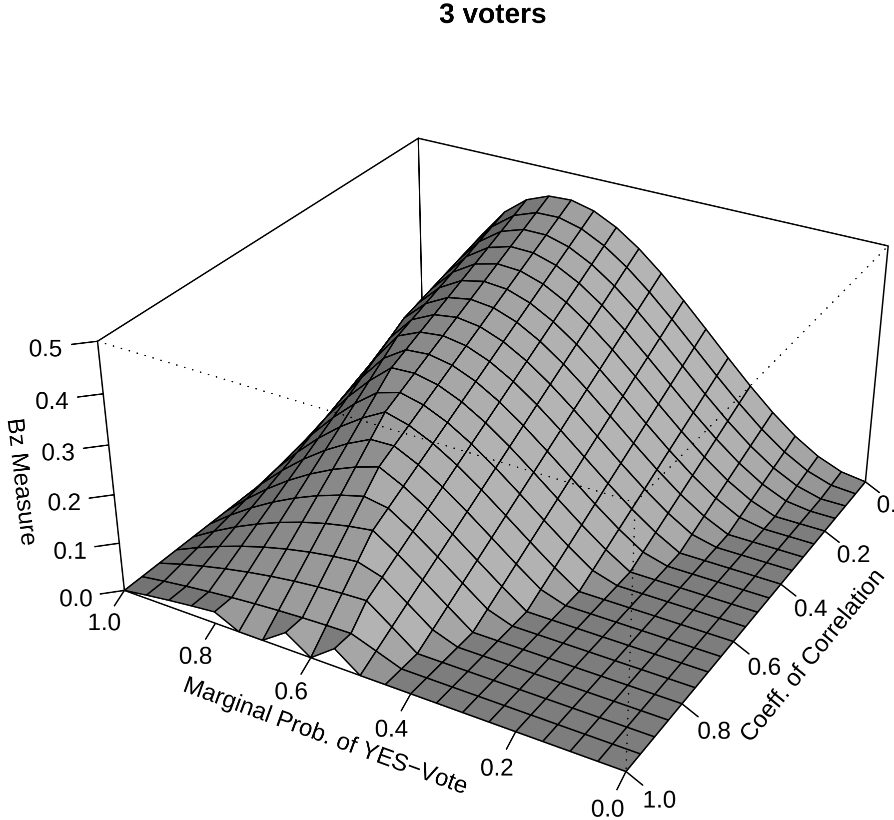
- $P(i \text{ is critical}|i \text{ votes YES}) = \frac{1}{p_i} \sum_{i \in \mathbf{S}} \pi_{\mathbf{S}} \left[w(\mathbf{S}) - w(\mathbf{S} \backslash \{i\}) \right]$

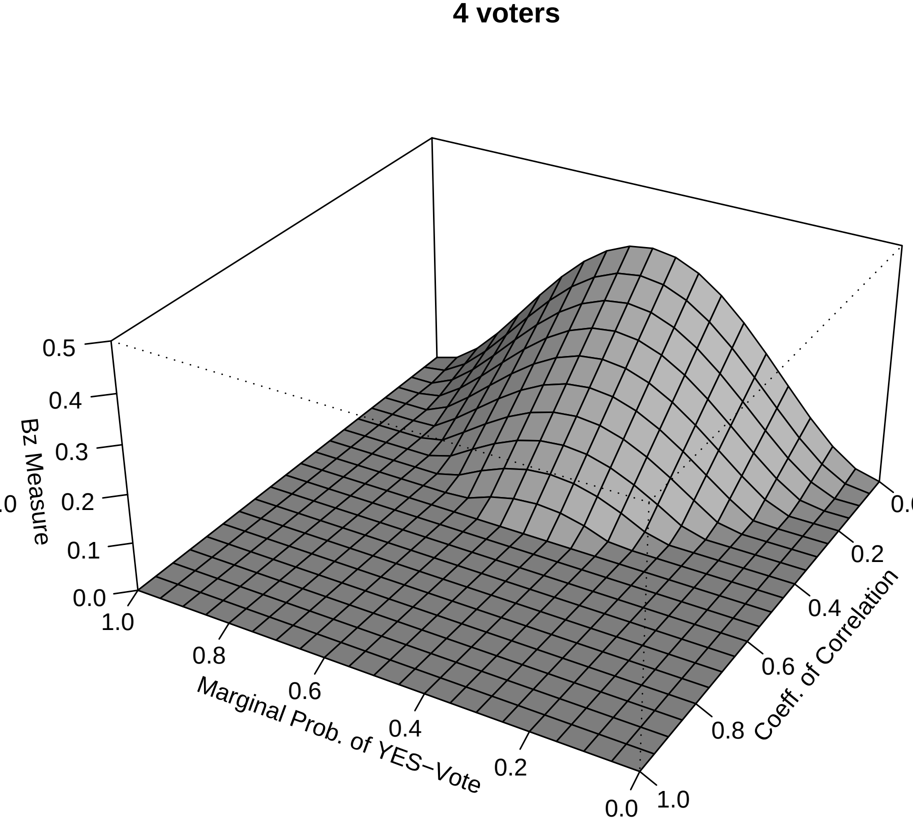
- i is independent of the rest, and the rest is independent of i

- $P(i \text{ is critical}) = \sum_{i \in \mathbf{S}} \pi_{\mathbf{S}} \left[w(\mathbf{S}) - w(\mathbf{S} \backslash \{i\}) \right]$

- The characteristic function w does not depend on the distribution π

The Bz measure of power in an unweighted simple-majority game





An Aggregation Problem

- $\pi_{\mathbf{s}} \geq 0 \quad \mathbf{s} = (v_1, v_2, \ldots, v_n) \quad v_i = \{1, 0\}$

- $\sum_{\mathbf{s} \in \mathbf{S}} \pi_{\mathbf{s}} = 1 \qquad \sum_{\mathbf{s} \in \mathbf{S}(i)} \pi_{\mathbf{s}} = p_i \qquad \sum_{\mathbf{s} \in \mathbf{S}(ij)} \pi_{\mathbf{s}} = p_i p_j + c_{ij} \sqrt{p_i q_i p_j q_j}$

- $\min_{\pi_{\mathbf{s}}} \frac{1}{2} \sum_{\mathbf{s}} \left[\pi_{\mathbf{s}} - \prod_{i=1}^{n} p_i^{v_i} q_i^{(1-v_i)} \right]^2$

- Proof of the existence of a probability distribution by construction

- Analytical solution

Applications

- Forecasting Bz probabilities using calibrated, or estimated models

- Scenario analysis in heterogeneous voting bodies, effects of strategies, information, preferences, opinion leadership, etc.

- The bounds approach: $\beta_i^l = \min_{\pi_{\mathbf{s}}}(\beta_i)$ and $\beta_i^h = \max_{\pi_{\mathbf{s}}}(\beta_i)$

- The heterogeneity-modified Penrose square-root rule: The more homogeneous the constituency is, the lower the voting power of its citizens will be, and the higher the voting power of their delegate ought to be if all citizens are to have equal powers

Correlations as Preferences

- Correlation is a pairwise property of random variables
- Two uncorrelated Bernoulli random variables are independent, but zero correlation does not imply independence in general
- Negative correlations pose a constraint in binary choice situations
- The property of being positively correlated is not transitive: $c_{x,y} > 0, c_{y,z} > 0 \Rightarrow c_{x,z} > 0$ iff $c_{x,y}^2 + c_{y,z}^2 > 1$