Voting Models for the Council of Ministers of the European Union

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Abstract

The Impartial Culture model predicts an unrealistically low probability of approval at the European Council of ministers. Here a Generalized Impartial Anonymous Culture model is introduced. In the asymptotic limit, reached when the number of voters goes to infinity, the probability of approval for both one and two key votes is computed. Using computer simulations, we check how these hypothesis apply to EU27.

JEL classification: D7

1 Introduction

In the last five years, a considerable body of research on the choice of the best voting rules for federal union has been inspired by the debates on the Treaty of Nice and the projects for an European Constitution [1, 2, 3, 5, 8]. In all these contributions the authors propose a voting model, and then search for the voting rule that has the best fit according to some normative criteria.

Recently a welcome and useful discussion has developed between a high level politician and voting specialists [9, 6]. The starting point of this discussion is a scientific analysis, based on the Impartial Culture (IC, hereafter) model, of the Treaty of Nice [6] claiming that the need of 255 (or 258) mandates (on a total of 345) will result in a serious deadlock at the council of ministers of the European Union, with an *a priori* probability of approval of 2%. A. Moberg disagrees strongly, pointing out that the result ignores the "strong consensual culture of the E.U.". The IC model, which states that each country chooses to vote 'yes' or 'no' independently with equal probability, is a possible one. But another exists, which is more subtle and less easy to compute. This model, called Impartial Anonymous Culture (IAC), asserts that all the distributions of the votes are equally likely. The aim of this study is to show that the use of a model related to the IAC condition is able to give answers which are closer to the reality of the European Union with 27 members (EU27 hereafter) and, in some way, takes into account the consensual character of the vote. By departing from the common IC assumption, we obtain a theoretical probability of passing a motion that turns out to be higher. Our result concerns not only the Treaty of Nice with its famous 74.8% majority rule (one key vote), but also one of the decision schemes that have been suggested during the debates for the European Constitution (double key vote: a motion is passed if it is supported by more than 50% of the countries gathering more than 50% of the population).

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2 The different models

We consider binary issue votes 'yes' or 'no' for the N states. In the simplest IC model, each vote is independent of the others and each voter says 'yes' or 'no' with equal probability $p = 1/2$. IC has serious drawbacks. It describes a vote where everybody is undecided (no exchange of points of view allowing the emergence of a majority has taken place); the vote will be won by a narrow margin going as $N^{1/2}$. This explains the low probability of approval with a quota of 258/345 *i.e.* 74.8% in the Treaty of Nice decision scheme. The idea is consequently to introduce a model where a probability p different from 1/2 has emerged. Moreover, our knowledge of p is itself of a probabilistic nature, it is mathematically described by the function $f(p)$ which is the probability distribution of p. The emergence of a probability p different from 1/2 seems rather natural in an assembly where certainly long discussions, explanations, compromises, package deals, etc... precede each vote (the "consensual culture" of A. Moberg). Notice that all these discussions are resumed in a $p \neq 1/2$ and that the subsequent votes are independent. Then, we introduce the Generalized Impartial Anonymous Culture (GIAC, Hereafter) model is characterized by a given $f(p)$ with $0 \leq p \leq 1$, $f(p) \geq 0$ and $\int_0^1 f(p)\mathrm{d}p = 1$. The function $f(p) = 1$ gives the IAC model. With this model, the average number of votes for which n voters of equal importance, on a total of N, have voted 'yes' reads

$$C_N^n \int_0^1 p^n \, (1-p)^{N-n} \mathrm{d}p = \frac{1}{N+1}. \tag{1}$$

All values of n (from 0 to N) have the same probability $1/(N+1)$, consequently, for the IAC model, the probability of having n 'yes' on a total of N voters is a flat curve. It is also easily proved that, if both n and N go to infinity while the ratio n/N is kept constant, the probability distribution of n is

$$F_N(n) = \frac{1}{N} \, f(\frac{n}{N}). \tag{2}$$

This result is a direct consequence of the possibility of interpreting a probability as a frequency when the number of random drawings goes to infinity.

Now let us suppose that each of the N voters has one mandate and that Q mandates are needed for an approval. Let $q = Q/N$. Then for the IAC model, the probability of approval is $1-q$, independent of N. For $q = .75$, for example, the IAC model gives a 25% chance, while the IC model gives 0.3% for $N = 27$.

3 The probability of approval in the asymptotic limit

In a more generalized case, voter i has a_i mandates; moreover, two kinds of mandates have been proposed in the E.U constitution : each voter has two mandates a_i and b_i, and his (her) vote ('yes' or 'no') is used in two qualified majority games $\mathcal{A}$ and $\mathcal{B}$, the respective quotas being Q_A and Q_B. The two quotas must be reached for final approval, each one being related to a certain type of legitimacy. The EU Constitution project proposes for country i to take $a_i = 1$ and b_i equal to the population of state i.

In order to be able to perform analytical computations, to see the role of $f(p)$ and the influence of the different quotas, we suppose that N is large enough to use asymptotic calculations. At the end,

we will compare the asymptotic results we obtain to numerical simulations[2].
For the GIAC model, characterized by $f(p)$, the distribution function for x mandates in favor of approval in the single key case reads

$$F_N(x) = \frac{1}{A} f(x/A), \tag{3}$$

while for the double key case, we obtain

$$F_N(x, y) = \frac{1}{AB}\, \delta(\frac{x}{A} - \frac{y}{B})\, f\left(p = \frac{x}{A} = \frac{y}{B}\right), \tag{4}$$

where δ is the Dirac distribution and x and y are the numbers of mandates in favor of approval for keys $\mathcal{A}$ and $\mathcal{B}$ respectively. Note that x and y are strictly correlated ($\frac{x}{A} = \frac{y}{B}$). In formulae (3) and (4), A and B are defined by $A = \sum_{i=1}^{N} a_i$ and $B = \sum_{i=1}^{N} b_i$.
For the IAC model ($f(p) = 1$), equation (3) means a flat density from $x = 0$ to $x = A$, while for the double key vote, the points are located on the segment joining $(x = 0, y = 0)$ and $(x = A, y = B)$ with a uniform distribution on this segment.
Note that these results hold for N going to infinity. It can be shown that the first correction (N large but not infinite) provides a diffusion around the points $(x = pA, y = pB)$ in $N^{-1/2}$. While this scattering slightly modifies the flatness of $F_N(x)$ for the one key vote, it transforms the segment of the two key vote into a long ellipse with a ratio long over small axes in $N^{1/2}$. Coming back to the segment structure, we see that in the double key vote case, with two unequal quotas, it is the one with the highest quota which will set up the frequency of 'yes' votes. Also, we must point out that equation (3) is a generalization of equation (2) obtained for a set of voters with one mandate each (in that case $A = N$). Notice also that integration of (4) respectively on y and x gives

$$F_N(x) = \frac{1}{A}\, f(\frac{x}{A}), \quad \text{and} \quad F_N(y) = \frac{1}{B}\, f(\frac{y}{B}), \tag{5}$$

in agreement with the results of the one key vote[3].
To end this section, a comparison with the results of the IC model is in order. The above treatment with $f(p) = \delta(p - 1/2)$ concentrates all the points at the central point $(x = A/2, y = B/2)$. This confirms the quickly decreasing probability of approval when the quotas are not very close to 1/2. For the double key vote, with the IC model, the authors have carried the computations of the next term to obtain the scattering around the central point. The approval probability P, obtained analytically for the two relative quotas equal to 1/2 reads

$$P = \frac{1}{\pi}\, \arctan(\frac{\sqrt{1+r}}{\sqrt{1-r}}) \tag{6}$$

[2]The details of the calculations are not given in this short text. They are available on the internet site of J.L. Rouet: www.univ-orleans.fr/SCIENCES/MAPMO/membres/jlrouet

[3]Concerning the double key vote, it is worth noticing that, for any N (not necessarily going to infinity) and for quotas equal to $A/2$ and $B/2$, the voting power of a state X (*i.e.* its probability of being pivotal) is given by $P(X) = \frac{P_A(X)+P_B(X)}{2}$ where $P_A(X)$ and $P_B(X)$ are the voting powers of X with (respectively) keys $\mathcal{A}$ and $\mathcal{B}$. This result is valid for IC and IAC or GIAC models when $f(p) = f(1-p)$.

where $r = \sum_{i=1}^{N} a_i b_i / \left[\sum_{i=1}^{N} a_i^2 \sum_{i=1}^{N} b_i^2\right]^{1/2}$ is the correlation factor between vote $\mathcal{A}$ and vote $\mathcal{B}$. P varies from $1/4$ ($r = 0$) to $1/2$ ($r = 1$, obtained for $b_i = ka_i$, in fact a single vote).

4 Numerical simulations

In this section, the results of numerical simulations will be shown for the IAC case. Because we want to reach the asymptotic limit which supposes both an important number of elections and a large number of voters, the Monte Carlo method should be used. Actually, it is not possible, when N is large, to enumerate, stock and compute the 2^N configurations because of lack of memories and computation time. In addition, the Monte Carlo technique will illustrate clearly the double probabilistic character of the IAC model. This method consists in making a random sampling among all the vote configurations, but without taking all of them.

The method has two steps. First a probability p is chosen at random in the distribution function $f(p)$. Second a vote configuration is chosen in accordance with this probability p : for each of the N voters, a random number is taken in a uniform distribution, if this number is lower than p, it is a 'yes' vote while it is a 'no' vote if the number is higher. This is in fact an acceptation-rejection method and if the number of voters is large, the number of 'yes' voters divided by N will tend toward p. This process is repeated for a large number of elections with, at each election, a choice of a new p into $f(p)$ and so on. Notice that the results of the IC model could also be obtained by this technique, with $f(p) = \delta(p - 1/2)$.

First we give the obtained results for a large number of voters ($N = 100$) and $M = 50\,000$ elections, both for a single key and a double key vote. For the single key case, figure 1 shows the histogram of the number of configurations, as a function of the related number of mandates. The histogram is flat in agreement with equation (3) and the probability of approval is very close to $(1 - q)$.

For the double key case, figure 2 gives the distribution of the M elections performed in the plane (x, y), one point representing one election. Because all the points have the same weight, their density gives the value of $F_N(x, y)$ (see equation (4)). As expected, the points are roughly distributed on the segment delimited by the two points $(0, 0)$ and (A, B). In addition to this global behavior, the distribution shows a certain scattering. It has been checked that, if the number of voters is increased, the scattering of the points decreases as expected. We have also checked that, for this case, the probability of approval is closely given by $1 - sup(q_A, q_B)$, where $q_A = Q_A/A$ and $q_B = Q_B/B$. For example, for $q_A = q_B = 70\%$, we get 29.3% of approval and for $q_A = 50\%$ and $q_B = 80\%$, we get 20.4%.

Now, the question is to know whether or not the asymptotic limit is a good approximation for the EU27. It is here possible to enumerate the 2^{27} vote configurations (taking care of their different weights). For the single key case, figure 3 shows the histogram of the number of configurations as a function of the related number of mandates, which have been taken proportional to the square root of the state populations. This choice is in accordance with the principle used in the EU15 and constitutes a good compromise between state and citizen legitimacies (see [2]). Again, the curve is rather flat, at least for q between 0.2 and 0.8, indicating that the asymptotic limit could be used for this single key vote as already stated in [4] in the IC model case.

For the double key case, we turn back to Monte Carlo simulations (although complete enumeration is possible) because each point has the same weight. Then, it is easier to interpret figure 4 which gives

Figure 1: One key vote. Distribution of the results of the votes for $N = 100$ voters and $50,000$ elections using the Monte Carlo technique. The mandates are chosen at random in a ratio 1 to 5 and the sum is normalized to 100.

Figure 2: Double key vote. distribution of the results of the votes for $N = 100$ voters and $50,000$ elections using the Monte Carlo technique. The mandates are chosen at random in a ratio 1 to 5 and the sum is normalized to 100. Each point represents an election.

the distribution of 2,700 vote configurations in the plane (x, y) (one point represents one election). For key $\mathcal{A}$, all the mandates are equal to 1 (state legitimacy) while for key $\mathcal{B}$, the number of mandates of a state is proportional to its population. Because of the discrete nature of the key $\mathcal{A}$ mandates, the points are aligned on vertical lines distant of 1. The scattering of the points, not negligible, is compatible with the $N^{-1/2}$ law as stated before. Nevertheless the rule $1 - sup(q_A, q_B)$ for the approval is fairly satisfied as shown by table 1.

5 Conclusion

Except if we take quotas closed to 1/2, IC and IAC (or GIAC) give results which differ by a large factor. Can we decide which model is the more appropriate ? The question is of great importance if we remember that the two main power indices (Banhzaf and Shapley–Shubik) are respectively based on IC and IAC models. In [3], the authors have criticized the IC model which describes so tied elections that they can be considered as not having fulfilled their role. In this paper, the GIAC model (with its arbitrary $f(p)$) was introduced and allows us to produce more sensible results. We have shown that the critics of A. Moberg were directed against the IC model but can be easily answered through the use of the IAC model.

Figure 3: One key vote. Distribution of the results of the votes for the EU27. The mandates are proportional to the square root of the populations and the sum is normalized to 100.

Figure 4: Double key vote. Distribution of the results of the votes for the EU27. For key A (x variable) all the mandates are equal to 1, for key B (y variable) the mandates are proportional to the populations and the sum is normalized to 100.

Finally, it is of interest to mention a recent study by Gelman *et alii* [7] that gives first insights on the nature of the relevant probability models. The chief merit of this study is that it analyzes data from American and European elections. It is shown that, for elections with a large number of voters N, the $N^{1/2}$ scale for the differences between two issues is not correct and must be replaced by an N^{α} scale. Using statistical technique, the authors arrive at $\alpha = .9$, but themselves insist that this value must be taken with caution and that a N scale may be correct. This confirms that the search for the adequate $f(p)$ (which must be reasonably stable from one election to the other) is of crucial importance. Of course, this difference between IAC and IC votes is much less important when the number of voters is small. But, with 27 members, the process of voting may become more frequent and more important, then realistic models must be used.

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		Key $\mathcal{A}$				
		14	17	19	22	25
	50%	45.44	38.27	31.92	21.42	10.71
	60%	39.56	35.60	30.86	21.33	10.71
Key $\mathcal{B}$	70%	31.56	30.20	27.73	20.68	10.71
	80%	22.75	22.54	21.86	18.40	10.55
	90%	13.64	13.64	13.61	12.80	9.03

Table 1: Double key vote. Percentage of approval for the EU27 as a function of the two quotas Q_A and Q_B. The results have been obtained by computing of all the vote configurations.

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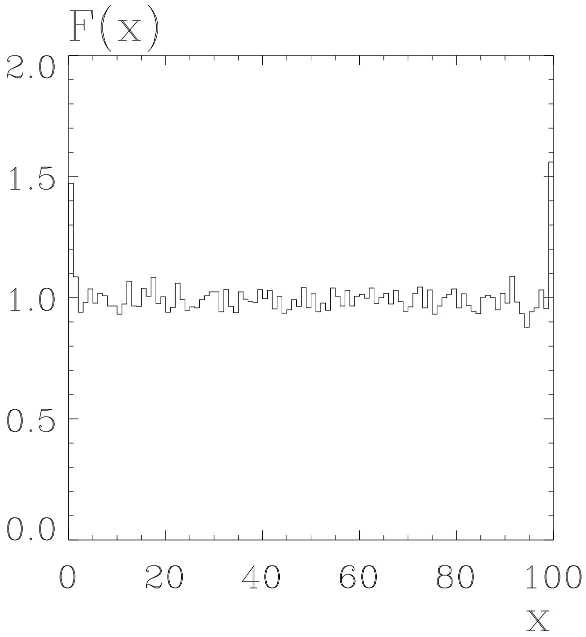

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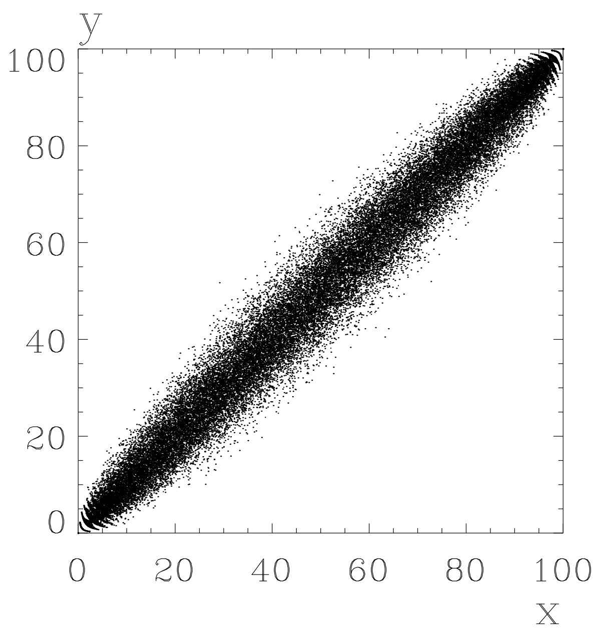
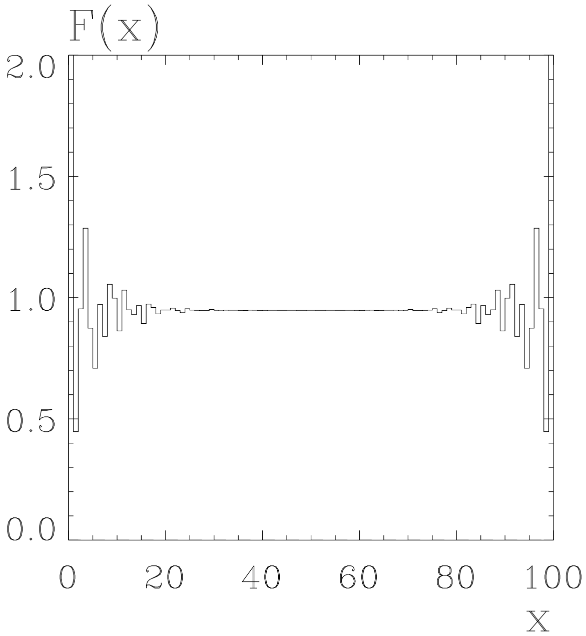


Figure 2: Double key vote. distribution of the results of the votes for $N = 100$ voters and $50,000$ elections using the Monte Carlo technique. The mandates are chosen at random in a ratio 1 to 5 and the sum is normalized to 100. Each point represents an election.



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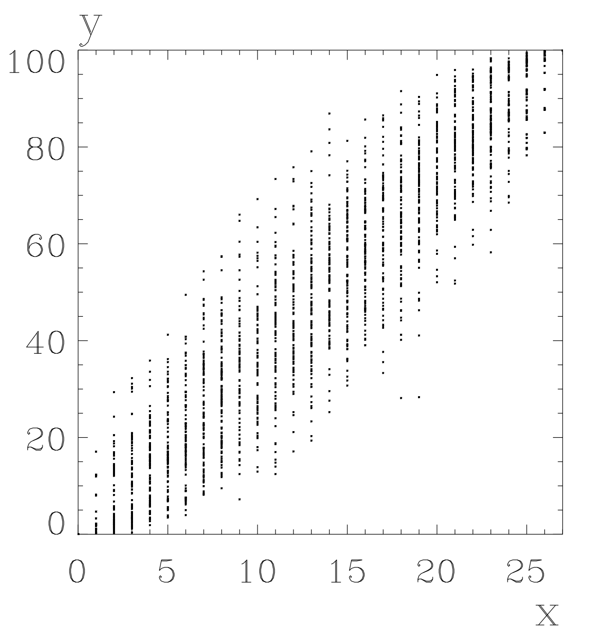

Figure 4: Double key vote. Distribution of the results of the votes for the EU27. For key A (x variable) all the mandates are equal to 1, for key B (y variable) the mandates are proportional to the populations and the sum is normalized to 100.