

LS Penrose's limit theorem:
Tests by simulation

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ABSTRACT

L S Penrose's limit theorem (PLT) – which is implicit in Penrose [5, p. 72] and for which he gave no rigorous proof – says that, in simple weighted voting games, if the number of voters increases indefinitely while existing voters retain their weights and the relative quota is pegged, then – under certain conditions – the ratio between the voting powers of any two voters converges to the ratio between their weights. Lindner and Machover [3] prove some special cases of PLT; and conjecture that the theorem holds, under rather general conditions, for large classes of weighted voting games, various values of the quota, and with respect to several measures of voting power. We use simulation to test this conjecture. It is corroborated w.r.t. the Penrose–Banzhaf index for a quota of 50% but not for other values; w.r.t. the Shapley–Shubik index the conjecture is corroborated for all values of the quota (short of 100%).

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1 Introduction

Throughout this paper, we shall be concerned with *weighted voting games* (WVGs). Let us recall briefly their definition. A WVG \mathcal{W} consists of a finite set N of together with an assignment of a non-negative real *weight* w_x to each $x \in N$; and a real $q \in (0, 1)$.

For our purposes it will be convenient, and will entail no loss of generality, to assume that all weights are positive. The *relative weight* of voter a in \mathcal{W} is given by

$$\bar{w}_a := \frac{w_a}{\sum_{x \in N} w_x}. \quad (1)$$

A subset $A \subseteq N$ (often referred to as a ‘coalition’) is said to be *winning* if

$$\sum_{x \in A} \bar{w}_x \geq q. \quad (2)$$

We refer to N as the *assembly* of \mathcal{W} , to the members of N as *voters*, and to q as the *quota*.¹

L S Penrose's limit theorem (PLT) is an assertion about the asymptotic behaviour of the voting power of voters in WVGs with a large number of voters. Here we shall consider the two major indices of voting power: the so-called *Banzhaf* index β (which is obtained by normalization from the absolute measure of voting power first proposed by Penrose [4]); and the *Shapley–Shubik* index ϕ proposed by these two authors in [6] (which is a special case of the Shapley value for co-operative games). For the definitions of these indices see, for example, Felsenthal and Machover [2].

For a precise statement of PLT we need the concept of a *q-chain* of WVGs, whose definition we borrow from [3].

Let

$$N^{(0)} \subsetneq N^{(1)} \subsetneq N^{(2)} \subsetneq \dots \quad (3)$$

be an infinite increasing chain of finite non-empty sets, and let

$$N = \bigcup_{n=0}^{\infty} N^{(n)}. \quad (4)$$

¹In the voting-power literature, q is often referred to as the *relative* quota, as distinct from the *absolute* quota, which equals $q \cdot \sum_{x \in N} w_x$.

Let w be a function that assigns to each $a \in N$ a positive real number w_a as *weight*; and let q be a real $\in (0, 1)$.

For each $n \in \mathbb{N}$ let $\mathcal{W}^{(n)}$ be the WVG whose assembly is $N^{(n)}$ – each voter $a \in N^{(n)}$ being endowed with the pre-assigned weight w_a – and whose quota is q .

We shall then say that $\{\mathcal{W}^{(n)}\}_{n=0}^{\infty}$ is a q -chain of WVGs.

Further, let ξ be an index of voting power. We shall say that *PLT holds for the q -chain $\{\mathcal{W}^{(n)}\}_{n=0}^{\infty}$ with respect to the index ξ* if for any $a, b \in N$

$$\lim_{n \rightarrow \infty} \frac{\xi_a[\mathcal{W}^{(n)}]}{\xi_b[\mathcal{W}^{(n)}]} = \frac{w_a}{w_b}. \quad (5)$$

Penrose [5, p. 72] gives an approximation formula for the voting power (as defined by him) of a voter in a WVG \mathcal{W} with quota $\frac{1}{2}$, according to which voters' powers are approximately proportional to their respective weights. He claims that this approximation is valid provided the number of voters in \mathcal{W} is large, and the relative weights of the voters in question are small. He offers no rigorous proof of his claim, merely an outline of an argument, obviously based on some version of the central limit theorem of probability theory.

Penrose's claim implies that PLT holds w.r.t. β (the normalized version of his measure of voting power) for any $\frac{1}{2}$ -chain, provided the relative weight of each $a \in N^{(n)}$ tends to 0 as n increases; that is,

$$\lim_{n \rightarrow \infty} \frac{w_a}{\sum_{x \in N^{(n)}} w_x} = 0.$$

However, Lindner and Machover [3] show by means of a simple counter-example that these conditions are insufficient for Penrose's approximation formula and the version of PLT implied by it. On the other hand, they prove the approximation formula (in a somewhat improved form) as well as PLT w.r.t. β for $\frac{1}{2}$ -chains satisfying more stringent conditions. (See [3, Theorem 3.6].) They also prove PLT w.r.t. the Shapley–Shubik index ϕ for a large class of q -chains with arbitrary $q \in (0, 1)$. (See [3, Theorem 2.3].)

Furthermore, they conjecture that PLT holds w.r.t. both β and ϕ for all $q \in (0, 1)$ and almost all q -chains. To be more precise: if a reasonable probability measure is defined on the space of all q -chains (or all q -chains with integer weights) then the probability that PLT holds equals 1.

In this paper we report the results of simulations designed to test their conjecture. Here is an outline of how we go about it. (A more detailed account will be given in Section 2.)

Obviously, we cannot select at random an entire q -chain, which is an infinite object. Instead, we proceed as follows. We fix some 'large' values of n , which will be the number of voters. (We allow n to get as large as feasibility of computation allows.) We also fix various values of the quota q , spaced at fairly close intervals.

Next, for each of our n , we select at random positive weights w_1, \dots, w_n . Replacing these by the corresponding relative weights, we get a random vector $\mathbf{w} = (\bar{w}_1, \dots, \bar{w}_n)$ in the $(n-1)$ -dimensional simplex $\Delta^{(n-1)}$ of all real n -vectors with non-negative components that add up to 1:

$$\Delta^{(n-1)} := \left\{ \mathbf{x} \in \mathbb{R}^n : x_i \geq 0, i = 1, \dots, n; \sum_{i=1}^n x_i = 1 \right\}. \quad (6)$$

This random selection is repeated a large number of times, so that for each of our n we obtain a large random sample of vectors $\mathbf{w} \in \Delta^{(n-1)}$.

For each randomly selected \mathbf{w} and fixed q we have a WVG \mathcal{W} , with assembly $N = \{1, \dots, n\}$, with \mathbf{w} as the vector of weights and q as quota.

For this WVG we compute the vector of values of the Banzhaf index $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$ and the Shapley–Shubik index $\boldsymbol{\phi} = (\phi_1, \dots, \phi_n)$. These vectors also belong to $\Delta^{(n-1)}$.

We are interested in the discrepancy (measured by a suitable metric) between the vector $\boldsymbol{\xi}$ – where $\boldsymbol{\xi}$ is $\boldsymbol{\beta}$ or $\boldsymbol{\phi}$ respectively – and the relative weight vector \mathbf{w} . For each of our n and q , we compute the mean and standard error of this distance, over our large sample of $\mathbf{w} \in \Delta^{(n-1)}$.

If, for a given value of q , the mean and standard deviation of the discrepancy between $\boldsymbol{\xi}$ and \mathbf{w} approach 0 as n increases, then this corroborates the hypothesis that PLT holds for almost all q -chains w.r.t. $\boldsymbol{\xi}$. If the mean discrepancy shows no tendency to approach 0 as n increases, this provides evidence against that hypothesis.

In Section 2 we fill in the details of the method outlined above. In Section 4 we present the results of our simulation. These are discussed in the concluding Section 3.

2 Description of the method

2.1. Random selection of weights In fact, we use two different methods of random selection. The first method selects n positive integer weights w_i independently of one another, with a Poisson probability distribution, shifted

so as to avoid 0 weights; thus, for each $i = 1, 2, \dots, n$ we have

$$\text{Prob}\{w_i = k\} = \frac{e^{-1}}{(k-1)!}, \quad k = 1, 2, \dots \quad (7)$$

Our second method selects the random vector \mathbf{w} from an $(n-1)$ -dimensional uniform distribution on the simplex $\Delta^{(n-1)}$. There are of course various ways for achieving this. We use the following method, which is very efficient computationally.² We select positive real weights w_i independently of one another, each with an exponential probability density

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

The exponential distribution is a special case of the gamma distribution; and the fact that this probability density for the w_i yields the required uniform distribution for the normalized vector \mathbf{w} follows from a property of the so-called Dirichlet composition. (For details, see [1, pp. 59f].)

Thus we have two separate series of samples. We shall refer to them as the *Poisson* and *uniform* samples, respectively.

In each of these two series, the size of our random sample of vectors $\mathbf{w} \in \Delta^{(n-1)}$ is 1,000 for every chosen value of n .

2.2. Choice of n As lowest value for n we took $n = 15$, because experience suggests that in cases where the asymptotic behaviour asserted by PLT occurs, it begins to manifest itself at about this value.

The choice of highest value for n was dictated by computational feasibility. The time needed to compute the vectors of values β and ϕ for given \mathbf{w} and q increases very steeply with n . We found that for the Poisson samples $n = 57$ was the highest practicable value. For the uniform samples we were able to go up to $n = 60$.

Fortunately, this range of values of n is sufficient for providing results from which quite firm conclusions can be drawn.

2.3. Choice of q Although the conjecture we are testing concerns values of q in the open interval $(0, 1)$, we need only consider values of q in the half-open interval $[0.5, 1)$.

To see this, note that both β and ϕ are *self-dual* (see [2, p. 180]). Thus, let \mathcal{W} be a WVG with weights w_x ($x \in N$) and quota q ; and let \mathcal{W}^* be the

²We are grateful to Friedrich Pukelsheim for suggesting this method to us.

simple voting game with the same assembly, N , whose winning coalitions are those $A \subseteq N$ such that

$$\sum_{x \in A} \bar{w}_x > 1 - q. \quad (9)$$

(\mathcal{W}^* is called the *dual* of \mathcal{W} . It is easy to see that it is a WVG with the same weights as \mathcal{W} and quota $1 - q + \varepsilon$, for any sufficiently small positive ε .) The self-duality of β and ϕ implies that

$$\beta_x[\mathcal{W}^*] = \beta_x[\mathcal{W}] \text{ and } \phi_x[\mathcal{W}^*] = \phi_x[\mathcal{W}] \text{ for all } x \in N. \quad (10)$$

Although in our definition (2) of a WVG with quota q we had a ‘blunt’ inequality (\geq), whereas here in (9) we have a sharp inequality ($>$), it is not difficult to see that this makes no difference to the asymptotic behaviour. More precisely, the asymptotic behaviour of a given q -chain w.r.t. both β and ϕ is the same as that of the $(1 - q)$ -chain having the same weights.³

In our simulation we fix q at the following values:

$$q = 0.50, 0.51, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95.$$

2.4. Measuring the discrepancy For each \mathbf{w} in our samples and each chosen value of q , we compute the vectors β and ϕ of the values of the Banzhaf and the Shapley–Shubik indices, respectively. In what follows, ‘ ξ ’ stands for either β or ϕ .

PLT w.r.t. ξ claims that asymptotically ξ approaches the normalized vector of weights \mathbf{w} . We measure the ‘discrepancy’ of ξ compared to \mathbf{w} in two ways.

First, we measure the *overall* discrepancy between ξ and \mathbf{w} by the well-known index of distortion D , commonly attributed to Loosemore and Hanby:⁴

$$D(\xi, \mathbf{w}) := \frac{1}{2} \sum_{i=1}^n |\xi_i - \bar{w}_i|. \quad (11)$$

Second, we measure the *local* (or *componentwise*) discrepancy between ξ and \mathbf{w} by

$$d(\xi, \mathbf{w}) := \max_{1 \leq i \leq n} \left| 1 - \frac{\xi_i}{\bar{w}_i} \right|. \quad (12)$$

Note that ξ is completely determined by \mathbf{w} and q . Therefore, if we fix n and q , and regard \mathbf{w} as a random variable, then $D(\xi, \mathbf{w})$ and $d(\xi, \mathbf{w})$ are also

³In this connection note that the results of Lindner and Machover [3] hold also – with virtually the same proofs – if in the definition (2) of WVG \geq is replaced by $>$.

⁴See however discussion by Taagepera and Grofman [7] of the authorship of this index.

random variables, whose distributions depend on that of \mathbf{w} . We are interested in the expected value and standard deviation of these random variable, as functions of n and q .

2.5. Output of computation In our simulation, we estimate the expected value and standard deviation of $D(\xi, \mathbf{w})$ and $d(\xi, \mathbf{w})$ by computing their mean and standard error for each of our samples. This yields the following outputs for all the chosen values of n and q :

$$\mu D(n, q), \quad \sigma D(n, q), \quad \mu d(n, q), \quad \sigma d(n, q).$$

Here ' μ ' and ' σ ' stand for mean and standard error, respectively.

More specifically, we have a set of four such outputs for each of the two indices and each of our sample series. Thus we have altogether:

$$\begin{aligned} &\mu_P D(\beta; n, q), \quad \sigma_P D(\beta; n, q), \quad \mu_P d(\beta; n, q), \quad \sigma_P d(\beta; n, q), \\ &\mu_U D(\beta; n, q), \quad \sigma_U D(\beta; n, q), \quad \mu_U d(\beta; n, q), \quad \sigma_U d(\beta; n, q), \\ &\mu_P D(\phi; n, q), \quad \sigma_P D(\phi; n, q), \quad \mu_P d(\phi; n, q), \quad \sigma_P d(\phi; n, q), \\ &\mu_U D(\phi; n, q), \quad \sigma_U D(\phi; n, q), \quad \mu_U d(\phi; n, q), \quad \sigma_U d(\phi; n, q). \end{aligned}$$

Here ' β ' and ' ϕ ' and the subscripts ' P ' and ' U ' are labels that refer to the Banzhaf and Shapley–Shubik indices, and the Poisson and uniform samples, respectively.

In Section 4 we present detailed tables (Tables 1&1a–8&8a) as well as 3-D graphs (Figures 1&1a–8&8a) of each of these sixteen statistics. In each diagram, the values of the statistic in question are plotted along the vertical axis; n is plotted along the \searrow axis and q along the \nearrow axis.

3 Conclusions

The tables and diagrams presented in Section 4 show a conspicuous difference between the behaviours of the two indices. We consider these indices in turn.

3.1. The Banzhaf index The data for the statistics labelled ' β ' do not corroborate the conjecture w.r.t. the Banzhaf index (that is, that PLT holds for almost all q -chains w.r.t. this index) except for $q = 0.5$ and perhaps for values of q very close to 0.5. Note that Penrose's original claim concerned only $q = 0.5$. This claim, as we know, does not hold in all cases even for $q = 0.5$, but it does now appear to hold in *almost* all such cases.

One interesting feature of these data ought to be pointed out. For fixed values of n near the bottom of our range, the mean discrepancy between β

and \mathbf{w} has a dip – a minimum, indicating the closest fit between β and \mathbf{w} – at a value of q considerably greater than 0.5. But as n increases the dip edges towards $q = 0.5$.

This general pattern applies to both the Poisson and the uniform samples, and to both measures of discrepancy. But there are some differences of detail.

In the case of $\mu_P D(\beta; n, q)$ (Table 1 and Figure 1), for $n = 15, \dots, 18$ the dip occurs at $q = 0.65$; for $n = 18, \dots, 48$ it occurs at $q = 0.60$; and for $n = 50, \dots, 57$ it occurs at $q = 0.55$.

In the case of $\mu_P d(\beta; n, q)$ (Table 2 and Figure 2), for $n = 15, \dots, 17$ the dip occurs at $q = 0.65$; for $n = 19, \dots, 50$ it occurs at $q = 0.60$; and for $n = 49, \dots, 57$ it occurs at $q = 0.55$.

In the case of $\mu_U D(\beta; n, q)$ (Table 3 and Figure 3), for $n = 15, \dots, 26$ the dip occurs at $q = 0.65$; and for $n = 27, \dots, 57$ it occurs at $q = 0.60$.

In the case of $\mu_U d(\beta; n, q)$ (Table 4 and Figure 4), for $n = 15, \dots, 23$ the dip occurs at $q = 0.65$; and for $n = 24, \dots, 57$ it occurs at $q = 0.60$.

Extrapolating from these data, it is reasonable to expect that for still greater values on n , beyond our range, the dip of all these four quantities – $\mu_P D(\beta; n, q)$, $\mu_P d(\beta; n, q)$, $\mu_U D(\beta; n, q)$ and $\mu_U d(\beta; n, q)$ – should occur at $q = 0.5$.⁵

Our negative findings w.r.t. β for $q > 0.5$ of course do not exclude the possibility that PLT holds for large classes of q -chains with $q > 0.5$. Finding ‘natural’ and sufficiently interesting such classes is an open problem.

3.2. The Shapley–Shubik index The data for the statistics labelled ‘ ϕ ’ corroborate the conjecture w.r.t. the Shapley–Shubik index for all $q \in (0, 1)$. For every chosen value of q , the mean discrepancy between ϕ and \mathbf{w} – whether measured by $\mu D(\phi, \mathbf{w})$ or $\mu d(\phi, \mathbf{w})$ – seems to approach 0 as n increases. It appears that the PLT w.r.t. ϕ does hold almost always.

Let us look at the behaviour of the mean discrepancy between ϕ and \mathbf{w} as a function of q and n .

Clearly, for any fixed n , as q gets very close to 1, we would expect a WVG with n voters to behave somewhat like a unanimity game, in which all voters have the same voting power, irrespective of their weights. Therefore it is reasonable to expect that as q approaches 1, the mean discrepancy between ϕ and \mathbf{w} should increase. Also, it is reasonable to expect that as q gets closer

⁵In an unpublished report that came to our attention after completing an earlier draft of this paper, Życzkowski and Słomczyński [8, Section 10] have made essentially the same observations and drew the same conclusion regarding the behaviour of the ‘dip’ (which they call ‘critical point’) in the mean discrepancy between β and \mathbf{w} . Our simulations – using much larger samples and considerably more values of n – provide an independent corroboration of their findings on this matter.

to 1, it would take greater values of n to overcome this ‘unanimity effect’. In other words, the closer q is to 1, the slower the convergence to 0 of the mean discrepancy as n increases.

Our data show that this is indeed the case. However, we wish to point out an additional interesting phenomenon concerning the dependence on n of the the mean discrepancy between ϕ and \mathbf{w} . For every fixed value of n in our range, this mean discrepancy increases monotonically with q ; but the rate of increase is by no means uniform. For each n , as q increases from 0.5 towards 1, we can discern two regimes: at first the increase in the mean discrepancy is very gentle, barely noticeable; then, quite abruptly, it becomes quite steep. In other words, the transition from the ‘PLT effect’ to the ‘unanimity effect’ is rather sharp. As may be expected, it seems that the greater the value of n , the higher is the value of q at which this transition takes place.

This general pattern applies to both the Poisson and the uniform samples, and to both measures of discrepancy. But again there are some difference of detail as to where (that is, at what value of q) the increase becomes steep for a given n . For these details we refer the reader to the relevant tables (Tables 5, 6, 7 and 8), in which values that seem to be affected by the ‘unanimity effect’ are highlighted.

4 Results

[See Tables and Figures on separate pages.]

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Table 1: $\mu_P D(\beta; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0122	0.0122	0.0107	0.0064	0.0028	0.0124	0.0259	0.0442	0.0690	0.0989	0.1439
16	0.0113	0.0113	0.0098	0.0056	0.0031	0.0130	0.0264	0.0444	0.0688	0.0976	0.1442
17	0.0105	0.0105	0.0089	0.0048	0.0034	0.0136	0.0265	0.0442	0.0686	0.0970	0.1429
18	0.0098	0.0098	0.0084	0.0042	0.0038	0.0140	0.0269	0.0447	0.0683	0.0976	0.1425
19	0.0093	0.0093	0.0079	0.0037	0.0042	0.0142	0.0272	0.0447	0.0677	0.0970	0.1404
20	0.0090	0.0090	0.0076	0.0034	0.0045	0.0149	0.0281	0.0460	0.0693	0.0985	0.1395
21	0.0085	0.0085	0.0071	0.0029	0.0048	0.0151	0.0284	0.0460	0.0690	0.0981	0.1384
22	0.0081	0.0081	0.0067	0.0024	0.0051	0.0155	0.0287	0.0464	0.0692	0.0983	0.1373
23	0.0077	0.0077	0.0063	0.0021	0.0054	0.0158	0.0291	0.0469	0.0701	0.0995	0.1386
24	0.0072	0.0072	0.0058	0.0017	0.0057	0.0157	0.0289	0.0462	0.0691	0.0979	0.1370
25	0.0069	0.0068	0.0055	0.0015	0.0059	0.0159	0.0289	0.0462	0.0694	0.0980	0.1383
26	0.0067	0.0066	0.0053	0.0012	0.0064	0.0164	0.0293	0.0468	0.0699	0.0981	0.1386
27	0.0064	0.0064	0.0050	0.0010	0.0066	0.0166	0.0298	0.0472	0.0701	0.0985	0.1392
28	0.0061	0.0060	0.0047	0.0008	0.0066	0.0166	0.0296	0.0469	0.0697	0.0983	0.1388
29	0.0060	0.0059	0.0046	0.0007	0.0070	0.0170	0.0301	0.0476	0.0706	0.0993	0.1392
30	0.0057	0.0056	0.0043	0.0006	0.0070	0.0169	0.0300	0.0473	0.0698	0.0985	0.1374
31	0.0055	0.0055	0.0042	0.0006	0.0071	0.0171	0.0301	0.0475	0.0700	0.0986	0.1373
32	0.0054	0.0053	0.0040	0.0005	0.0073	0.0173	0.0304	0.0479	0.0703	0.0989	0.1371
33	0.0052	0.0051	0.0038	0.0006	0.0074	0.0174	0.0305	0.0480	0.0705	0.0990	0.1374
34	0.0050	0.0050	0.0036	0.0007	0.0076	0.0176	0.0305	0.0478	0.0703	0.0985	0.1371
35	0.0048	0.0048	0.0035	0.0007	0.0077	0.0176	0.0305	0.0478	0.0705	0.0985	0.1369
36	0.0048	0.0047	0.0034	0.0008	0.0079	0.0179	0.0309	0.0482	0.0707	0.0988	0.1374
37	0.0046	0.0046	0.0032	0.0009	0.0080	0.0180	0.0310	0.0484	0.0710	0.0990	0.1375
38	0.0045	0.0044	0.0031	0.0010	0.0082	0.0181	0.0313	0.0485	0.0711	0.0991	0.1373
39	0.0044	0.0043	0.0030	0.0011	0.0082	0.0183	0.0314	0.0488	0.0713	0.0995	0.1376
40	0.0042	0.0042	0.0029	0.0012	0.0083	0.0183	0.0312	0.0484	0.0707	0.0989	0.1365
41	0.0042	0.0041	0.0028	0.0013	0.0084	0.0183	0.0315	0.0486	0.0711	0.0992	0.1365
42	0.0041	0.0040	0.0027	0.0014	0.0085	0.0185	0.0316	0.0489	0.0713	0.0995	0.1365
43	0.0040	0.0039	0.0026	0.0015	0.0086	0.0185	0.0317	0.0490	0.0714	0.0996	0.1370
44	0.0039	0.0038	0.0025	0.0016	0.0087	0.0187	0.0318	0.0491	0.0715	0.0995	0.1368
45	0.0038	0.0037	0.0024	0.0017	0.0087	0.0187	0.0318	0.0492	0.0716	0.0998	0.1379
46	0.0037	0.0037	0.0023	0.0017	0.0089	0.0189	0.0321	0.0494	0.0719	0.0999	0.1375
47	0.0036	0.0036	0.0022	0.0018	0.0089	0.0189	0.0321	0.0494	0.0719	0.1000	0.1375
48	0.0035	0.0035	0.0021	0.0019	0.0089	0.0188	0.0319	0.0492	0.0713	0.0994	0.1367
49	0.0035	0.0034	0.0021	0.0019	0.0090	0.0190	0.0320	0.0493	0.0716	0.0995	0.1367
50	0.0034	0.0034	0.0020	0.0020	0.0091	0.0191	0.0323	0.0497	0.0721	0.1002	0.1375
51	0.0033	0.0033	0.0020	0.0021	0.0092	0.0192	0.0324	0.0498	0.0721	0.1003	0.1374
52	0.0032	0.0032	0.0019	0.0021	0.0091	0.0191	0.0322	0.0494	0.0717	0.0997	0.1365
53	0.0032	0.0031	0.0018	0.0022	0.0092	0.0192	0.0323	0.0496	0.0719	0.1000	0.1370
54	0.0032	0.0031	0.0018	0.0023	0.0094	0.0193	0.0324	0.0498	0.0722	0.1001	0.1375
55	0.0031	0.0030	0.0017	0.0023	0.0094	0.0193	0.0324	0.0497	0.0720	0.1000	0.1374
56	0.0030	0.0029	0.0017	0.0024	0.0094	0.0193	0.0324	0.0497	0.0720	0.1000	0.1371
57	0.0030	0.0029	0.0016	0.0024	0.0095	0.0195	0.0326	0.0500	0.0724	0.1005	0.1376

Table 1a: $\sigma_p D(\beta; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{smallmatrix} q \\ n \end{smallmatrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0062	0.0062	0.0052	0.0032	0.0023	0.0060	0.0095	0.0143	0.0198	0.0270	0.0350
16	0.0057	0.0057	0.0047	0.0027	0.0027	0.0059	0.0096	0.0136	0.0189	0.0244	0.0333
17	0.0047	0.0047	0.0041	0.0023	0.0024	0.0054	0.0084	0.0129	0.0189	0.0250	0.0357
18	0.0044	0.0044	0.0037	0.0020	0.0025	0.0055	0.0086	0.0128	0.0177	0.0240	0.0332
19	0.0042	0.0042	0.0035	0.0020	0.0027	0.0052	0.0086	0.0124	0.0170	0.0235	0.0320
20	0.0036	0.0036	0.0031	0.0018	0.0022	0.0050	0.0084	0.0124	0.0174	0.0239	0.0318
21	0.0035	0.0035	0.0030	0.0017	0.0024	0.0049	0.0082	0.0121	0.0166	0.0220	0.0294
22	0.0033	0.0033	0.0026	0.0013	0.0024	0.0050	0.0077	0.0111	0.0152	0.0204	0.0282
23	0.0026	0.0026	0.0022	0.0012	0.0019	0.0044	0.0072	0.0106	0.0149	0.0201	0.0270
24	0.0025	0.0025	0.0021	0.0010	0.0020	0.0045	0.0071	0.0106	0.0150	0.0204	0.0276
25	0.0025	0.0025	0.0020	0.0010	0.0020	0.0043	0.0071	0.0105	0.0148	0.0198	0.0273
26	0.0024	0.0024	0.0018	0.0008	0.0021	0.0042	0.0067	0.0099	0.0138	0.0186	0.0257
27	0.0021	0.0021	0.0016	0.0007	0.0020	0.0043	0.0067	0.0099	0.0137	0.0180	0.0250
28	0.0019	0.0018	0.0015	0.0006	0.0019	0.0039	0.0064	0.0094	0.0132	0.0178	0.0246
29	0.0019	0.0019	0.0014	0.0005	0.0020	0.0039	0.0062	0.0091	0.0127	0.0170	0.0232
30	0.0017	0.0017	0.0013	0.0004	0.0018	0.0039	0.0063	0.0093	0.0130	0.0174	0.0231
31	0.0018	0.0017	0.0013	0.0004	0.0020	0.0039	0.0061	0.0090	0.0125	0.0169	0.0223
32	0.0015	0.0015	0.0012	0.0004	0.0018	0.0036	0.0060	0.0087	0.0121	0.0167	0.0220
33	0.0014	0.0014	0.0011	0.0003	0.0017	0.0036	0.0058	0.0084	0.0118	0.0158	0.0212
34	0.0014	0.0014	0.0011	0.0004	0.0018	0.0037	0.0059	0.0086	0.0121	0.0162	0.0221
35	0.0013	0.0012	0.0010	0.0004	0.0017	0.0035	0.0057	0.0083	0.0116	0.0155	0.0218
36	0.0013	0.0013	0.0010	0.0005	0.0018	0.0037	0.0059	0.0088	0.0121	0.0162	0.0223
37	0.0013	0.0013	0.0009	0.0005	0.0018	0.0036	0.0057	0.0084	0.0117	0.0155	0.0211
38	0.0011	0.0011	0.0008	0.0005	0.0017	0.0034	0.0054	0.0079	0.0109	0.0150	0.0204
39	0.0012	0.0012	0.0009	0.0005	0.0018	0.0036	0.0056	0.0083	0.0115	0.0153	0.0205
40	0.0011	0.0011	0.0008	0.0005	0.0017	0.0035	0.0056	0.0081	0.0110	0.0146	0.0194
41	0.0010	0.0010	0.0007	0.0005	0.0016	0.0032	0.0051	0.0075	0.0105	0.0146	0.0194
42	0.0010	0.0010	0.0007	0.0004	0.0016	0.0032	0.0051	0.0074	0.0104	0.0142	0.0191
43	0.0010	0.0010	0.0007	0.0005	0.0017	0.0033	0.0052	0.0077	0.0106	0.0142	0.0192
44	0.0010	0.0010	0.0007	0.0005	0.0017	0.0034	0.0052	0.0077	0.0106	0.0144	0.0193
45	0.0009	0.0008	0.0006	0.0004	0.0016	0.0031	0.0050	0.0072	0.0101	0.0138	0.0194
46	0.0008	0.0008	0.0006	0.0004	0.0016	0.0031	0.0049	0.0070	0.0098	0.0130	0.0180
47	0.0009	0.0009	0.0006	0.0005	0.0017	0.0033	0.0052	0.0074	0.0103	0.0137	0.0184
48	0.0008	0.0007	0.0005	0.0004	0.0015	0.0030	0.0047	0.0069	0.0097	0.0135	0.0188
49	0.0008	0.0008	0.0005	0.0005	0.0016	0.0031	0.0048	0.0071	0.0099	0.0133	0.0180
50	0.0008	0.0007	0.0005	0.0004	0.0016	0.0031	0.0050	0.0073	0.0103	0.0138	0.0184
51	0.0007	0.0007	0.0005	0.0005	0.0015	0.0030	0.0047	0.0069	0.0095	0.0128	0.0172
52	0.0007	0.0007	0.0004	0.0004	0.0015	0.0029	0.0047	0.0068	0.0094	0.0128	0.0173
53	0.0007	0.0007	0.0004	0.0004	0.0015	0.0029	0.0046	0.0067	0.0094	0.0128	0.0174
54	0.0007	0.0007	0.0004	0.0005	0.0016	0.0030	0.0046	0.0067	0.0094	0.0126	0.0172
55	0.0006	0.0006	0.0004	0.0004	0.0015	0.0028	0.0045	0.0066	0.0092	0.0124	0.0169
56	0.0006	0.0006	0.0004	0.0004	0.0014	0.0028	0.0044	0.0066	0.0092	0.0125	0.0170
57	0.0006	0.0006	0.0004	0.0004	0.0015	0.0028	0.0045	0.0066	0.0092	0.0126	0.0172

Table 2: $\mu_{pd}(\beta; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0578	0.0578	0.0498	0.0284	0.0155	0.0615	0.1197	0.1899	0.2755	0.3700	0.5370
16	0.0545	0.0545	0.0464	0.0246	0.0173	0.0651	0.1230	0.1930	0.2759	0.3669	0.5544
17	0.0524	0.0524	0.0443	0.0220	0.0201	0.0695	0.1272	0.1985	0.2832	0.3731	0.5674
18	0.0505	0.0505	0.0424	0.0190	0.0223	0.0728	0.1312	0.2026	0.2847	0.3771	0.5784
19	0.0480	0.0480	0.0401	0.0168	0.0245	0.0739	0.1331	0.2031	0.2838	0.3759	0.5696
20	0.0482	0.0482	0.0400	0.0154	0.0265	0.0798	0.1405	0.2129	0.2943	0.3851	0.5632
21	0.0460	0.0460	0.0377	0.0131	0.0286	0.0813	0.1425	0.2141	0.2940	0.3851	0.5504
22	0.0448	0.0447	0.0361	0.0110	0.0305	0.0848	0.1462	0.2187	0.2983	0.3889	0.5487
23	0.0428	0.0427	0.0342	0.0095	0.0323	0.0862	0.1485	0.2209	0.3016	0.3908	0.5541
24	0.0404	0.0401	0.0318	0.0079	0.0337	0.0862	0.1474	0.2185	0.2991	0.3874	0.5571
25	0.0388	0.0385	0.0304	0.0068	0.0353	0.0873	0.1486	0.2197	0.3009	0.3892	0.5630
26	0.0387	0.0383	0.0297	0.0055	0.0386	0.0919	0.1534	0.2258	0.3064	0.3937	0.5684
27	0.0382	0.0377	0.0290	0.0046	0.0408	0.0946	0.1580	0.2306	0.3111	0.3982	0.5711
28	0.0355	0.0350	0.0268	0.0038	0.0400	0.0926	0.1545	0.2259	0.3059	0.3943	0.5669
29	0.0368	0.0363	0.0274	0.0036	0.0442	0.0995	0.1639	0.2375	0.3179	0.4041	0.5674
30	0.0342	0.0337	0.0254	0.0033	0.0430	0.0970	0.1604	0.2332	0.3122	0.3988	0.5584
31	0.0336	0.0331	0.0246	0.0035	0.0440	0.0986	0.1620	0.2345	0.3138	0.4003	0.5584
32	0.0331	0.0327	0.0239	0.0036	0.0458	0.1014	0.1656	0.2389	0.3180	0.4043	0.5557
33	0.0322	0.0317	0.0230	0.0039	0.0467	0.1020	0.1661	0.2394	0.3186	0.4050	0.5568
34	0.0320	0.0316	0.0225	0.0050	0.0487	0.1050	0.1697	0.2433	0.3231	0.4082	0.5613
35	0.0305	0.0301	0.0213	0.0051	0.0488	0.1044	0.1681	0.2414	0.3214	0.4065	0.5621
36	0.0310	0.0306	0.0214	0.0063	0.0516	0.1086	0.1744	0.2488	0.3286	0.4126	0.5670
37	0.0301	0.0297	0.0205	0.0070	0.0520	0.1091	0.1743	0.2483	0.3284	0.4125	0.5677
38	0.0294	0.0291	0.0198	0.0075	0.0531	0.1101	0.1762	0.2501	0.3302	0.4146	0.5670
39	0.0290	0.0287	0.0193	0.0085	0.0540	0.1119	0.1779	0.2522	0.3316	0.4157	0.5667
40	0.0289	0.0286	0.0190	0.0094	0.0557	0.1146	0.1812	0.2561	0.3358	0.4196	0.5638
41	0.0286	0.0283	0.0187	0.0102	0.0569	0.1161	0.1839	0.2586	0.3385	0.4222	0.5635
42	0.0275	0.0272	0.0177	0.0106	0.0565	0.1155	0.1821	0.2568	0.3362	0.4197	0.5620
43	0.0271	0.0268	0.0171	0.0114	0.0577	0.1164	0.1838	0.2590	0.3388	0.4226	0.5639
44	0.0271	0.0268	0.0168	0.0120	0.0594	0.1188	0.1865	0.2616	0.3414	0.4241	0.5653
45	0.0258	0.0255	0.0159	0.0126	0.0584	0.1169	0.1838	0.2589	0.3385	0.4219	0.5693
46	0.0262	0.0259	0.0158	0.0134	0.0613	0.1211	0.1898	0.2654	0.3455	0.4282	0.5703
47	0.0259	0.0257	0.0154	0.0142	0.0621	0.1223	0.1910	0.2669	0.3470	0.4290	0.5704
48	0.0249	0.0246	0.0146	0.0144	0.0619	0.1214	0.1900	0.2663	0.3458	0.4280	0.5678
49	0.0245	0.0241	0.0140	0.0149	0.0624	0.1221	0.1903	0.2660	0.3458	0.4278	0.5674
50	0.0241	0.0237	0.0136	0.0152	0.0625	0.1223	0.1907	0.2666	0.3463	0.4285	0.5685
51	0.0237	0.0233	0.0133	0.0159	0.0634	0.1231	0.1918	0.2675	0.3470	0.4298	0.5678
52	0.0232	0.0228	0.0128	0.0162	0.0635	0.1237	0.1923	0.2682	0.3480	0.4294	0.5656
53	0.0234	0.0229	0.0127	0.0171	0.0654	0.1263	0.1955	0.2719	0.3516	0.4335	0.5684
54	0.0235	0.0230	0.0126	0.0179	0.0673	0.1290	0.1991	0.2762	0.3563	0.4372	0.5707
55	0.0227	0.0222	0.0121	0.0180	0.0669	0.1278	0.1975	0.2747	0.3545	0.4360	0.5690
56	0.0220	0.0215	0.0116	0.0183	0.0662	0.1268	0.1959	0.2726	0.3520	0.4338	0.5689
57	0.0221	0.0216	0.0114	0.0189	0.0680	0.1291	0.1991	0.2760	0.3558	0.4375	0.5713

Table 2a: $\sigma_{pd}(\beta; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0340	0.0340	0.0286	0.0158	0.0145	0.0353	0.0562	0.0738	0.0842	0.0882	0.1465
16	0.0334	0.0334	0.0275	0.0142	0.0157	0.0377	0.0592	0.0744	0.0848	0.0832	0.1372
17	0.0302	0.0302	0.0254	0.0123	0.0158	0.0368	0.0560	0.0744	0.0844	0.0874	0.1393
18	0.0296	0.0296	0.0244	0.0105	0.0172	0.0398	0.0593	0.0768	0.0845	0.0859	0.1202
19	0.0289	0.0289	0.0235	0.0100	0.0186	0.0389	0.0596	0.0758	0.0842	0.0826	0.1058
20	0.0271	0.0271	0.0220	0.0098	0.0171	0.0402	0.0608	0.0765	0.0843	0.0782	0.0926
21	0.0266	0.0266	0.0216	0.0087	0.0188	0.0402	0.0607	0.0772	0.0844	0.0780	0.0827
22	0.0260	0.0260	0.0202	0.0072	0.0196	0.0418	0.0611	0.0762	0.0835	0.0765	0.0837
23	0.0223	0.0223	0.0175	0.0066	0.0178	0.0388	0.0577	0.0726	0.0799	0.0728	0.0830
24	0.0220	0.0219	0.0170	0.0056	0.0187	0.0406	0.0587	0.0746	0.0820	0.0752	0.0864
25	0.0209	0.0208	0.0162	0.0052	0.0187	0.0387	0.0573	0.0719	0.0789	0.0720	0.0803
26	0.0215	0.0214	0.0161	0.0041	0.0213	0.0422	0.0609	0.0761	0.0821	0.0747	0.0721
27	0.0206	0.0203	0.0151	0.0037	0.0212	0.0425	0.0616	0.0761	0.0826	0.0745	0.0701
28	0.0186	0.0183	0.0140	0.0033	0.0204	0.0406	0.0591	0.0734	0.0798	0.0716	0.0660
29	0.0192	0.0189	0.0139	0.0029	0.0221	0.0428	0.0605	0.0748	0.0812	0.0731	0.0590
30	0.0169	0.0167	0.0124	0.0027	0.0203	0.0401	0.0578	0.0715	0.0785	0.0717	0.0587
31	0.0182	0.0179	0.0128	0.0034	0.0225	0.0420	0.0603	0.0738	0.0797	0.0725	0.0563
32	0.0170	0.0167	0.0121	0.0034	0.0220	0.0415	0.0597	0.0728	0.0791	0.0716	0.0560
33	0.0159	0.0157	0.0113	0.0034	0.0214	0.0417	0.0593	0.0730	0.0796	0.0718	0.0566
34	0.0164	0.0161	0.0111	0.0044	0.0230	0.0434	0.0612	0.0746	0.0808	0.0734	0.0606
35	0.0151	0.0148	0.0105	0.0043	0.0221	0.0423	0.0598	0.0733	0.0792	0.0715	0.0589
36	0.0154	0.0152	0.0104	0.0049	0.0235	0.0431	0.0605	0.0741	0.0799	0.0738	0.0595
37	0.0155	0.0152	0.0104	0.0053	0.0240	0.0444	0.0619	0.0747	0.0811	0.0738	0.0541
38	0.0144	0.0143	0.0096	0.0050	0.0239	0.0429	0.0601	0.0726	0.0784	0.0710	0.0504
39	0.0148	0.0145	0.0097	0.0056	0.0246	0.0450	0.0621	0.0751	0.0812	0.0737	0.0513
40	0.0140	0.0138	0.0090	0.0058	0.0242	0.0440	0.0615	0.0745	0.0801	0.0733	0.0486
41	0.0138	0.0136	0.0089	0.0059	0.0245	0.0447	0.0620	0.0749	0.0804	0.0738	0.0470
42	0.0135	0.0134	0.0085	0.0059	0.0249	0.0447	0.0616	0.0740	0.0797	0.0734	0.0491
43	0.0128	0.0127	0.0081	0.0062	0.0246	0.0437	0.0607	0.0730	0.0786	0.0716	0.0521
44	0.0134	0.0133	0.0082	0.0064	0.0262	0.0470	0.0642	0.0775	0.0832	0.0768	0.0525
45	0.0122	0.0121	0.0075	0.0066	0.0249	0.0444	0.0614	0.0735	0.0786	0.0721	0.0533
46	0.0122	0.0120	0.0072	0.0069	0.0257	0.0449	0.0619	0.0742	0.0795	0.0729	0.0479
47	0.0127	0.0126	0.0075	0.0072	0.0265	0.0457	0.0622	0.0745	0.0796	0.0740	0.0485
48	0.0110	0.0108	0.0063	0.0068	0.0243	0.0423	0.0575	0.0690	0.0739	0.0688	0.0466
49	0.0113	0.0111	0.0063	0.0071	0.0255	0.0446	0.0602	0.0720	0.0776	0.0713	0.0444
50	0.0109	0.0107	0.0061	0.0071	0.0252	0.0440	0.0605	0.0728	0.0785	0.0728	0.0458
51	0.0109	0.0107	0.0061	0.0078	0.0259	0.0449	0.0613	0.0732	0.0786	0.0721	0.0445
52	0.0101	0.0100	0.0057	0.0075	0.0248	0.0432	0.0593	0.0714	0.0765	0.0716	0.0449
53	0.0107	0.0105	0.0059	0.0079	0.0264	0.0456	0.0622	0.0740	0.0791	0.0733	0.0473
54	0.0111	0.0108	0.0059	0.0085	0.0276	0.0466	0.0627	0.0744	0.0790	0.0734	0.0477
55	0.0100	0.0098	0.0053	0.0080	0.0261	0.0442	0.0600	0.0718	0.0768	0.0710	0.0458
56	0.0097	0.0095	0.0052	0.0081	0.0252	0.0438	0.0594	0.0711	0.0766	0.0709	0.0449
57	0.0099	0.0097	0.0051	0.0084	0.0267	0.0450	0.0608	0.0726	0.0775	0.0717	0.0448

Table 3: $\mu_{VD}(\beta; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0507	0.0500	0.0420	0.0275	0.0160	0.0278	0.0499	0.0792	0.1173	0.1664	0.2320
16	0.0467	0.0461	0.0390	0.0249	0.0140	0.0271	0.0497	0.0804	0.1178	0.1656	0.2311
17	0.0434	0.0429	0.0362	0.0225	0.0121	0.0263	0.0498	0.0798	0.1178	0.1661	0.2319
18	0.0406	0.0402	0.0338	0.0204	0.0105	0.0260	0.0502	0.0800	0.1169	0.1644	0.2299
19	0.0359	0.0356	0.0301	0.0183	0.0099	0.0248	0.0487	0.0789	0.1158	0.1625	0.2278
20	0.0347	0.0345	0.0288	0.0164	0.0093	0.0260	0.0499	0.0794	0.1162	0.1632	0.2277
21	0.0304	0.0302	0.0253	0.0147	0.0091	0.0251	0.0487	0.0784	0.1152	0.1618	0.2265
22	0.0299	0.0296	0.0247	0.0134	0.0087	0.0259	0.0494	0.0793	0.1163	0.1634	0.2278
23	0.0295	0.0293	0.0241	0.0124	0.0088	0.0267	0.0508	0.0804	0.1167	0.1632	0.2275
24	0.0260	0.0258	0.0214	0.0112	0.0086	0.0261	0.0499	0.0798	0.1167	0.1632	0.2267
25	0.0247	0.0245	0.0203	0.0103	0.0087	0.0263	0.0500	0.0793	0.1159	0.1626	0.2265
26	0.0232	0.0230	0.0190	0.0093	0.0089	0.0265	0.0501	0.0795	0.1162	0.1628	0.2270
27	0.0217	0.0215	0.0178	0.0086	0.0090	0.0264	0.0501	0.0795	0.1160	0.1626	0.2270
28	0.0212	0.0211	0.0172	0.0077	0.0093	0.0269	0.0504	0.0797	0.1162	0.1630	0.2275
29	0.0209	0.0207	0.0168	0.0072	0.0096	0.0277	0.0512	0.0804	0.1166	0.1629	0.2267
30	0.0196	0.0194	0.0157	0.0064	0.0096	0.0276	0.0507	0.0797	0.1159	0.1621	0.2259
31	0.0190	0.0189	0.0152	0.0058	0.0101	0.0280	0.0511	0.0800	0.1164	0.1629	0.2267
32	0.0182	0.0180	0.0145	0.0053	0.0102	0.0282	0.0513	0.0800	0.1160	0.1622	0.2259
33	0.0179	0.0177	0.0141	0.0048	0.0107	0.0288	0.0517	0.0804	0.1165	0.1627	0.2265
34	0.0171	0.0170	0.0134	0.0043	0.0109	0.0288	0.0518	0.0804	0.1162	0.1623	0.2259
35	0.0163	0.0161	0.0128	0.0041	0.0110	0.0290	0.0520	0.0807	0.1166	0.1628	0.2262
36	0.0155	0.0154	0.0122	0.0036	0.0112	0.0292	0.0521	0.0806	0.1163	0.1623	0.2259
37	0.0155	0.0154	0.0121	0.0034	0.0116	0.0296	0.0526	0.0813	0.1170	0.1630	0.2265
38	0.0146	0.0145	0.0113	0.0029	0.0117	0.0296	0.0523	0.0808	0.1166	0.1626	0.2262
39	0.0142	0.0140	0.0109	0.0027	0.0117	0.0295	0.0523	0.0808	0.1166	0.1626	0.2257
40	0.0141	0.0140	0.0107	0.0024	0.0122	0.0299	0.0525	0.0807	0.1163	0.1621	0.2255
41	0.0140	0.0139	0.0106	0.0022	0.0126	0.0303	0.0528	0.0811	0.1166	0.1624	0.2258
42	0.0128	0.0127	0.0098	0.0021	0.0124	0.0300	0.0526	0.0810	0.1168	0.1628	0.2260
43	0.0129	0.0127	0.0096	0.0019	0.0127	0.0303	0.0527	0.0809	0.1164	0.1623	0.2259
44	0.0125	0.0124	0.0093	0.0018	0.0129	0.0305	0.0530	0.0813	0.1169	0.1630	0.2264
45	0.0123	0.0122	0.0091	0.0018	0.0131	0.0307	0.0532	0.0813	0.1169	0.1627	0.2259
46	0.0119	0.0118	0.0087	0.0017	0.0133	0.0308	0.0532	0.0813	0.1168	0.1626	0.2259
47	0.0114	0.0113	0.0084	0.0016	0.0133	0.0307	0.0531	0.0813	0.1169	0.1628	0.2262
48	0.0112	0.0111	0.0082	0.0016	0.0135	0.0309	0.0532	0.0813	0.1168	0.1626	0.2260
49	0.0111	0.0109	0.0080	0.0017	0.0137	0.0311	0.0535	0.0815	0.1169	0.1627	0.2261
50	0.0107	0.0106	0.0077	0.0017	0.0138	0.0312	0.0536	0.0817	0.1170	0.1627	0.2258
51	0.0105	0.0104	0.0075	0.0017	0.0139	0.0313	0.0535	0.0816	0.1171	0.1628	0.2259
52	0.0102	0.0101	0.0073	0.0018	0.0141	0.0315	0.0539	0.0821	0.1175	0.1632	0.2262
53	0.0100	0.0099	0.0071	0.0019	0.0142	0.0315	0.0538	0.0818	0.1170	0.1627	0.2259
54	0.0097	0.0096	0.0068	0.0019	0.0143	0.0316	0.0538	0.0819	0.1172	0.1628	0.2258
55	0.0096	0.0095	0.0067	0.0020	0.0145	0.0319	0.0542	0.0822	0.1175	0.1631	0.2261
56	0.0092	0.0091	0.0064	0.0020	0.0145	0.0318	0.0540	0.0820	0.1173	0.1627	0.2258
57	0.0091	0.0090	0.0063	0.0022	0.0145	0.0317	0.0539	0.0818	0.1170	0.1624	0.2254
58	0.0089	0.0088	0.0061	0.0023	0.0147	0.0320	0.0542	0.0821	0.1173	0.1627	0.2255
59	0.0088	0.0087	0.0060	0.0024	0.0148	0.0320	0.0541	0.0819	0.1170	0.1625	0.2255
60	0.0086	0.0085	0.0058	0.0025	0.0149	0.0322	0.0543	0.0822	0.1173	0.1627	0.2256

Table 3a: $\sigma_U D(\beta; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0649	0.0645	0.0525	0.0283	0.0246	0.0258	0.0342	0.0416	0.0480	0.0531	0.0553
16	0.0591	0.0586	0.0476	0.0248	0.0204	0.0240	0.0335	0.0409	0.0471	0.0513	0.0533
17	0.0580	0.0575	0.0469	0.0258	0.0182	0.0226	0.0324	0.0392	0.0449	0.0495	0.0520
18	0.0568	0.0564	0.0453	0.0229	0.0142	0.0221	0.0325	0.0392	0.0447	0.0490	0.0510
19	0.0513	0.0509	0.0410	0.0227	0.0146	0.0206	0.0300	0.0375	0.0444	0.0495	0.0508
20	0.0489	0.0484	0.0379	0.0177	0.0125	0.0211	0.0295	0.0359	0.0414	0.0465	0.0492
21	0.0408	0.0405	0.0320	0.0161	0.0131	0.0190	0.0270	0.0338	0.0394	0.0445	0.0479
22	0.0407	0.0402	0.0308	0.0136	0.0116	0.0196	0.0272	0.0331	0.0392	0.0450	0.0473
23	0.0425	0.0420	0.0317	0.0131	0.0104	0.0199	0.0272	0.0329	0.0382	0.0429	0.0459
24	0.0338	0.0334	0.0253	0.0114	0.0101	0.0180	0.0249	0.0312	0.0367	0.0414	0.0444
25	0.0342	0.0338	0.0252	0.0110	0.0096	0.0177	0.0248	0.0312	0.0368	0.0419	0.0451
26	0.0296	0.0293	0.0216	0.0098	0.0102	0.0171	0.0236	0.0296	0.0356	0.0408	0.0440
27	0.0269	0.0265	0.0197	0.0093	0.0100	0.0160	0.0224	0.0288	0.0346	0.0394	0.0427
28	0.0266	0.0263	0.0190	0.0082	0.0093	0.0159	0.0218	0.0277	0.0336	0.0388	0.0428
29	0.0278	0.0274	0.0196	0.0077	0.0093	0.0164	0.0222	0.0275	0.0330	0.0378	0.0421
30	0.0252	0.0249	0.0178	0.0068	0.0087	0.0157	0.0214	0.0268	0.0325	0.0375	0.0411
31	0.0232	0.0229	0.0162	0.0059	0.0089	0.0156	0.0208	0.0257	0.0311	0.0366	0.0408
32	0.0220	0.0217	0.0153	0.0053	0.0084	0.0151	0.0204	0.0253	0.0302	0.0352	0.0394
33	0.0225	0.0221	0.0154	0.0049	0.0086	0.0152	0.0201	0.0246	0.0296	0.0346	0.0388
34	0.0216	0.0212	0.0145	0.0047	0.0086	0.0148	0.0198	0.0245	0.0291	0.0336	0.0373
35	0.0202	0.0199	0.0134	0.0052	0.0082	0.0139	0.0192	0.0239	0.0285	0.0334	0.0378
36	0.0178	0.0175	0.0119	0.0047	0.0079	0.0135	0.0187	0.0235	0.0280	0.0325	0.0372
37	0.0195	0.0192	0.0128	0.0044	0.0081	0.0139	0.0190	0.0239	0.0283	0.0328	0.0368
38	0.0149	0.0146	0.0101	0.0034	0.0077	0.0130	0.0176	0.0221	0.0269	0.0315	0.0357
39	0.0171	0.0168	0.0111	0.0036	0.0076	0.0129	0.0178	0.0225	0.0274	0.0324	0.0362
40	0.0162	0.0159	0.0104	0.0029	0.0080	0.0133	0.0178	0.0222	0.0266	0.0308	0.0347
41	0.0170	0.0167	0.0108	0.0023	0.0083	0.0134	0.0175	0.0215	0.0258	0.0306	0.0351
42	0.0115	0.0113	0.0077	0.0031	0.0070	0.0119	0.0165	0.0211	0.0257	0.0304	0.0348
43	0.0151	0.0148	0.0094	0.0022	0.0076	0.0124	0.0167	0.0208	0.0250	0.0293	0.0335
44	0.0127	0.0124	0.0080	0.0025	0.0074	0.0119	0.0161	0.0203	0.0246	0.0290	0.0327
45	0.0133	0.0131	0.0082	0.0026	0.0076	0.0123	0.0164	0.0203	0.0244	0.0285	0.0323
46	0.0124	0.0121	0.0075	0.0024	0.0073	0.0117	0.0157	0.0198	0.0239	0.0281	0.0320
47	0.0093	0.0091	0.0063	0.0019	0.0067	0.0112	0.0153	0.0197	0.0243	0.0289	0.0327
48	0.0110	0.0108	0.0067	0.0020	0.0071	0.0113	0.0152	0.0191	0.0230	0.0271	0.0310
49	0.0099	0.0097	0.0062	0.0020	0.0069	0.0111	0.0152	0.0192	0.0233	0.0274	0.0316
50	0.0094	0.0092	0.0059	0.0018	0.0068	0.0110	0.0150	0.0192	0.0235	0.0277	0.0313
51	0.0097	0.0096	0.0060	0.0019	0.0069	0.0112	0.0150	0.0190	0.0230	0.0270	0.0307
52	0.0086	0.0085	0.0055	0.0019	0.0065	0.0108	0.0148	0.0187	0.0227	0.0267	0.0304
53	0.0080	0.0078	0.0050	0.0020	0.0065	0.0105	0.0144	0.0182	0.0219	0.0258	0.0298
54	0.0070	0.0069	0.0045	0.0018	0.0062	0.0101	0.0139	0.0178	0.0220	0.0261	0.0300
55	0.0072	0.0071	0.0045	0.0019	0.0062	0.0102	0.0138	0.0174	0.0212	0.0252	0.0294
56	0.0063	0.0062	0.0040	0.0017	0.0057	0.0095	0.0133	0.0170	0.0209	0.0248	0.0286
57	0.0067	0.0066	0.0042	0.0018	0.0060	0.0100	0.0140	0.0179	0.0218	0.0255	0.0288
58	0.0060	0.0059	0.0038	0.0019	0.0057	0.0096	0.0134	0.0171	0.0210	0.0250	0.0287
59	0.0064	0.0063	0.0038	0.0019	0.0059	0.0096	0.0133	0.0169	0.0205	0.0243	0.0280
60	0.0060	0.0059	0.0037	0.0019	0.0058	0.0096	0.0133	0.0169	0.0205	0.0242	0.0279

Table 4: $\mu_U d(\beta; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.2380	0.2116	0.1842	0.1488	0.1370	0.1793	0.2639	0.3667	0.5451	0.9039	1.8070
16	0.2036	0.1885	0.1635	0.1239	0.1064	0.1559	0.2356	0.3493	0.4914	0.8225	1.6838
17	0.1841	0.1758	0.1507	0.1086	0.0906	0.1379	0.2234	0.3293	0.4857	0.7601	1.6640
18	0.1684	0.1607	0.1380	0.0894	0.0734	0.1268	0.2192	0.3297	0.4533	0.7120	1.5392
19	0.1496	0.1449	0.1240	0.0768	0.0618	0.1197	0.2122	0.3172	0.4405	0.6879	1.5328
20	0.1446	0.1414	0.1198	0.0678	0.0533	0.1207	0.2172	0.3170	0.4394	0.6644	1.4250
21	0.1319	0.1303	0.1088	0.0591	0.0503	0.1202	0.2159	0.3178	0.4296	0.6508	1.4118
22	0.1298	0.1278	0.1065	0.0535	0.0470	0.1223	0.2181	0.3195	0.4310	0.6317	1.3786
23	0.1292	0.1280	0.1058	0.0498	0.0470	0.1276	0.2267	0.3275	0.4312	0.6185	1.3122
24	0.1191	0.1180	0.0974	0.0437	0.0470	0.1280	0.2264	0.3274	0.4302	0.6116	1.2766
25	0.1146	0.1136	0.0936	0.0404	0.0471	0.1300	0.2280	0.3282	0.4305	0.5985	1.2572
26	0.1110	0.1099	0.0899	0.0367	0.0485	0.1332	0.2316	0.3323	0.4324	0.5918	1.2416
27	0.1061	0.1052	0.0858	0.0335	0.0494	0.1353	0.2341	0.3349	0.4341	0.5899	1.2168
28	0.1060	0.1051	0.0848	0.0306	0.0523	0.1404	0.2398	0.3406	0.4396	0.5898	1.2008
29	0.1051	0.1042	0.0836	0.0282	0.0549	0.1450	0.2451	0.3461	0.4444	0.5880	1.1841
30	0.1010	0.1001	0.0800	0.0252	0.0562	0.1471	0.2463	0.3467	0.4446	0.5852	1.1791
31	0.0998	0.0989	0.0784	0.0226	0.0591	0.1505	0.2502	0.3508	0.4486	0.5865	1.1642
32	0.0971	0.0963	0.0761	0.0207	0.0611	0.1537	0.2540	0.3547	0.4522	0.5849	1.1578
33	0.0968	0.0959	0.0751	0.0189	0.0641	0.1579	0.2581	0.3588	0.4561	0.5876	1.1458
34	0.0945	0.0936	0.0728	0.0173	0.0665	0.1603	0.2607	0.3610	0.4580	0.5884	1.1423
35	0.0917	0.0908	0.0704	0.0163	0.0684	0.1630	0.2645	0.3655	0.4623	0.5900	1.1391
36	0.0894	0.0885	0.0684	0.0150	0.0700	0.1651	0.2666	0.3675	0.4640	0.5904	1.1379
37	0.0895	0.0886	0.0679	0.0143	0.0728	0.1690	0.2710	0.3722	0.4685	0.5936	1.1347
38	0.0863	0.0854	0.0650	0.0127	0.0744	0.1700	0.2715	0.3724	0.4689	0.5937	1.1302
39	0.0837	0.0828	0.0627	0.0121	0.0750	0.1702	0.2717	0.3727	0.4691	0.5944	1.1257
40	0.0841	0.0831	0.0622	0.0112	0.0780	0.1738	0.2752	0.3757	0.4716	0.5946	1.1257
41	0.0849	0.0840	0.0623	0.0110	0.0815	0.1788	0.2809	0.3818	0.4775	0.5985	1.1284
42	0.0795	0.0787	0.0585	0.0109	0.0809	0.1772	0.2792	0.3802	0.4761	0.5988	1.1279
43	0.0795	0.0786	0.0577	0.0103	0.0829	0.1793	0.2810	0.3818	0.4777	0.5990	1.1280
44	0.0790	0.0781	0.0570	0.0105	0.0858	0.1833	0.2860	0.3871	0.4828	0.6018	1.1279
45	0.0785	0.0775	0.0561	0.0108	0.0881	0.1861	0.2888	0.3899	0.4851	0.6032	1.1272
46	0.0766	0.0756	0.0544	0.0106	0.0889	0.1868	0.2895	0.3904	0.4855	0.6036	1.1285
47	0.0750	0.0741	0.0532	0.0113	0.0906	0.1888	0.2916	0.3926	0.4878	0.6055	1.1304
48	0.0738	0.0729	0.0518	0.0116	0.0918	0.1898	0.2926	0.3936	0.4885	0.6062	1.1310
49	0.0740	0.0731	0.0515	0.0125	0.0946	0.1937	0.2971	0.3981	0.4927	0.6088	1.1322
50	0.0722	0.0713	0.0499	0.0132	0.0953	0.1942	0.2976	0.3988	0.4936	0.6091	1.1321
51	0.0710	0.0701	0.0487	0.0137	0.0963	0.1948	0.2978	0.3987	0.4935	0.6097	1.1333
52	0.0700	0.0691	0.0479	0.0144	0.0979	0.1974	0.3011	0.4023	0.4969	0.6118	1.1358
53	0.0689	0.0680	0.0467	0.0152	0.0992	0.1985	0.3022	0.4032	0.4977	0.6121	1.1355
54	0.0683	0.0674	0.0459	0.0163	0.1013	0.2012	0.3051	0.4063	0.5006	0.6135	1.1364
55	0.0682	0.0673	0.0455	0.0172	0.1035	0.2042	0.3085	0.4097	0.5039	0.6162	1.1380
56	0.0659	0.0650	0.0438	0.0176	0.1033	0.2037	0.3080	0.4094	0.5036	0.6158	1.1388
57	0.0650	0.0641	0.0428	0.0186	0.1041	0.2042	0.3083	0.4093	0.5034	0.6149	1.1386
58	0.0649	0.0640	0.0424	0.0198	0.1066	0.2077	0.3125	0.4140	0.5078	0.6175	1.1395
59	0.0642	0.0633	0.0415	0.0206	0.1077	0.2085	0.3128	0.4138	0.5075	0.6184	1.1408
60	0.0634	0.0624	0.0408	0.0215	0.1089	0.2104	0.3155	0.4168	0.5104	0.6195	1.1420

Table 4a: $\sigma_{\mathcal{A}}(\beta; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.1965	0.1838	0.1507	0.1296	0.1419	0.1553	0.1804	0.2163	0.3275	0.7686	1.4704
16	0.1656	0.1634	0.1329	0.1026	0.1026	0.1333	0.1388	0.1705	0.2245	0.5433	1.0038
17	0.1616	0.1564	0.1273	0.0953	0.0933	0.0942	0.1134	0.1354	0.2399	0.3923	0.9997
18	0.1551	0.1531	0.1211	0.0709	0.0691	0.0782	0.1090	0.1301	0.1698	0.2665	0.7931
19	0.1429	0.1405	0.1123	0.0631	0.0609	0.0784	0.0999	0.1161	0.1473	0.2496	1.1170
20	0.1414	0.1396	0.1088	0.0514	0.0507	0.0731	0.0997	0.1140	0.1393	0.2415	0.6141
21	0.1233	0.1223	0.0958	0.0448	0.0494	0.0710	0.0972	0.1109	0.1148	0.2239	0.6701
22	0.1269	0.1258	0.0964	0.0399	0.0453	0.0729	0.0993	0.1103	0.1108	0.1637	0.5684
23	0.1314	0.1299	0.0983	0.0390	0.0432	0.0751	0.1000	0.1113	0.1101	0.1429	0.4647
24	0.1129	0.1117	0.0848	0.0335	0.0433	0.0725	0.0985	0.1097	0.1054	0.1310	0.3730
25	0.1137	0.1125	0.0844	0.0325	0.0422	0.0728	0.0989	0.1115	0.1089	0.1150	0.3517
26	0.1048	0.1036	0.0773	0.0295	0.0431	0.0742	0.0985	0.1096	0.1063	0.1011	0.3609
27	0.0978	0.0966	0.0723	0.0278	0.0419	0.0725	0.0969	0.1085	0.1055	0.0947	0.2832
28	0.0982	0.0969	0.0711	0.0257	0.0416	0.0740	0.0978	0.1084	0.1049	0.0931	0.2731
29	0.1018	0.1004	0.0729	0.0240	0.0428	0.0753	0.0987	0.1089	0.1052	0.0905	0.2286
30	0.0955	0.0942	0.0686	0.0222	0.0412	0.0750	0.0986	0.1090	0.1057	0.0850	0.2544
31	0.0935	0.0923	0.0660	0.0191	0.0434	0.0775	0.0998	0.1091	0.1054	0.0826	0.1959
32	0.0893	0.0881	0.0632	0.0180	0.0416	0.0758	0.0977	0.1065	0.1026	0.0783	0.1851
33	0.0908	0.0895	0.0635	0.0170	0.0438	0.0787	0.1004	0.1086	0.1040	0.0792	0.1588
34	0.0889	0.0876	0.0611	0.0167	0.0447	0.0795	0.1014	0.1100	0.1054	0.0765	0.1523
35	0.0840	0.0827	0.0572	0.0176	0.0436	0.0760	0.0972	0.1055	0.1015	0.0755	0.1532
36	0.0778	0.0767	0.0534	0.0165	0.0436	0.0760	0.0972	0.1057	0.1016	0.0749	0.1474
37	0.0823	0.0811	0.0557	0.0162	0.0450	0.0781	0.0987	0.1065	0.1022	0.0756	0.1381
38	0.0722	0.0711	0.0495	0.0134	0.0451	0.0780	0.0983	0.1057	0.1011	0.0732	0.1332
39	0.0759	0.0747	0.0506	0.0139	0.0446	0.0769	0.0971	0.1047	0.1006	0.0741	0.1305
40	0.0763	0.0751	0.0506	0.0127	0.0474	0.0801	0.0998	0.1070	0.1025	0.0726	0.1290
41	0.0789	0.0776	0.0514	0.0118	0.0482	0.0804	0.0991	0.1051	0.1001	0.0730	0.1291
42	0.0628	0.0618	0.0424	0.0135	0.0460	0.0778	0.0971	0.1039	0.0994	0.0721	0.1279
43	0.0722	0.0710	0.0464	0.0116	0.0479	0.0795	0.0984	0.1046	0.0993	0.0703	0.1224
44	0.0666	0.0655	0.0431	0.0119	0.0480	0.0789	0.0970	0.1028	0.0975	0.0695	0.1186
45	0.0690	0.0677	0.0436	0.0134	0.0491	0.0801	0.0981	0.1036	0.0984	0.0697	0.1174
46	0.0656	0.0645	0.0413	0.0118	0.0490	0.0797	0.0979	0.1037	0.0988	0.0694	0.1156
47	0.0568	0.0560	0.0379	0.0115	0.0483	0.0793	0.0973	0.1028	0.0975	0.0692	0.1181
48	0.0619	0.0608	0.0384	0.0117	0.0498	0.0799	0.0975	0.1030	0.0978	0.0673	0.1124
49	0.0590	0.0580	0.0373	0.0128	0.0504	0.0806	0.0980	0.1031	0.0977	0.0676	0.1149
50	0.0571	0.0562	0.0361	0.0129	0.0507	0.0804	0.0974	0.1023	0.0967	0.0677	0.1122
51	0.0584	0.0574	0.0362	0.0130	0.0515	0.0816	0.0989	0.1040	0.0981	0.0679	0.1115
52	0.0541	0.0532	0.0341	0.0130	0.0504	0.0803	0.0974	0.1022	0.0968	0.0671	0.1109
53	0.0525	0.0516	0.0328	0.0139	0.0514	0.0808	0.0975	0.1022	0.0965	0.0656	0.1089
54	0.0495	0.0486	0.0309	0.0141	0.0516	0.0810	0.0973	0.1015	0.0956	0.0669	0.1087
55	0.0500	0.0492	0.0309	0.0145	0.0521	0.0816	0.0980	0.1021	0.0960	0.0653	0.1078
56	0.0459	0.0451	0.0284	0.0138	0.0505	0.0793	0.0956	0.0999	0.0943	0.0635	0.1049
57	0.0473	0.0465	0.0292	0.0147	0.0510	0.0799	0.0963	0.1008	0.0951	0.0653	0.1051
58	0.0450	0.0442	0.0277	0.0151	0.0516	0.0798	0.0954	0.0990	0.0931	0.0641	0.1045
59	0.0467	0.0459	0.0277	0.0158	0.0532	0.0820	0.0980	0.1019	0.0955	0.0644	0.1025
60	0.0447	0.0439	0.0268	0.0160	0.0520	0.0800	0.0955	0.0993	0.0934	0.0637	0.1015

Table 5: $\mu_p D(\phi; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

$\frac{q}{n}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0160	0.0160	0.0160	0.0160	0.0160	0.0161	0.0163	0.0181	0.0241	0.0598	0.1333
16	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0151	0.0160	0.0207	0.0504	0.1320
17	0.0137	0.0137	0.0137	0.0137	0.0137	0.0138	0.0139	0.0147	0.0190	0.0429	0.1285
18	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0135	0.0164	0.0358	0.1250
19	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0126	0.0150	0.0314	0.1166
20	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0119	0.0120	0.0137	0.0276	0.1086
21	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112	0.0114	0.0126	0.0231	0.0969
22	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0108	0.0116	0.0205	0.0891
23	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0103	0.0109	0.0180	0.0836
24	0.0096	0.0096	0.0096	0.0096	0.0096	0.0096	0.0096	0.0097	0.0102	0.0159	0.0804
25	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0095	0.0142	0.0778
26	0.0089	0.0089	0.0089	0.0089	0.0089	0.0089	0.0089	0.0089	0.0091	0.0130	0.0758
27	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0088	0.0122	0.0724
28	0.0082	0.0082	0.0082	0.0082	0.0082	0.0082	0.0082	0.0082	0.0083	0.0110	0.0669
29	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0102	0.0615
30	0.0077	0.0077	0.0077	0.0077	0.0077	0.0077	0.0077	0.0077	0.0077	0.0093	0.0538
31	0.0074	0.0074	0.0074	0.0074	0.0074	0.0074	0.0074	0.0074	0.0075	0.0087	0.0485
32	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0083	0.0434
33	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0078	0.0389
34	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0074	0.0367
35	0.0065	0.0065	0.0065	0.0065	0.0065	0.0065	0.0065	0.0065	0.0066	0.0069	0.0352
36	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0067	0.0326
37	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0064	0.0310
38	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061	0.0062	0.0280
39	0.0059	0.0059	0.0059	0.0059	0.0059	0.0059	0.0059	0.0059	0.0059	0.0061	0.0258
40	0.0057	0.0057	0.0057	0.0057	0.0057	0.0057	0.0057	0.0057	0.0057	0.0059	0.0223
41	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	0.0056	0.0057	0.0201
42	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055	0.0056	0.0186
43	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0173
44	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0053	0.0158
45	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0150
46	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0141
47	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0050	0.0132
48	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0123
49	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0111
50	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0046	0.0104
51	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0046	0.0098
52	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0089
53	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0086
54	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0082
55	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0076
56	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0071
57	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0067

Table 6: $\mu_{pd}(\phi; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

[illegible]

Table 6a: $\sigma_{pd}(\phi; n, q)$ for n from 15 to 57 and selected values of q from 0.50 to 0.95 (1000 trials)

q n	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0246	0.0246	0.0247	0.0248	0.0252	0.0259	0.0299	0.0404	0.0816	0.0786	0.1397
16	0.0237	0.0237	0.0236	0.0236	0.0237	0.0239	0.0258	0.0307	0.0688	0.0760	0.1274
17	0.0218	0.0218	0.0218	0.0220	0.0221	0.0227	0.0240	0.0314	0.0606	0.0797	0.1134
18	0.0208	0.0208	0.0208	0.0207	0.0207	0.0207	0.0221	0.0268	0.0465	0.0816	0.0929
19	0.0205	0.0205	0.0206	0.0206	0.0205	0.0206	0.0210	0.0230	0.0405	0.0813	0.0879
20	0.0196	0.0196	0.0196	0.0196	0.0197	0.0196	0.0200	0.0209	0.0353	0.0796	0.0775
21	0.0190	0.0190	0.0190	0.0190	0.0190	0.0190	0.0190	0.0205	0.0293	0.0772	0.0697
22	0.0180	0.0180	0.0180	0.0180	0.0180	0.0181	0.0180	0.0192	0.0256	0.0750	0.0646
23	0.0162	0.0162	0.0162	0.0162	0.0162	0.0162	0.0161	0.0161	0.0193	0.0699	0.0551
24	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0159	0.0163	0.0206	0.0642	0.0571
25	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0150	0.0182	0.0541	0.0541
26	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150	0.0168	0.0479	0.0494
27	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0146	0.0146	0.0160	0.0449	0.0484
28	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0146	0.0364	0.0450
29	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0138	0.0336	0.0420
30	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0122	0.0124	0.0270	0.0408
31	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0126	0.0125	0.0230	0.0415
32	0.0120	0.0120	0.0120	0.0120	0.0120	0.0120	0.0120	0.0120	0.0121	0.0215	0.0426
33	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0116	0.0180	0.0418
34	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0115	0.0116	0.0197	0.0474
35	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0109	0.0145	0.0437
36	0.0109	0.0109	0.0109	0.0109	0.0109	0.0109	0.0109	0.0109	0.0110	0.0169	0.0489
37	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0108	0.0139	0.0474
38	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0124	0.0503
39	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0124	0.0571
40	0.0098	0.0098	0.0098	0.0098	0.0098	0.0098	0.0098	0.0098	0.0099	0.0130	0.0576
41	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0096	0.0108	0.0606
42	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0095	0.0112	0.0636
43	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0097	0.0635
44	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0104	0.0650
45	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0086	0.0092	0.0649
46	0.0085	0.0085	0.0085	0.0085	0.0085	0.0085	0.0085	0.0085	0.0085	0.0090	0.0641
47	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0641
48	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0077	0.0079	0.0622
49	0.0079	0.0079	0.0079	0.0079	0.0079	0.0079	0.0079	0.0079	0.0079	0.0082	0.0608
50	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0078	0.0077	0.0079	0.0597
51	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0079	0.0565
52	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0072	0.0534
53	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0076	0.0525
54	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0075	0.0077	0.0492
55	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0070	0.0467
56	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0429
57	0.0069	0.0069	0.0069	0.0069	0.0069	0.0069	0.0069	0.0069	0.0069	0.0070	0.0401

Table 7: $\mu_U D(\phi; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\frac{q}{n}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0452	0.0449	0.0444	0.0444	0.0447	0.0454	0.0472	0.0510	0.0708	0.1090	0.1863
16	0.0416	0.0415	0.0416	0.0413	0.0418	0.0429	0.0435	0.0471	0.0632	0.1018	0.1794
17	0.0392	0.0392	0.0391	0.0386	0.0389	0.0388	0.0400	0.0439	0.0582	0.0942	0.1717
18	0.0367	0.0367	0.0366	0.0363	0.0360	0.0357	0.0367	0.0407	0.0523	0.0898	0.1651
19	0.0336	0.0335	0.0335	0.0334	0.0331	0.0328	0.0343	0.0371	0.0488	0.0816	0.1577
20	0.0319	0.0319	0.0319	0.0317	0.0317	0.0317	0.0319	0.0342	0.0437	0.0760	0.1525
21	0.0291	0.0291	0.0291	0.0291	0.0291	0.0291	0.0297	0.0316	0.0401	0.0702	0.1470
22	0.0282	0.0282	0.0282	0.0282	0.0282	0.0283	0.0280	0.0299	0.0382	0.0658	0.1399
23	0.0273	0.0273	0.0273	0.0273	0.0273	0.0272	0.0266	0.0283	0.0355	0.0600	0.1351
24	0.0253	0.0253	0.0253	0.0254	0.0254	0.0253	0.0254	0.0267	0.0338	0.0562	0.1303
25	0.0242	0.0242	0.0242	0.0242	0.0242	0.0243	0.0241	0.0249	0.0310	0.0526	0.1240
26	0.0230	0.0230	0.0230	0.0230	0.0231	0.0231	0.0232	0.0239	0.0293	0.0490	0.1197
27	0.0220	0.0220	0.0220	0.0220	0.0220	0.0220	0.0220	0.0231	0.0265	0.0454	0.1144
28	0.0212	0.0212	0.0212	0.0212	0.0212	0.0213	0.0212	0.0219	0.0255	0.0423	0.1109
29	0.0206	0.0206	0.0206	0.0206	0.0207	0.0206	0.0206	0.0206	0.0243	0.0401	0.1070
30	0.0196	0.0196	0.0196	0.0196	0.0196	0.0196	0.0197	0.0194	0.0227	0.0361	0.1011
31	0.0191	0.0191	0.0191	0.0191	0.0191	0.0191	0.0191	0.0188	0.0214	0.0358	0.0970
32	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0182	0.0199	0.0331	0.0932
33	0.0179	0.0179	0.0179	0.0179	0.0179	0.0178	0.0178	0.0178	0.0192	0.0315	0.0899
34	0.0172	0.0172	0.0172	0.0172	0.0172	0.0172	0.0172	0.0172	0.0184	0.0295	0.0871
35	0.0167	0.0167	0.0167	0.0167	0.0167	0.0167	0.0167	0.0166	0.0178	0.0276	0.0834
36	0.0161	0.0161	0.0161	0.0161	0.0161	0.0161	0.0161	0.0161	0.0170	0.0262	0.0806
37	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0158	0.0159	0.0166	0.0250	0.0772
38	0.0152	0.0152	0.0152	0.0152	0.0152	0.0152	0.0153	0.0152	0.0158	0.0237	0.0739
39	0.0148	0.0148	0.0148	0.0148	0.0148	0.0148	0.0147	0.0148	0.0152	0.0227	0.0703
40	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0215	0.0683
41	0.0142	0.0142	0.0142	0.0142	0.0142	0.0142	0.0141	0.0141	0.0140	0.0207	0.0657
42	0.0136	0.0136	0.0136	0.0136	0.0136	0.0136	0.0137	0.0138	0.0139	0.0193	0.0634
43	0.0133	0.0133	0.0133	0.0133	0.0133	0.0133	0.0133	0.0133	0.0134	0.0185	0.0606
44	0.0130	0.0130	0.0130	0.0130	0.0130	0.0131	0.0130	0.0130	0.0131	0.0172	0.0585
45	0.0127	0.0127	0.0127	0.0127	0.0127	0.0127	0.0127	0.0128	0.0128	0.0165	0.0560
46	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0124	0.0126	0.0159	0.0539
47	0.0121	0.0121	0.0121	0.0121	0.0121	0.0121	0.0121	0.0122	0.0122	0.0151	0.0520
48	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0147	0.0499
49	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0116	0.0143	0.0483
50	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0114	0.0134	0.0460
51	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0111	0.0131	0.0441
52	0.0110	0.0110	0.0110	0.0110	0.0110	0.0110	0.0110	0.0110	0.0110	0.0128	0.0426
53	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0123	0.0406
54	0.0105	0.0105	0.0105	0.0105	0.0105	0.0105	0.0105	0.0105	0.0105	0.0117	0.0390
55	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0103	0.0114	0.0373
56	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0101	0.0111	0.0358
57	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099	0.0109	0.0346
58	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0097	0.0098	0.0105	0.0330
59	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095	0.0095	0.0102	0.0319
60	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0094	0.0100	0.0307

Table 7a: $\sigma_U D(\phi; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.0402	0.0394	0.0375	0.0365	0.0343	0.0329	0.0327	0.0316	0.0343	0.0382	0.0466
16	0.0339	0.0339	0.0328	0.0314	0.0313	0.0315	0.0298	0.0313	0.0311	0.0375	0.0435
17	0.0331	0.0331	0.0341	0.0301	0.0288	0.0266	0.0278	0.0285	0.0325	0.0367	0.0431
18	0.0307	0.0312	0.0306	0.0282	0.0257	0.0233	0.0247	0.0272	0.0298	0.0349	0.0415
19	0.0287	0.0286	0.0288	0.0263	0.0240	0.0213	0.0238	0.0249	0.0293	0.0335	0.0409
20	0.0243	0.0244	0.0247	0.0239	0.0227	0.0209	0.0207	0.0236	0.0270	0.0329	0.0386
21	0.0206	0.0205	0.0210	0.0201	0.0195	0.0181	0.0199	0.0220	0.0248	0.0295	0.0375
22	0.0195	0.0195	0.0195	0.0195	0.0191	0.0185	0.0179	0.0199	0.0245	0.0298	0.0357
23	0.0192	0.0192	0.0192	0.0191	0.0186	0.0174	0.0160	0.0186	0.0232	0.0290	0.0352
24	0.0157	0.0157	0.0157	0.0162	0.0155	0.0150	0.0154	0.0176	0.0228	0.0270	0.0346
25	0.0155	0.0155	0.0155	0.0155	0.0152	0.0154	0.0140	0.0163	0.0208	0.0273	0.0334
26	0.0136	0.0136	0.0136	0.0135	0.0139	0.0139	0.0135	0.0157	0.0193	0.0250	0.0316
27	0.0122	0.0122	0.0122	0.0122	0.0123	0.0121	0.0124	0.0146	0.0179	0.0244	0.0310
28	0.0117	0.0117	0.0116	0.0116	0.0119	0.0116	0.0113	0.0137	0.0173	0.0235	0.0304
29	0.0118	0.0118	0.0118	0.0118	0.0121	0.0113	0.0115	0.0118	0.0164	0.0232	0.0296
30	0.0108	0.0108	0.0108	0.0109	0.0108	0.0107	0.0109	0.0104	0.0149	0.0223	0.0287
31	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0101	0.0099	0.0139	0.0222	0.0281
32	0.0094	0.0094	0.0094	0.0094	0.0094	0.0096	0.0094	0.0092	0.0125	0.0207	0.0273
33	0.0091	0.0091	0.0091	0.0091	0.0091	0.0088	0.0086	0.0087	0.0123	0.0202	0.0265
34	0.0085	0.0085	0.0085	0.0085	0.0085	0.0086	0.0083	0.0087	0.0117	0.0197	0.0263
35	0.0080	0.0080	0.0080	0.0080	0.0081	0.0084	0.0079	0.0080	0.0111	0.0185	0.0261
36	0.0073	0.0073	0.0073	0.0073	0.0073	0.0075	0.0070	0.0078	0.0100	0.0167	0.0249
37	0.0076	0.0076	0.0076	0.0076	0.0076	0.0077	0.0072	0.0080	0.0098	0.0174	0.0251
38	0.0066	0.0066	0.0066	0.0066	0.0066	0.0066	0.0069	0.0071	0.0088	0.0152	0.0236
39	0.0068	0.0068	0.0068	0.0068	0.0068	0.0067	0.0065	0.0068	0.0084	0.0159	0.0237
40	0.0065	0.0065	0.0065	0.0065	0.0065	0.0065	0.0064	0.0063	0.0070	0.0152	0.0228
41	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0062	0.0061	0.0065	0.0144	0.0225
42	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0060	0.0071	0.0133	0.0224
43	0.0057	0.0057	0.0057	0.0057	0.0057	0.0057	0.0056	0.0052	0.0064	0.0129	0.0216
44	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0059	0.0121	0.0213
45	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0052	0.0053	0.0060	0.0110	0.0212
46	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0049	0.0047	0.0059	0.0105	0.0204
47	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0045	0.0049	0.0056	0.0101	0.0200
48	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0047	0.0100	0.0192
49	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0043	0.0044	0.0046	0.0096	0.0196
50	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0045	0.0046	0.0086	0.0189
51	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041	0.0043	0.0043	0.0087	0.0185
52	0.0039	0.0039	0.0039	0.0039	0.0039	0.0039	0.0039	0.0041	0.0045	0.0083	0.0189
53	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0042	0.0075	0.0176
54	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0039	0.0071	0.0170
55	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0034	0.0038	0.0065	0.0167
56	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0035	0.0063	0.0159
57	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0036	0.0067	0.0166
58	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031	0.0037	0.0056	0.0160
59	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0032	0.0053	0.0150
60	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029	0.0030	0.0034	0.0054	0.0155

Table 8: $\mu_{\mathcal{U}}d(\phi; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{matrix} q \\ n \end{matrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.2407	0.2094	0.2095	0.2196	0.2479	0.2790	0.3403	0.4011	0.5585	0.7495	1.2201
16	0.2002	0.1817	0.1874	0.1992	0.2140	0.2352	0.2882	0.3516	0.4921	0.7216	1.1095
17	0.1802	0.1724	0.1696	0.1788	0.1889	0.2158	0.2566	0.3416	0.4821	0.6873	1.1229
18	0.1665	0.1529	0.1552	0.1573	0.1739	0.1951	0.2300	0.3032	0.4359	0.6703	1.0603
19	0.1451	0.1415	0.1421	0.1449	0.1515	0.1709	0.2079	0.2705	0.3766	0.5923	1.0307
20	0.1362	0.1320	0.1327	0.1359	0.1435	0.1554	0.1844	0.2530	0.3508	0.5662	0.9862
21	0.1243	0.1231	0.1230	0.1249	0.1325	0.1488	0.1710	0.2235	0.3574	0.5501	0.9806
22	0.1202	0.1181	0.1176	0.1199	0.1276	0.1310	0.1596	0.1979	0.3015	0.5051	0.9217
23	0.1152	0.1148	0.1150	0.1167	0.1191	0.1247	0.1372	0.1855	0.2796	0.5054	0.8908
24	0.1088	0.1081	0.1085	0.1088	0.1112	0.1161	0.1282	0.1682	0.3029	0.4680	0.8859
25	0.1044	0.1039	0.1041	0.1046	0.1056	0.1111	0.1200	0.1495	0.2525	0.4181	0.8434
26	0.1005	0.1002	0.1003	0.1005	0.1019	0.1050	0.1153	0.1447	0.2304	0.3876	0.8192
27	0.0965	0.0964	0.0964	0.0966	0.0988	0.0999	0.1136	0.1344	0.2043	0.3821	0.8134
28	0.0947	0.0946	0.0946	0.0948	0.0953	0.0974	0.1021	0.1257	0.2030	0.3532	0.7760
29	0.0925	0.0925	0.0925	0.0926	0.0930	0.0940	0.0985	0.1162	0.1873	0.3530	0.7559
30	0.0892	0.0892	0.0892	0.0892	0.0895	0.0902	0.0935	0.1094	0.1677	0.3202	0.7358
31	0.0873	0.0873	0.0873	0.0873	0.0875	0.0882	0.0917	0.1045	0.1546	0.3084	0.7065
32	0.0849	0.0849	0.0849	0.0850	0.0850	0.0855	0.0876	0.0975	0.1389	0.2977	0.7110
33	0.0835	0.0835	0.0835	0.0836	0.0836	0.0838	0.0853	0.0923	0.1369	0.2639	0.6576
34	0.0815	0.0815	0.0815	0.0815	0.0815	0.0818	0.0831	0.0885	0.1252	0.2762	0.6228
35	0.0795	0.0795	0.0795	0.0795	0.0795	0.0798	0.0805	0.0848	0.1162	0.2602	0.6364
36	0.0775	0.0775	0.0775	0.0775	0.0776	0.0777	0.0782	0.0811	0.1071	0.2442	0.6489
37	0.0766	0.0766	0.0766	0.0766	0.0766	0.0767	0.0769	0.0804	0.1022	0.2147	0.6086
38	0.0742	0.0742	0.0742	0.0742	0.0742	0.0743	0.0768	0.0775	0.1018	0.2018	0.5601
39	0.0722	0.0722	0.0722	0.0722	0.0722	0.0722	0.0724	0.0754	0.0925	0.2186	0.5754
40	0.0713	0.0713	0.0713	0.0713	0.0713	0.0713	0.0714	0.0730	0.0899	0.1870	0.5483
41	0.0708	0.0708	0.0708	0.0708	0.0708	0.0708	0.0709	0.0729	0.0808	0.1704	0.5342
42	0.0683	0.0683	0.0683	0.0683	0.0683	0.0683	0.0684	0.0694	0.0806	0.1711	0.5117
43	0.0671	0.0671	0.0671	0.0671	0.0671	0.0671	0.0672	0.0675	0.0732	0.1570	0.5201
44	0.0664	0.0664	0.0664	0.0664	0.0664	0.0665	0.0665	0.0681	0.0720	0.1441	0.4814
45	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0655	0.0663	0.0710	0.1419	0.4865
46	0.0641	0.0641	0.0641	0.0641	0.0641	0.0641	0.0642	0.0642	0.0697	0.1332	0.4780
47	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0635	0.0665	0.1295	0.4466
48	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0618	0.0637	0.1312	0.4142
49	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0615	0.0639	0.1276	0.4402
50	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0602	0.0619	0.1078	0.4389
51	0.0589	0.0589	0.0589	0.0589	0.0589	0.0589	0.0589	0.0591	0.0598	0.1060	0.4258
52	0.0583	0.0583	0.0583	0.0583	0.0583	0.0583	0.0583	0.0584	0.0603	0.1008	0.3879
53	0.0573	0.0573	0.0573	0.0573	0.0573	0.0573	0.0573	0.0573	0.0577	0.0912	0.4008
54	0.0566	0.0566	0.0566	0.0566	0.0566	0.0566	0.0566	0.0566	0.0571	0.0895	0.3707
55	0.0562	0.0562	0.0562	0.0562	0.0562	0.0562	0.0562	0.0563	0.0572	0.0863	0.3709
56	0.0548	0.0548	0.0548	0.0548	0.0548	0.0548	0.0548	0.0549	0.0552	0.0811	0.3634
57	0.0539	0.0539	0.0539	0.0539	0.0539	0.0539	0.0539	0.0540	0.0540	0.0839	0.3668
58	0.0536	0.0536	0.0536	0.0536	0.0536	0.0536	0.0536	0.0536	0.0542	0.0732	0.3388
59	0.0528	0.0528	0.0528	0.0528	0.0528	0.0528	0.0528	0.0528	0.0530	0.0750	0.3419
60	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0531	0.0692	0.3148

Table 8a: $\sigma_{\mathcal{A}}(\phi; n, q)$ for n from 15 to 60 and selected values of q from 0.50 to 0.95 (1000 trials)

$\begin{smallmatrix} q \\ n \end{smallmatrix}$	0.50	0.51	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
15	0.1724	0.1425	0.1339	0.1450	0.2648	0.2195	0.3275	0.4654	0.5520	0.9951	0.8078
16	0.1261	0.1071	0.1183	0.1705	0.1712	0.1456	0.2315	0.2321	0.5023	0.6545	0.5795
17	0.1090	0.1070	0.1009	0.1099	0.1165	0.1760	0.1776	0.3247	0.5394	0.8358	0.7214
18	0.1270	0.0869	0.0837	0.0815	0.1454	0.1672	0.1776	0.3430	0.7665	0.6283	0.6627
19	0.0805	0.0818	0.0849	0.0789	0.0759	0.1018	0.2078	0.2180	0.3276	0.4633	0.7902
20	0.0721	0.0686	0.0694	0.0711	0.0835	0.0922	0.1351	0.3122	0.3230	0.5143	0.6757
21	0.0617	0.0608	0.0612	0.0604	0.1258	0.1590	0.1852	0.2473	0.8614	0.7472	0.8092
22	0.0602	0.0599	0.0595	0.0597	0.1184	0.0673	0.2216	0.1358	0.4277	0.5048	0.6281
23	0.0593	0.0594	0.0593	0.0594	0.0595	0.0624	0.0761	0.1802	0.2815	0.6583	0.6522
24	0.0525	0.0525	0.0534	0.0532	0.0522	0.0541	0.0647	0.1232	0.7564	0.6885	1.1029
25	0.0518	0.0519	0.0519	0.0527	0.0513	0.0616	0.0633	0.1110	0.3553	0.3306	0.8810
26	0.0483	0.0483	0.0482	0.0481	0.0488	0.0502	0.0684	0.1549	0.3856	0.2816	0.8072
27	0.0454	0.0454	0.0454	0.0453	0.0609	0.0450	0.2258	0.1078	0.2064	0.3721	1.0214
28	0.0444	0.0444	0.0444	0.0443	0.0448	0.0443	0.0460	0.0854	0.2210	0.3741	0.8024
29	0.0447	0.0447	0.0447	0.0446	0.0452	0.0433	0.0479	0.0904	0.1738	0.3997	0.6185
30	0.0427	0.0427	0.0427	0.0427	0.0426	0.0426	0.0431	0.1080	0.1514	0.4064	1.2762
31	0.0413	0.0413	0.0413	0.0412	0.0412	0.0408	0.0432	0.0924	0.2042	0.3534	0.6941
32	0.0397	0.0397	0.0397	0.0397	0.0397	0.0399	0.0397	0.0781	0.1293	0.5796	1.0411
33	0.0393	0.0393	0.0393	0.0393	0.0393	0.0386	0.0410	0.0514	0.1892	0.2097	0.5634
34	0.0377	0.0377	0.0377	0.0377	0.0377	0.0377	0.0401	0.0473	0.1020	0.8151	0.5774
35	0.0358	0.0358	0.0358	0.0358	0.0358	0.0364	0.0355	0.0496	0.1130	0.3714	1.0673
36	0.0345	0.0345	0.0345	0.0345	0.0345	0.0349	0.0336	0.0386	0.0820	0.4236	1.0397
37	0.0350	0.0350	0.0350	0.0349	0.0349	0.0351	0.0339	0.0474	0.0803	0.1738	1.0334
38	0.0327	0.0327	0.0327	0.0327	0.0327	0.0326	0.0767	0.0432	0.0952	0.1568	0.6252
39	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0316	0.0458	0.0755	0.7461	0.7249
40	0.0325	0.0325	0.0325	0.0325	0.0325	0.0324	0.0321	0.0355	0.1658	0.1747	0.8461
41	0.0325	0.0325	0.0325	0.0325	0.0325	0.0325	0.0319	0.0546	0.0547	0.1345	0.4989
42	0.0294	0.0294	0.0294	0.0294	0.0294	0.0294	0.0294	0.0313	0.0637	0.2319	0.6402
43	0.0300	0.0300	0.0300	0.0300	0.0300	0.0299	0.0296	0.0289	0.0349	0.1366	0.5385
44	0.0290	0.0290	0.0290	0.0290	0.0290	0.0290	0.0289	0.0564	0.0386	0.1016	0.3652
45	0.0288	0.0288	0.0288	0.0288	0.0288	0.0288	0.0287	0.0337	0.0383	0.1557	0.5110
46	0.0278	0.0278	0.0278	0.0278	0.0278	0.0278	0.0277	0.0269	0.0472	0.1216	0.5385
47	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0270	0.0288	0.0346	0.1614	0.4342
48	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0.0260	0.0290	0.2219	0.2569
49	0.0262	0.0262	0.0262	0.0262	0.0262	0.0262	0.0262	0.0263	0.0386	0.2276	0.6092
50	0.0255	0.0255	0.0255	0.0255	0.0255	0.0255	0.0255	0.0269	0.0300	0.0920	0.4657
51	0.0253	0.0253	0.0253	0.0253	0.0253	0.0253	0.0253	0.0259	0.0265	0.1124	0.4584
52	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0244	0.0250	0.0374	0.1100	0.3822
53	0.0239	0.0239	0.0239	0.0239	0.0239	0.0239	0.0239	0.0238	0.0244	0.0765	0.6349
54	0.0231	0.0231	0.0231	0.0231	0.0231	0.0231	0.0231	0.0229	0.0242	0.1010	0.3162
55	0.0228	0.0228	0.0228	0.0228	0.0228	0.0228	0.0228	0.0229	0.0273	0.0922	0.4384
56	0.0217	0.0217	0.0217	0.0217	0.0217	0.0217	0.0217	0.0217	0.0230	0.0956	0.3686
57	0.0219	0.0219	0.0219	0.0219	0.0219	0.0219	0.0219	0.0220	0.0218	0.2453	0.5197
58	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0230	0.0555	0.3310
59	0.0211	0.0211	0.0211	0.0211	0.0211	0.0211	0.0211	0.0211	0.0219	0.1073	0.3364
60	0.0206	0.0206	0.0206	0.0206	0.0206	0.0206	0.0205	0.0206	0.0287	0.0562	0.2659