Local Monotonicity of Voting Power: A Conceptual Analysis

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Abstract This paper examines a fundamental and on-going debate in the literature on voting power about what constitutes a 'reasonable' measure of *a priori* voting power. We focus on one of the central axioms or postulates known as local montonicity which says that voting power should be ranked in the same order as the order of voting weights. By examining a general violation of local montonicity under Straffin's partial homogeneity approach we show that this postulate lacks convincing justification. However, and somewhat paradoxically, we argue that the previous arguments against local montonicity are flawed, and the intuition behind the postulate is essentially correct. The problem lies with the definition of *a prioricity* and the nature of the voting game.

Keywords local monotonicity, voting power, resources, Straffin's partial homogeneity approach

JEL classifications

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1. Introduction

The problem of designing fair voting systems is a central topic in social choice theory. Within this topic there is now an established body of theory that concerns the measurement of voting power, or how much influence over outcomes a voting rule assigns to each member of the voting body. There is, however, a fundamental and on-going debate in this literature – not unlike that found in the freedom of choice literature[1] – about what constitutes a 'reasonable' measure of *a priori* voting power, i.e. the power that each player has ex ante. The reason is in part due to the fact that there is as yet no intuitively *compelling* and *complete* set of axioms that *uniquely* characterize a measure and in part due to the fact that the different measures that not only give different values but also different rankings of the players.

A central topic in this debate is whether or not a reasonable measure of voting power should fulfil *local monotonicity*. This is a postulates which says that in weighted voting systems – voting systems characterized by a vector of voting weights attached to each player and a quota – if a player i has at least as much weight as a player j, then player i should have as least as much power as player j. While the Shapley-Shubik index (Shapley and Shubik 1954) and the Penrose (1946), Banzhaf (1965) or Coleman (1971) measures are locally monotonic, the Deegan-Packel index (Deegan and Packel 1978) and the Public-Good Index (PGI)

* The research on which this paper is based has greatly benefited from intensive discussions with Manfred Holler and Moshé Machover. An early forerunner of the paper was presented at the Institute of SocioEconomics research seminar in June 2001.

[1] We are referring here to the debate about the about appropriate axioms for characterizing a measure of freedom. See, for example Jones and Sugden (1982), Pattanaik and Xu (1990, Pattanaik and Xu 1998), Sen (1991), (Sugden 1985), van Hees (2000).

(Holler 1985, Holler and Packel 1983) are not, i.e. a small player can have more power than a large one.

Freixas and Gambarelli (1997) and Felsenthal and Machover (1998) have taken the position that local monotonicity is such an intuitively compelling postulate that any measure that violates cannot be used as a reasonable yardstick of voting power.[2] This would mean that the Deegan-Packel index and PGI in a sense suffer 'pathological' defects. In the strong language of Barry (1980a: 194), these indices can be seen as being like a broken thermometer: the fact that it does not register a higher temperature when put in a flame does not tell us anything new and interesting (albeit counter intuitive) about the nature if heat. In this view, 'reasonable' obviously means agreement with some intuitively acceptable property, which local montonicity is believed to be.

On the other hand, Deegan-Packel (1978, 1983) and Holler (1997, 1998) as well as Brams and Fishburn (1995) take the position that if the rationale or 'story' of a measure is reasonable and acceptable, then we are forced to accept that power is not locally monotone and that this is an inescapable fact of power being a social phenomenon (they cite empirical evidence to this effect). The underlying argument being that it is mistaken to take an axiomatic approach to the analysis of social interaction. In this view, 'reasonable' obviously refers to the coherence of the rationale of the model of voting power.

The aim of this paper is to sketch a way to resolve this debate; in particular to determine whether or not local montonicity can justifiably be used to select out a 'reasonable' measure of *a priori* voting power. We will do this by examining a general violation of local montonicity by another set of measures derived from Straffin (1977), which has so far been ignored in the debate. Straffin's *partial homogeneity approach* is a probabalistic interpretation of voting power based on Owen's (1972, 1975) *multilinear extension* (MLE) of a game. As is well known, the Shapley-Shubik and absolute Banzhaf indicies can be derived as special cases of the MLE given a probability model of player behaviour, as can the PGI (Brueckner 2001). Straffin's approach allows us to mix these probability models so that we can derive an infinite set of families of power measures.

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[2] Actually the importance of local montonicity as axiom or 'postulate' of power was noted already by Allingham (1975), although he did not take such a 'strong' position to that of Freixas and Gambarelli (1997) and Felsenthal and Machover (1998).

The reason for the violation of local montonicity by the family of power measures derived by Straffin's approach is wholly different to that of the violation by the Deegan-Packel index and the PGI. In the latter case the reason is due to the fact that the measures are based only on *minimal winning coalitions* (coalitions in which no proper subset are winning). That is, these measures ignore certain coalitions in which *i* is critical (i.e. without *i* the coalition is losing) either on the grounds that these coalitions will not form or because they should be ignored (the rationale for this is given in section 2). While in the former case, the violation of local montonicity is due to the fact that the partial homogeneity approach does not treat each coalition as equally likely. Under Straffin's approach, the power of a player *i* depends not only upon the coalitions in which *i* is critical but also upon the probability that such a coalition arises, which is a function of voter propensities to vote 'yes' or 'no'. The greater the probability of a coalition arising in which *i* is critical, the greater is *i*'s power.

Although at first sight it appears quite reasonable to measure a player's voting power as a function of being critical *and* of the probability such a critical coalition arising it is in fact considered to be inconsistent with the conventional notion of *a prioricity* as it is used in the voting power literature. A measure of voting power is deemed to be *a priori* if it does not include any information exogenous to the rule itself (the set of 'winning coalitions'). This suggests that Straffin's approach is not *a priori*, because the information about voter propensities is exogenous to the rule, and thus the violation of local montonicity under this method is irrelevant to that nature of *a priori voting power*. It is our contention that this line of reasoning is mistaken. We will show that the measures derived under Straffin's approach can be as *a priori* as the classical approach and thus the violation of local montonicity under this approach does say something important about the nature of a priori voting power: there is no convincing justification to use local montonicity to pick out a 'reasonable' measure of voting power. It is not as an intuitively plausible axiom or postulate as believed to be.

However, having done away with local monotonicity, we argue – some what paradoxically it may seem – that the position taken by Deegan-Packel (1978, 1983), Holler (1997, 1998) and Brams and Fishburn (1995) that power must be accepted to be not locally monotonic is not entirely correct either. Their position is essentially one of argument by analogy. Drawing on experimental and political and social evidence they say that the fact that a player *j* who has less weight than a

player *i* in a weighted voting game can have more power than player *i* is simply an instance of the violation of local monotonicity in resources that we observe all the time. Put differently, these authors believe that power is not necessarily increasing in the 'resources', or to borrow Dahl's (1957) terminology, in the 'base of power'. Voting weights are just a particular kind of resource or 'base of power'.

The problem here is that once we recognize that calculating voting power actually presupposes a probability model and a decision-making structure, the resources or 'base of power' is no longer restricted to only the voting weights. These weights may be augmented by the assumptions about how players behave (whether or not their behaviour is correlated), which is contingent on the *a priori* incentive structures given by the decision-making structure. Hence a player *i* may have more weight than a player *j* but due to the incentive structures that govern formation of winning coalitions, *i* may be in a weaker position because certain coalitions where *i* is critical may have a smaller probability of occurring than the coalitions in which *j* is critical. In a sense the value a player *i*'s weight in a game is modified by the number of other players with whom *i* is correlated. Thus a violation of local monotonicity as defined by voting weights only does not imply a violation of local montonicity when defined in terms of resources or the 'base of power' more generally. Our intuition is that power *is* locally monotonic in this latter sense. As one can imagine, this throws up all sorts of theoretical problems because it requires a much more complicated definition of resources and a method for calculating the value of these resources in a voting game.

This paper is organized as follows. In section 2 we reproduce the basic formal framework for simple games and the measurement of voting power (readers familiar with this material may wish to skip this section). In section 3, we outline the five basic postulates – local monotonicity is one of them – that are generally taken to be necessary for defining a reasonable measure of *a priori* voting power. Section 4 considers the derivation of local monotonicity from the more general desirability (also called dominance) relation. In section 5 we examine the connection of local montonicity to the definition of *a prioricity*. Section 6 considers *a prioricity* as a normative criterion. In section 7 we consider a primitive definition of power and its relationship to resources. It is this section that we show that local montonicity in voting weights is a special case of a more general local montonicity based upon resources or the 'base of power' and that a violation of the former does not entail a violation of the latter. Section 8 winds up the paper.

2. Simple Games and the Measurement of Voting Power

In order to develop our argument we need to restate the basic definitions of the theory of simple games and voting power. We refer the reader to Shapley (1962), Felsenthal and Machover (1998), and Taylor and Zwicker (1999) for additional background and results.

The most important definition that we require is that of a *decision rule* which we will first formulate informally as follows. Let a *n*-member decision-making body be denoted by a set *N*. A decision rule specifies which subsets of *N* can ensure the acceptance of a proposal. Formally:

Let $N = \{1, 2, \ldots, n\}$ be the set of players. $\wp(N) = \{0,1\}^n$ is the set of feasible coalitions. The *simple game v* is characterized by the set $\mathcal{W}(v) \subseteq \wp(N)$ of *winning coalitions*. $\mathcal{W}(v)$ satisfies $\varnothing \notin \mathcal{W}(v)$; $N \in \mathcal{W}(v)$; and if $S \in \mathcal{W}(v)$ and $S \subseteq T$ then $T \in \mathcal{W}(v)$. In other words, *v* can be represented as a pair $(N, \mathcal{W})$. Further, *v* can also be described by a characteristic function, $v: \wp(n) \to \{0,1\}$ with $v(S) = 1$ iff $S \in \mathcal{W}$ and 0 otherwise.

By $\mathcal{G}^N$ we denote the set of all such *n*-person simple games. *Weighted voting games* are a special sub-class of simple games characterized by a non-negative real vector $(w_1, w_2, \ldots, w_n)$ where w_i represents player *i*'s voting weight and a quota *q* which is the quota of votes necessary to establish a winning coalition, such that *quota* $0 < q \le \sum_{i \in N} w_i$. A weighted voting game is represented by $[q; w_1, w_2, \ldots, w_n]$.

Power, in the generic sense of an ability or capacity to determine an outcome, is represented in a simple game as the ability of a player *i* to change the outcome of a play of the game. We say that a player *i* who by leaving a winning coalition $S \in \mathcal{W}(v)$ turns it into a losing coalition $S \setminus \{i\} \notin \mathcal{W}(v)$ has a *swing* in *S* and is called a *critical member* of *S*. Coalitions where *i* has a swing are called *critical coalitions with respect to i*. Let us denote the set of critical coalitions w.r.t *i* as $\mathcal{C}_i$. A concise description of *v* can be given by a set $\mathcal{M}(v)$, where $S \in \mathcal{W}(v)$ but no subset of *S* is in $\mathcal{W}(v)$, i.e. all members of *S* are critical. We call such a coalition a *minimal winning coalition* (MWC). Further, we denote by η_i the number of swings of player *i* in a game *v*. Thus, $\eta_i =_{def} |\mathcal{C}_i(v)|$. A player *i* for which $\eta_i(v) = 0$ is called a *dummy* in *v*, i.e. it is never the case that *i* can turn a winning coalition into a losing coalition (it is easy to see that *i* is a dummy iff it is never a member of an MWC; and *i* is a *dictator* if $\{i\}$ is the sole MWC).

A *measure of voting power* is a mapping $\xi: \mathcal{G}^N \rightarrow \mathbb{R}_+^n$ that assigns to each player $i \in N$ a number $\xi_i(v)$ that indicates i's power in the game v. As we have already mentioned in the introduction, there are a number of well known measures, namely, the Shapley-Shubik index, the Banzhaf index, the Deegan-Packel index, and the Holler-Packel or Public Good Index.

The Shapley-Shubik (1954) index (S-S) is a special case of the Shapley (1953) value for cooperative games. In this measure power equals the relative number of pivotal ('swing') positions of a player i in a simple game v assuming all player permutations are equally probable. The idea (or 'story') is that the players line up to vote yes and the player that turns a losing coalition into a winning coalition is the pivot ('swing'). The S-S is given by:

$$\phi_i(v) =_{def} \sum_{\substack{S \in \mathcal{W} \\ i \in S}} \frac{(|S|-1)!(n-|S|)!}{n!}$$

Whereas the S-S is concerned with the order in which a winning coalition may form, the Banzhaf (1965) index (Bz) examines any winning coalition, irrespective of the order in which it may be formed and considers any voter to have power from having a swing in it.[3] The 'absolute' or non-normalized Bz measure is given by:

$$\beta_i'(v) =_{def} \frac{\eta_i(v)}{2^{n-1}}$$

The Bz index is obtained by normalization:[4]

$$\beta_i'(v) =_{def} \frac{\eta_i(v)}{\sum_{j=1}^{n} n_j(v)}$$

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[3] The Banzhaf measure is in fact a rediscovery of Penrose (1946) and was later independently rediscovered by Rae (1969) and Coleman (1971). A history of the measure of voting power is contained in Felsenthal and Machover (1998).

[4] Here we follow Felsenthal and Machover (1998) and reserve the term 'index' for measures in which $\sum_{i \in N} \xi_i(v) = 1$. See section 3.

The Deegan-Packel (1978) index (D-P) is based on three assumptions: that only MWCs will form; all MWCs are equally likely; and the MWC that is formed will divide the payoff equally among its members. Subject to these assumptions, the D-P assigns to each player power proportional to the player's expected payoff. Denote by $\mathcal{M}_i(v)$ the set of MWCs to which player *i* belongs. The D-P is given by:

$$\rho_i(v) =_{def} \frac{1}{\mathcal{M}(v)} \frac{1}{\sum_{S \in \mathcal{M}_i(v)} |S|}$$

The Holler-Packel (Holler 1982, Holler and Packel 1983) or the Public Good Index (PGI), is also based on MWCs, although the story is different. Whereas the D-P is based sharing the spoils of victory, the PGI is based upon the essential characteristic of a public good: non-rivalry in consumption and non-excludability in access. Thus if the outcome of a game *v* is the provision of a public good, each member of the winning coalition will receive the undivided value of the coalition. Only MWCs are taken into account not because winning coalitions with excess players will not form, but when it comes to the provision of a public good they will only form by sheer 'luck' because of the potential for free-riding.[5] Assuming that that all MWCs are equally likely, the PGI is given by:

$$h_i(v) =_{def} \frac{|\mathcal{M}_i(v)|}{\sum_{j=1}^{n} |\mathcal{M}_j(v)|}$$

The non-normalized or 'absolute' PGI is given by:

$$h_i'(v) =_{def} \frac{|\mathcal{M}_i(v)|}{|\mathcal{M}(v)|}$$

Power in simple games can also be modelled in probabalistic setting. As we have already said in section 1, this is what Straffin's (1977, 1988) *partial*

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[5] This rationale is based on Barry (1980a, 1980b).

homogeneity approach is all about. It is a particular interpretation and extension of Owen's (1972, 1975) *multilinear extension* (MLE) of a game v.[6]

Instead of deterministic coalitions $S \subseteq N$ that correspond to corner points $s \in \{0,1\}^n$ of the n-dimensional unit cube, one considers random coalitions $\mathcal{S}$ represented by the points $p_i \in [0,1]^n$ anywhere in the cube. Each p_i is interpreted as the probability of a player i deciding in favour of a random proposal or participating in a random coalition; p_i is also known as a player's *acceptance rate*.

Assuming that acceptance decisions are independent, the probability **P** of a given coalition $S \subseteq N$ is $\mathbf{P}(\mathcal{S} = S) = \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j)$. If we extend the characteristic function v of a simple game by weighting each $v(S)$ with the respective probability of formation, we obtain the MLE $f : [0,1]^n \rightarrow [0,1]$ of a game v:

$$\begin{aligned} f(p_1, \ldots, p_n) &= \sum_{S \subseteq N} \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) v(S) \\ &= \sum_{S \in \mathcal{W}(v)} \prod_{i \in S} p_i \prod_{j \notin S} (1 - p_j) \end{aligned}$$

For fixed acceptance rates, the MLE gives the probability that a winning coalition S will form in v, and thus the expected value of v. The partial derivative $\partial f / \partial p_i$ of v's MLE w.r.t to p_i is called by Straffin (1977, 1988) a player's *power polynomial*, which we denote by f_i.

$f_i(p_1, \ldots, p_n)$ is, then, the probability of i having a swing (i.e. having power in the generic sense) in a random coalition in a game v. If a player's acceptance rates are themselves random variables with a joint distribution P, the expectation $Ef_i = \int f_i(p_1, \ldots p_n) dP$ is i's power in a game v. The probablistic *measure of power* $Ef_i(v)$ coincides with some of the classical measures under different probability models. (Note that we will use $Ef_i(v)$ when referring to a measure derived from Straffin's approach and $\xi_i(v)$ for measures in general).

Independence $\quad p_i \sim U(0,1) \qquad \forall\, i \in N$ (A1)

i.e. the decision of i has nothing to do with decision of j.[7]

[6] See also Laruelle and Valenciano (2001a) for an attempted synthesis of the probabalistic models.

[7] Actually one does not necessarily need the uniform distribution. Leech (1990) has shown that distribution must only have a mean of 0.5.

Homogeneity $t \sim U(0,1)$, $p_i = t$ $\forall\ i \in N$ (A2)

i.e. each i approves or rejects a proposal with the same probability t but t varies from proposal to proposal.

It is a well-known result from Straffin that applying (A1) we obtain the non-normalized or absolute Bz measure; applying (A2) we obtain the S-S; and as Brueckner (2001) has shown (A1) in combination with counting only MWCs (i.e. $\mathcal{M}_i$) gives the non-normalized or absolute PGI.

It is easy to see that this probability model is extremely flexible and allows us to create families of power measures that lie between the extremes of (A1) and (A2) by mixing these assumptions. This is what Straffin meant by *partial homogeneity structure* on N which is a partition $\mathcal{P}=\{G_1,\ldots,G_m\}$ of N into disjoint subsets. We call such a game *a game with a partition structure* and it is given by the triple $(N,\mathcal{W},\mathcal{P})$. Formally,

Partial homogeneity $\mathcal{P}=\{G_1,\ldots,G_m\}$ (A3)

$$G_k \cap G_l = \varnothing \quad if \quad k \neq l, \quad \bigcup G_k = N$$

$$t \sim U(0,1), \quad p_i = t_k \quad \forall\ i \in G_k, \quad k = 1,\ldots,m$$

It is important to note that even if it is not explicitly given, every simple game assumes a partition structure $\mathcal{P}$. That is, if $\mathcal{P}$ is the discrete partition of N into one-player subsets we have (A1); if $\mathcal{P}$ is the indiscrete partition $\mathcal{P}=\{N\}$, we have (A2).[8]

3. Postulates

Given the variety voting power measures and the fact there is as yet no intuitively *complete* and *compelling* set of axioms that uniquely characterizes a measure but only *individual* axioms – some of which are compelling and others opaque unconvincing[9] – there has been a number of attempts to reduce the set of measures by eliminating those that violate certain properties that are considered intuitively reasonable for a measure of *a priori* power. Here the literature very much concurs

[8] Note, a *partition structure* is not to be confused with a *coalition structure* as in Aumann and Dreze (1974). There are major conceptual differences in the two structures. Partitions are not coalitions, but the correlation of player behaviour.

[9] See, for example, Straffin's (1983: 292–297) discussion of the axiomation of the S-S and Bz.

on three basic postulates or desiderata.[10] An *a priori* measure of voting power $\xi_i(v)$ should at the very least satisfy:

Iso-invariance (P1)
if there is an isomorphism of v to v' that maps a player i to i', then $\xi_i(v) = \xi_{i'}(v')$.

Ignoring dummies (P2)
if v and v' have exactly the same MWCs, i.e. $\mathcal{M}(v) = \mathcal{M}(v')$, then $\xi_i(v) = \xi_i(v')$ for any player i common to both.

Vanishing for dummies (P3)
$\xi_i(v) = 0$ if i is a dummy in v.

(P1) generally trades under the name of *symmetry*, which is a special case of iso-invariance in which we have an automorphism of v (i.e. an isomorphism of v to itself). This postulate requires that $\xi_i(v)$ be symmetric (i.e. invariant under any automorphism): if players i and j have symmetric positions w.r.t to v they have equal power. Note that (P1) implies that $\xi_i(v)$ should depend only on the collection $\mathcal{W}(v)$ of winning coalitions and nothing more. In this sense it can also be called an *anonymity* or *neutrality condition*, i.e. $\xi_i(v)$ does not depend upon the identity of the players. Felsenthal and Machover (1995: 204) claim that to deny (P1) would be tantamount to denying that simple games provide an adequate framework for theorizing about *a priori* voting power. Felsenthal and Machover buttress their position by saying that all authors dealing with voting power within the framework of simple games implicitly if not explicitly accept (P1). However, as we will discuss later, (P1) is a very restrictive way of defining *a prioricity* – a restriction that is a cause for much confusion about the nature of *a priori* voting power per se. That is, *a priori* voting power can be shown to presuppose (P1) only under very specific conditions; these are shown up under Straffin's partial homogeneity approach. This leads us to the conclusion that the framework of a simple game, $(N, \mathcal{W})$, may not in itself be adequate for theorizing about *a priori* voting power.

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[10] Postulates (P1)–(P3) appear in all axiomatisations of measures of voting power as well as in comparisons of the different measures, e.g. Allingham (1975), Felsenthal and Machover (1998), Freixas and Gambarelli (1997), Laruelle (1999), Straffin (1983).

(P2) means that the value of ξ_i for any voter i in the simple game v is unchanged if v is extended to v' by adding new dummy players (or equivalently, removing a dummy player from v will not alter the value of ξ_i. (P3) is obvious: dummy players have no power.

A forth postulate, that of normalization has also frequently been put forward:

Normalization (P4)

$\sum_{i\in N}\xi_i(v)=1.$

The meaning of (P4) is straightforward: it is a way of answering questions like 'What fraction of the power in this game do I hold?' However, as a postulate of power it is not without technical difficulties because for the Bz it is not necessarily meaningful (Dubey and Shapley 1979, Shapley 1977) and in particular distorts the probabalistic interpretation of the Bz measure. Furthermore, in contrast to (P1)–(P3), (P4) is not without conceptual problems, in that there is no intuitive justification for saying that a measure of voting power ought – either naturally or artifically – to sum to unity (Laruelle and Valenciano 1999, Laruelle and Valenciano 2001b) and thus should not be used to eliminate a measure as being unreasonable.[11]

Finally, we come to the fifth and central postulate of this paper:

Local montonicity (P5)

If in a weighted voting game v, $w_i \geq w_j$ then $\xi_i(v)\geq\xi_j(v)$.

(P5) can be expressed in two ways: that a $\xi_i(v)$ should preserve the order of weights; or more rhetorically, 'having extra votes cannot hurt you, although will not necessarily help you.'

All the classical measures that we have listed above satisfy the first three postulates; normalization is naturally satisfied by the S-S and D-P indices (the Bz and PGI are 'normalized'); and the D-P and the PGI violate (P5) as will in general the family of measures $Ef_i(v)$ that can be derived from Straffin's partial

[11] Actually the problem is not restricted to voting power. There was a fair amount of controversy among political scientists and sociologists from the 1950s to the 1970s about whether or not power had a constant sum property. See. for instance, Nagel (1973) and Wrong (1979: 237ff). The question has in fact been reincarnated in Felsenthal and Machovers's (1998) distinction between what they call 'power as influence' (I-power) and 'power as prize' (P-power), the latter is considered to be a zero-sum game while the former not.

homogeneity structure as represented by (A3). For illustration, consider the following three examples:

Example 3.1 Assume the weighted voting game [51; 30, 26, 16, 12, 9, 7]. (i) The D-P values are $\rho_1 = 0.23$, $\rho_2 = 0.18$, $\rho_3 = 0.21$, $\rho_4 = \rho_5 = 0.16$, and $\rho_6 = 0.07$. (ii) The PGI values are $h_1 = 0.21$, $h_2 = 0.17$, $h_3 = 0.21$, $h_4 = h_5 = 0.17$, $h_6 = 0.08$. (iii) Assume (A3) as follows: player 1 behaves independently, while players 2, 3, and 4 form a standard t and players 5 and 6 form a standard $(1–t)$. Then we have $Ef_1(v) = 0.40$, $Ef_2(v) = 0.47$ $Ef_3(v) = 0.38$, $Ef_4(v) = 0.30$, $Ef_5(v) = 0.12$, and $Ef_6(v) = 0.03$.

Example 3.2 Assume the weighted voting game [51; 30, 30, 18, 10, 9, 3]. (ii) The D-P values are $\rho_1 = \rho_2 = 0.19$, $\rho_3 = 0.22$, and $\rho_4 = \rho_5 = \rho_6 = 0.13$. (ii) The PGI values are $h_1 = h_2 = h_3 = h_4 = h_5 = 0.18$, and $h_6 = 0.09$. (iii) Assume (A3) as follows: player 1 behaves independently, while players 2, 3, and 4 form a standard t and players 5 and 6 form a standard $(1–t)$. Then we have $Ef_i(v)$ are $Ef_1(v) = 0.37$, $Ef_2(v) = 0.50$ $Ef_3(v) = 0.42$, $Ef_4(v) = 0.25$, and $Ef_5(v) = Ef_6(v) = 0.08$.

It is easy to see that the violation of (P5) – which we will henceforth denote as LM – by the D-P and PGI is for entirely different reasons than the violation resulting from the application of (A3) to Straffin's probablistic approach. In the first case the reason lies with the fact that both measures are based only on MWCs. According to the Deegan-Packel 'story' only MWCs will form; according to the PGI 'story' only MWCs form intentionally (excess sized coalitions are a matter of 'luck') and express power so that only they should be counted in the calculation of power. This means that a certain number of a player's swings are not counted in the final measure of voting power, i.e. those in $\mathcal{C}_i \backslash \mathcal{M}_i$. It can be the case that a 'large' player is 'crowded out' by many smaller players, who may have far more opportunities to form MWCs. In the second case, the violation of LM is a result of the coalitions no longer being equally probable. Player 2 gets a 'boost' in power over and above player 1 because it is critical in a winning coalition that occurs with a probability of 0.0833, which is significantly larger than the probability of any of winning coalitions in which player 1 is critical. This compensates for the fact that player 2 has four less swings than player 1. Table 1 gives the probabilities of each of the winning coalitions.

Table 1: Winning Coalitions for Example 3.1

				$\mathscr{W}$ Player i						
S	1	2	3	4	5	6	$\sum_{i \in S} wi$	*MWC*	$\mathbf{P}(S)_{(A1)}$	$\mathbf{P}(S)_{(A3)}$
1	30	26					56	yes	0.0156	0.0083
2		26	16		9		51	yes	0.0156	0.0083
3		26	16	12			54	yes	0.0156	0.0833
4	30			12	9		51	yes	0.0156	0.0083
5	30		16			7	53	yes	0.0156	0.0083
6	30		16		9		55	yes	0.0156	0.0083
7	30		16	12			58	yes	0.0156	0.0167
8	30	26				7	63	no	0.0156	0.0083
9	30	26			9		65	no	0.0156	0.0083
10	30	26		12			68	no	0.0156	0.0167
11	30	26	16				72	no	0.0156	0.0167
12		26		12	9	7	54	yes	0.0156	0.0083
13		26	16		9	7	58	no	0.0156	0.0083
14		26	16	12		7	61	no	0.0156	0.0167
15		26	16	12	9		63	no	0.0156	0.0167
16	30			12	9	7	58	no	0.0156	0.0167
17	30		16		9	7	62	no	0.0156	0.0167
18	30		16	12		7	65	no	0.0156	0.0083
19	30		16	12	9		67	no	0.0156	0.0083
20	30	26			9	7	72	no	0.0156	0.0167

.../ Table 1 cont.

				$\mathcal{W}$ Player i						
S	1	2	3	4	5	6	$\sum_{i \in S} wi$	*MWC*	$\mathbf{P}(S)_{(A1)}$	$\mathbf{P}(S)_{(A3)}$
21	<u>30</u>	<u>26</u>		12		7	75	no	0.0156	0.0083
22	<u>30</u>	26		12	9		77	no	0.0156	0.0083
23	<u>30</u>	26	16			7	79	no	0.0156	0.0083
24	30	26	16		9		81	no	0.0156	0.0083
25	30	26	16	12			84	no	0.0156	0.0833
26		<u>26</u>	16	12	9	7	70	no	0.0156	0.0083
27	<u>30</u>		16	12	9	7	74	no	0.0156	0.0083
28	30	26		12	9	7	84	no	0.0156	0.0083
29	30	26	16		9	7	88	no	0.0156	0.0083
30	30	26	16	12		7	91	no	0.0156	0.0167
31	30	26	16	12	9		93	no	0.0156	0.0167
32	30	26	16	12	9	7	100	no	0.0156	0.0083
$\lvert\mathcal{C}_i\rvert$	18	14	10	6	6	2			0.5000	0.5000
$\lvert\mathcal{M}_i\rvert$	5	4	5	4	4	2				

Note: Critical player is underlined.

4. The Desirability Relation

Given the conviction that LM is an intuitively compelling postulate of *a priori* voting power it is necessary to recap its justification in some detail. That is, we will now lay out the argument in favour of LM, and then show that it rests on rather precarious conceptual foundations.

As a number of authors have pointed out (Felsenthal and Machover 1995, Felsenthal and Machover 1998: 241–246, Freixas and Gambarelli 1997), LM is a special case of the *desirability* (also called *dominance*) relation, $\succeq$, which is a pre-ordering (i.e. it is transitive and reflexive) of the players in a simple game *v*.[12] The idea is that we can order the players in terms of their contribution to a coalition *S*. Formally,

$$i \succeq j \quad \text{iff} \quad S \cup \{j\} \in \mathcal{W}(v) \quad \text{implies} \quad S \cup \{i\} \in \mathcal{W}(v).$$

In words, player *i* is at least as desirable as *j* in coalition *S* in a game *v* if interchanging *i* and *j* does not change *S* from winning to losing, i.e. *i* and *j* can be regarded as substitutes. If we have $i \succeq j$ but not $j \succeq i$, then $i \succ j$, i.e. player *i* is strictly more desirable than player *j*. Thus,

$$i \succ j \quad \text{then} \quad \xi_i(v) > \xi_j(v).$$

It is also easy to see that if players *i* and *j* in *v* are interchangeable (i.e substitutes), then by symmetry $\xi_i(v) = \xi_j(v)$, and,

$$i \succeq j \quad \text{then} \quad \xi_i(v) \geq \xi_j(v).$$

For a weighted voting game it clearly follows that if $w_i \geq w_j$ then $i \succeq j$, i.e. anything that w_j can do, w_i can also do because a winning coalition cannot become a losing coalition if it gains more weight (but it does not necessarily follow that if $w_i > w_j$ then $i \succ j$). It is therefore straightforward that if $w_i \geq w_{j\,j}$ then $\xi_i(v) \geq \xi_j(v)$, viz. precisely LM as in (P5).

It is evidently hard to quarrel with this argument. Felsenthal and Machover (1998: 245) have expressed it forcefully: ‘In our view, any reasonable measure of

[12] The desirability relation was first introduced by Isbell (1958) and later generalized by Maschler and Peleg (1966). See also Taylor and Zwicker (1999: 86–92).

a priori power … must respect dominance [desirability]. The case for this postulate is so strong that it hardly needs spelling out.' If, whatever j can contribute to the passing of a bill i can do as well (is at least as desirable) and in some cases more (is more desirable), i must be considered to have greater influence than j. That is, if one accepts the logic of the desirability relation, one is logically forced to accept LM. The implication is that any *a priori* measure of voting power that violates LM is 'pathological' and should be disqualified as serving as a valid yardstick. The violation of LM is taken to be a wholly unacceptable negation of the intuitively compelling proposition that if a player i can do more than a player j, then i not only has more power than j but also must have means available that j does not have, i.e. power is monotone in the 'bases' or resources of power. The greater the resources, the greater the power (or at least not decreasing). This position was summed up by Bertrand Russell (1938) when he wrote in a classic essay on the nature of power that, 'Nevertheless, it is easy to say, roughly, that A has more power than B, if A achieves many intended effects and B only a few' (p. 37). In a WVG, a player's resource is clearly the voting weight.

The rest of this paper will now build an argument to show that the strong position taken, for instance, by Felsenthal and Machover as regards the violation of LM is unwarranted; and that it is quite possible to have a violation of LM without violating the idea that power is monotone in resources.

5. A Prioricity

It is little – if at all – recognized that the desirability relation and thus LM is closely related to the informational restrictions implicit in the particular notion of *a prioricity* that has traditionally been used in defining *a priori* voting power. In this section this relationship will be brought out in such a way as to make the informational constraints transparent. Once this is done, we can show that widening the notion of *a prioricity* – actually bring it into line with its more common usage – will seriously undermine the appeal of LM as defined above.

The conventional meaning of *a priori* voting power is that a measure of power should rule out by default all information not provided by the framework of the collection of subsets $\mathcal{W}(v)$ (or the characteristic function). Properly speaking it is the idea that a player's *a priori* voting power is endogenous to the rule. This is

clearly a very a sparse informational framework, but it is one that can be found in the original papers by Shapley and Shubik (1954), Banzhaf (1965) and Coleman (1971). As Roth (Roth 1988: 9) puts it:

> Analyzing voting rules that are modelled as simple games abstracts from the particular personalities and political interests present in particular voting environments, but this abstraction is what makes the analysis focus on the rules themselves rather than on the other aspects of the political environment. This kind of analysis seems to be just what is needed to analyze the voting rules in a new constitution, for example, long before the specific issues to be voted on arise or the specific factions and personalities that will be involved can be identified.

Commenting on the passage quoted, Felsenthal and Machover (1998: 20) point out that a simple game is an 'abstract shell, *uninhabited* by real agents, with real likes and dislikes, attractions, and repulsions'. It is for this reason that they insist that a truly *a priori* measure of voting power must not presuppose any specific information as to the interests of the voters or the affinities and disaffinities between them. The upshot is that an *a priori* measure of voting power should treat each coalition as equally likely. This, it should be noted, is precisely what is required for the desirability relation because it concerns only the contribution that a player i makes to a coalition S and not the likelihood that this coalition will actually occur.

There is, however, a major problem with this definition of *a prioricity*: it is not quite accurate to say that an *a priori* measure of voting power is based only on $\mathcal{W}(v)$. In two very important respects *any* measure of voting power contains, or at least presupposes, further information.

Firstly, it is in fact not possible to calculate voting power in absence of an assumption of how the players behave.[13] If we assume that all coalitions are equally probable we are in effect assuming that for a random bill put before the assembly each player votes 'yes' or 'no' with equal probability.[14] This is precisely the idea underpinning the Bz and can also be used to derive the S-S (Felsenthal and Machover 1998: 187–190). Thus the *a prioricity* of a measure of voting power

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[13] This point was already recognized by Dubey and Shapley (1979: 103). Also discussed in Laruelle and Valenciano (2002: 10).

[14] We are ignoring the case of abstentions. See Felsenthal and Machover (1997) and Braham and Steffen (Forthcoming 2002).

is contingent upon a probability model of voting behaviour. The general belief is that a probability model that treats all players in the same way is *a priori* while one that does not is *a posteriori*, i.e. an *a priori* measure must be iso-invariant (P1) in that it does not distinguish between players with symmetric weights. Further, it is also believed that the only legitimate *a prioristic* model is one derived from the Bernoullian principle of insufficient reason which assigns equi-probabilities to each choice that a player faces, i.e. each player votes 'yes' or 'no' with equi-probability.

Secondly, despite the claim that $(N,\mathcal{W})$ ignores the social organization of the players, this is also not true. If we assume symmetric probabilities on voting (whatever they may be) we are in effect assuming *a priori* a particular type of social organization: that the structure in which the players are embedded is not socially differentiated in any significant manner. This is not a lack of social organization, but a specific type of one. Social anthropology denotes this as a 'segmentary' or 'acephalous' social structure. For example, if we assume that N is made up of discrete individuals each concerned only with his or her own likes and dislikes – a model that Rae (1969: 42) calls *political individualism* – then we have a partition structure $\mathcal{P}$ of one-player subsets. This clearly a form of social organization, viz., an individualistic one. Thus any simple game is a game with a partition structure $(N,\mathcal{W},\mathcal{P})$; symmetric probabilities occur either under the independence assumption (A1) or the homogeneity assumption (A2) as discussed in section 2. In a sense, can denote the case of all players behaving independently or all behaving homogenously as a 'flat' structure.

In many instances this is clearly true: voting in a parliament for instance is flat in the sense that there are *a priori* no structural differences between its formal members, although there obviously will be temporal differences in its real members as a matter of political and personal predilection resulting in a correlation of voting behaviour – but this is not 'structure' as we mean it here.[15] We take 'structure' to a recurring pattern of social behaviour that is relatively

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[15] Thus our position is in no way to be confused with that of Brams (1975: 202) who takes the environmental constraints or decision-making structure that we are dealing with to be preference based: 'One such constraint is the organitzational ties of players, which may limit their freedom to select other players as coalition partners. In many legislatures, for example, the structure of the party system is all-important in determining what coalitions form. When strict party discipline prevails, a legislator always votes with his party and has no opportunity to seek out potential coalition partners among nonparty members.'

static; it is the set of norms, statuses, and roles that are received and acted out by the players. Structure, in this Lévi-Straussian interpretation, is *abstracted* from actual behaviour ('surface phenomena) as determined by individual preferences.[16]

We can characterise a flat or undifferentiated decision structure by saying that each member has *a priori* complete freedom of choice.[17] That is to say, a flat structure is one in which there are no descriptive (as against normative) reasons to believe that the players will vote in one way or another other than for reasons related to the particulars of the players.[18] For example, knowledge of a particular voting rule, say the Council of Ministers of the European Union, does not entail the signing of contracts between the players that assign the players particular roles. Hence, for such a structure, a symmetric probability model that assigns equal likelihood to each of the options for each player would seem for all intents and purposes to be the appropriate; and the Bz will probably turn out to be the measure to use.[19] But now note that the fact that such a measure obeys iso-invariance, the desirability relation, and thus LM is a happy coincidence and represents *that* type of decision-making structure. There is no way we can conclude from this that voting power either *is*, or *ought* to be, locally monotone as defined by (P5).

The reason is that there are also cases, perhaps more common than realized in the voting power literature, in which the decision-making structure in which voting takes place is in a socially differentiated and structure. That is, there *are* reasons beyond the personalities of the players which will determine voting behaviour, i.e. the players may have signed contracts that determine their behaviour to some extent. This is obviously the case of a bureaucracy or firm; or more generally a hierarchy made up of authority relations. In such a setting the

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16 See for example, Lévi-Strauss' introductory essay 'The Scope of Anthropology' in his *Structural Anthropology* vol. 2 (1978), in particular, pp. 15–21. See also Scheffler (1970) and Sewell (1992) for a more recent analysis of the meaning of 'structure' in the social sciences.

17 This concurs with Dubey and Shapley's (1979: 103) discussion of the Bz.

18 Note here that our position here is diametrically opposed to the position taken by Laurelle and Valenciano (2002) when they write that their '…formulation gives at once a clear foundation to the purely normative use of some classical measures [of voting power], and a clear understaning of their obvious lack of descriptive value' (p. 14). We disagree: according to our discussion classical measures do have descriptive content when applied to appropriate situations.

19 Whether this is the case, i.e. that the independence assumption is the most appropriate *a priori* assumption of a flat structure, or whether it is *a priori* impossible to differentiate between the independence and the homogeneity assumption as the two extreme cases of partial homogeneity is discussed in Steffen (2000) and Braham and Steffen (Forthcoming 2002).

players occupy positions and have to make choices that pertain to the aims of the department or that part of the organization to which they belong. In contrast to a flat structure, a player's freedom of choice is constrained by the system of incentives rewards used to make sure that the each player makes choices that are concordant with their department or section and that of the organization as a whole. If we assume, for example, economically rational players and optimal contracts (i.e. the principal-agent problem is solved), players belonging to the same department or section of the organization will have highly correlated voting behaviour. That is, an organization is a series of arrangements between individuals with possibly differing goals.[20] For instance, a bank will have staff that are responsible for expanding credit and staff responsible for managing risk. The granting of a large and risky loan will usually require consent of both sections and can easily be modelled as voting game. It is reasonable to assume that the staff responsible for expanding credit will all have one standard of behaviour, while those responsible for managing risk will have an opposing standard.[21] Examples 3.1(iii) and 3.2(iii), above, captures this structure in the definition of the partition structure, $\mathscr{P}$, in terms of two opposing standards of t and $(1-t)$. Note, also, that in these examples we have not actually defined p_i at all; we have only assumed certain patterns of correlated voting behaviour. Hence any reasonable model of voting power – in the sense of the rationale of the model – associated with committee voting in such structures requires that we take into account these different behavioural standards, i.e. apply the partial homogeneity structure of (A3). But if we do so we will not only violate iso-invariance but probably also LM as well, although it should be obvious that both postulates will be respected within each element G_k of a partition $\mathscr{P}$.

[20] Shubik (1962) discussed this issue some forty years ago. See also the much earlier attempt to formalize this issue by Morgenstern (1951).

[21] See Steffen (2002) for a detailed example and Braham and Steffen (2001) for a more general investigation of this case which also includes another example, that of a United Nations field office responsible for development projects that are a part of a refugee repatriation programme. In many instances, such projects have to be approved by the finance section of the agency headquarters which may have interests completely at odds with those of the field office. The field office is concerned with the welfare of particular refugees; the goal of the finance office is maximising donor contributions, which often leads the to a tendency to support 'high visibility' projects that are popular with donors but have little value to the refugees. Alternatively put, the finance office has a tendency to turn down useful 'low visibility' projects proposed by the field office. See also Martens *et. al.* (2002: 46–47).

Now, if we would follow the position taken by Felsenthal and Machover, then our measure will either be *a posteriori* – because it does not accord with (A1); or if considered *a priori*, the reasons for which we will give below, it will be unreasonable, because of the violation of LM.

It is not hard to see that it is the way in which *a prioricity* is being used in the literature that is at fault here. The general meaning of *a priori* is knowing on the basis of reflection and reasoning without appeal to experience. If we consider our bank example, it should be clear that we are entirely in accord with this. The application of (A3) does not presuppose factual information in terms of 'flesh and blood' individuals: all the sociological, psychological, and political – and dare say even the psychiatric – aspects of the players can be ignored in this structure. It is not names that are attached to the votes, but only the 'objective' interests of the positions in an organization.[22] The structure is still, to use Felsenthal and Machover's (1998: 20) own words, an 'abstract shell, *uninhabited* by real agents, with real likes and dislikes, attractions, and repulsions' and is therefore totally in accord with the position taken by Roth (1988) that we cited above. In other words, there is a mistaken belief that *a prioricity* means disregard for all forms of social organization; when it fact only means disregarding that information pertaining to the particulars of the players. It does not follow at all to say that if $p_i \neq p_j$ then we have necessarily included information about player preferences. This would be to conflate behavioural standards (such as legal rules) with personal preferences.[23]

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[22] Straffin was in fact lead astray here: 'Partial homogeneity assumptions are by their nature ad hoc; they would be out of place in theoretical analysis of abstract political structures where the level of abstraction requires symmetrical treatment of the players' (Straffin 1978: 493). The position is repeated again in (Straffin 1988: 77–78). Straffin is of course correct if he is referring only to parliamentary decision-making structures.

[23] Laruelle and Valenciano (2002) appear to make this mistake. They write: 'In this paper we propose a more general model which includes the two separate ingredients in a voting situation: the voting rule and the voters. The voting rule, specifies for a given set of seats when a proposal is to be accepted or rejected depending on the resulting vote configuration. Voters, the second ingredient in a voting situation, are included via their voting behavior, which is summarized by a distribution of probability over the vote configurations. *This distribution of probability depends on the preferences of the actual voters over the issues they will have to decide upon*, the liklihood of these issues being proposed, the agenda-setting issue, etc. This general model, unlike the traditional one, is apt for positive or descriptive purposes' (p. 2). (Emphasis added.) Our argument is that the distribution of probability *does not* necessarily depend upon the preferences of the actual voters in order to give voting power descriptive content.

To bring this point out further, consider the following. Assume that we have two games *v* and *u* both of which are characterized by exactly the same vector of voting weights and quota. Assume further that *u* is embedded in a structure such that some of the players will *a priori* be correlated due to the existence of incentive contracts that cannot be separated from the existence of the voting situation itself, i.e. if there were no contracts there would be no voting. The game *v* has no such structure. Then the conventional notion of *a prioricity* says that we should ignore the structural differences so that *v* and *u* are isomorphic, i.e. players in *v* who have the same weight as players in *u* will have the same power (cf. Examples 7.1 and 7.2 with 7.3–5.7 below). If we did not ignore the differences the resulting measure of power would either be classified as *a posteriori* or if *a priori* then as 'unreasonable' or 'pathological' because of its violation of LM, although perfectly reasonable in terms of the rationale of the model.

Essentially what our argument is boiling down to is that the belief that (A3) *necessarily* contains *a posteriori* information is mistaken; (A3) *contains more structure*. What is true, however, is that more structure implies more information, but this does not imply that the information is *a posteriori*. Put another way, a three dimensional space can contain more information than a two dimensional space; but this does not make the information in a two dimensional space more *a priori* than in the three dimensional space. Thus we see that the conventional meaning of *a priori* voting power refers to the quantity of information (which should be as little as possible) and not whether that information refers to matters of fact about the particular players. This is a very restrictive use of the term *a priori* and as we have shown a cause for much confusion.

There is a final consideration about the definition of *a prioricity* in terms only the voting rule that we must remark upon and which has important implications for understanding the violation of LM by MWC measures such as the D-P and the PGI. These measures are based upon a model of coalition formation, essentially derived from Riker's (1962) *size principle*, viz., only MWCs will form. This implies a theory of rational behaviour, which is information that is exogenous to the rule. MWC measures are based on the assumption that the players are *homo oeconomicus*. No such theory is necessarily implied by the Bz. In a sense the Bz has no generic players, but only the seats where players sit, although Rae (1969) in the derivation of his index which turns out to be the Bz in disguise and Felsenthal and Machover (1998) in defining their notion of I-power (power as influence)

have attempted to add some flesh to the model in the form of 'political individualism' and 'policy seeking' respectively. The idea in both cases is that the player is only concerned with voting for the outcome he or she prefers, regardless of what others do. However, this is a thin overlay compared to the theory behind the D-P and the PGI. Now, it is easy to see that the conventional notion of *a prioricity* would deem both the D-P and PGI as *a posteriori*, or at least less *a priori* than the Bz. This would seem a strange conclusion because like the Bz neither of these measures presuppose any factual information about the players.

It should also be evident that our bank example above also assumes rational players; otherwise there would not be a principal-agent problem, and optimal contracts designed to direct the behaviour of the players. If the D-P and the PGI are considered as *a priori*, why not a measure derived by Straffin's partial homogeneity approach which can also be motivated by an assumption of rational players.

In fact, it is worth noting that a partition structure $\mathcal{P}$ can be derived from the assumption of a generic player. To see this, imagine that we have a partition structure but no model of a generic player, such as *homo oeconomicus*. That is, the players are simply entities that vote. They may be rational or irrational, *homo sapiens* or chimpanzees, or extraterrestrials. If this is the case, any partition structure in which a voting rule is embedded becomes irrelevant because under such circumstances we are forced to assume that the players vote 'yes' or 'no' with equiprobability. Partial homogeneity either cannot survive, or makes no sense, without a model of a generic voter.

What we see here is that the moment we introduce a theory of rational behaviour, or better said, add more structure to the voting game, we obtain measures of *a priori* voting power that are perfectly reasonable in the rationale behind the model structure but unreasonable in the sense that they violate LM. Turning the problem around we could conclude that MWC measures and all measures derived under Straffin's partial homogeneity approach are *a posteriori* and that the violation of LM is a fact of *a posteriori* voting power. But this conclusion is not without a major problem: why is it acceptable that *a posteriori* power is not locally monotone? Simply because it is a property of the model? Why should we accept the properties of the model for this type of power but not for a priori power? This is entirely inconsistent with the idea of the 'reasonableness' of LM in the sense that it accords with the presupposition that power is locally

monotone in resources. Why should this 'reasonableness' criterion be valid only for measures that fulfil a very special notion of *a prioricity*?

Something clearly must be eschewed. Our belief is it is (a) the restrictive meaning of *a prioricity*, and (b) a presupposition that LM generally reflects a relationship between resources and power; that is, a violation of LM does not necessarily imply that power is not (weakly) increasing in resources.

6. Normativity

We have extensively discussed the grounds for eschewing the definition of *a prioricity* as conventionally used in the voting power literature. Before turning to the resource–power relationship we need to deal with another issue related to the *a prioricity* discussion that of the normative appeal of *a priori* analysis and of LM.

While we have chosen to eschew a particular definition of *a prioricity* – information contained in the rule and the rule only – one could argue that we could equally discard the notion of *a prioricity* altogether. The hair splitting classification of power into *a priori* and *a posteriori* would seem to be of little substantial interest other than possibly to philosophers. This position would be mistaken because *a prioricity is* an important classificatory device and one that is recognized by most scholars in the field of voting power. Its value has already been hinted at in the earlier quote by Roth. The vital part is the last sentence, which we repeat here: 'This kind of analysis seems to be just what is needed to analyze the voting rules in a new constitution, for example, long before the specific issues to be voted on arise or the specific factions and personalities that will be involved can be identified.' The rub of methodological argument is that *a prioricity* has normative appeal for institutional design: it corresponds to a 'veil of ignorance' argument á la Harsanyi (1955) and Rawls (1971).[24] Phrased differently, a valid analysis of constitutional structures requires that we exclude all controversial information and forms of reasoning; beliefs about the ideals of the good are not part of an even handed analysis of a constitution.

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[24] The importance of the veil of ignorance character of *a prioricity* to the analysis of voting power is stressed is stressed in Lane and Berg (1999), Holler and Widgrén (1999) and Felsenthal and Machover (2000) in their reply to the critical attack on voting power measures by Garrett and Tsebelis (1999) who argue for a preference-based approach to the measure of voting power.

This last point is not to be taken lightly. In fact it would seem that the normative appeal of the veil of ignorance is what underlies the position taken by Felsenthal, Machover and Zwicker (1998: 105–106) when they say that the choice between assumptions in a model of voting power 'should not be made according to their degree of verisimilitude, the extent to which they are truly descriptive of real-life voting situations.' Although Felsenthal, Machover and Zwicker do not explicitly justify this methodological imperative, we can only guess that it is implicitly an ethical one. The alternative epistemological justification that comes to mind is the instrumentalist position that says an assumption in a model is not to be judged by its realism but by its predictive usefulness (either in its range or accuracy). But this argument has little, if any, cutting edge in this context because it the theory and measurement of *a priori* voting power being more akin to the theory of social welfare functions does not fall at all within the class of empirical theory.

It should be evident from the forgoing discussion that going behind a veil of ignorance *does not* force us into a veil of ignorance that is described only by the independence assumption (A1), or essentially the Bz model. That model does not necessarily concur with a Harsanyian or Rawlsian veil of ignorance. It is true that both Harsanyi's and Rawls's ethics ask us to abstract from our particular circumstances when choosing social states or constitutions, but the solutions that Harsanyi and Rawls arrive at in terms of what people will agree upon in the 'original position' differ and do so because of the behavioural model that they use. Harsanyi arrives at the conclusion that rational individuals faced with the choice between alternative social states will choose that which maximizes the mean utility. He achieves this by employing the a model of a risk neutral utility maximizer. Rawls in contrast concludes that rational maximizers will choose that social state in which the worst off is maxmized because his rational maximizers happen to be risk averse. Thus from the perspective of ethical theory, a veil of ignorance does not mean absence of behavioural theory. Far from it: there is no ethical theory without it. What is important is that the behavioural theory is not based upon information about particular individuals.[25] We should not, therefore, believe that a constitutional analysis implies the need to eschew entirely social

[25] See Raz (1986: 110–133) for a discussion of the role of rationality in Rawls' ethical theory.

structure and behaviour.[26] To belabour the point and even be a little rhetorical, is it reasonable to take the position that from behind a veil of ignorance the world should be treated as structurally undifferentiated even if we have information to the contrary? This might generate a result in which voting power is locally monotone, but this does not make the result any more normatively valid than if it violated LM.

7. Power and Resources

Although we have shown via a discussion of Straffin's partial homogeneity approach that in its present form LM is untenable as a postulate of *a priori* power if *a prioricity* is not taken in such a restrictive sense that Felsenthal and Machover for instance do, this does not imply that we can take the position of Deegan and Packel [, 1983 752], Brams and Fishburn (1995), and especially Holler (1997, 1998) that the violation of LM simply reflects a social and political fact that there is an inverse relationship between power (in whatever form) and resources. This may seem paradoxical, but this is not too difficult to resolve. As it turns out, the underlying intuition of LM is not necessarily wrong; only its definition is too restrictive.

If we abstract from the particular definition of LM to that of monotonicity *simpliciter* we find a very general principle which states that as the underlying data of a problem changes, so does its solution. LM merely takes as its underlying data the vector of voting weights $(w_1, w_2, \ldots, w_n)$. There lies the problem. As we have argued in the previous section, the underlying data of voting game is actually more than this: it is made up of (i) the voting weights and (ii) the players positions within the decision-making structure. The interaction of both these components are what we can call the resources or, to use Dahl's (1957) terminology, the 'base' of (voting) power. Under what we have called a flat structure, the position of each

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[26] Although in their attempt at a probabalistic refoundation of power measures Laruelle and Valenciano (2001a) recognise that the decision rule $\mathcal{W}(v)$ is not a 'game' and requires a specification of a probability model (actually this insight can be found explicitly in Straffin (1983, 1988, 1994)) they do not push their analysis far enough. The result is that they err in their conclusion that '… from a normative point of view the Banzhaf index is no doubt the best candidate as a reference for the design of voting procedures, where any information about the voters should be ignored even if available' (p. 26).

‘vote’ of a player’s voting weight (which is merely the sum of a players ‘votes’) is by definition symmetric and therefore each ‘vote’ has the same ability to make a difference to the outcome irrespective of who possesses these votes. In a flat structure, resources (or base of power) and weight happen to coincide; in a differentiated structure they do not. This is shown up in examples 3.1 and 3.2, above. But to bring out the point even more, we will examibe a simpler set of examples. Consider now a committee of five players and a simple majority rule, which can be represented as the weighted voting game [3; 1, 1, 1, 1, 1]. We can construct the following seven scenarios.

Example 5.1 Assume (A1) for all players, we have $Ef_1 = Ef_2 = Ef_3 = Ef_4 = Ef_5 = 0.38$. (This is the Penrose/Banzhaf measure β').

Example 5.2 Assume (A2) for all players, we have $Ef_1 = Ef_2 = Ef_3 = Ef_4 = Ef_5 = 0.20$ (This is the S-S ϕ).

Example 5.3 Assume (A3) as follows: players 1, 2, 3 form a standard t and players 4 and 5 a standard $(1–t)$. We have $Ef_1 = Ef_2 = Ef_3 = 0.53$ and $Ef_4 = Ef_5 = 0.30$.

Example 5.4 Assume (A3) as follows: players 1, 2 form a standard t and players 3 and 4 a standard $(1–t)$ and player 5 behaves independently. We have $Ef_1 = Ef_2 = = Ef_3 = Ef_4 = 0.42$ and $Ef_5 = 0.53$.

Example 5.5 Assume (A3) as follows: players 1, 2, 3, 4 form a standard t and player 5 behaves independently. We have $Ef_1 = Ef_2 = Ef_3 = Ef_4 = 0.25$ and $Ef_5 = 0.20$.

Example 5.6 Assume (A3) as follows: players 1, 2, 3 form a standard t and players 4 and 5 behave independently. We have $Ef_1 = Ef_2 = Ef_3 = 0.33$ and $Ef_4 = Ef_5 = 0.25$.

Example 5.7 Assume (A3) as follows: players 1, and 2 form a standard t and players 3, 4, 5 behave independently. We have $Ef_1 = Ef_2 = 0.38$ and $Ef_3 = Ef_4 = Ef_5 = 0.33$.

Observe that except for the extreme cases of applying (A1) and (A2), Ef_i is always less for the independent players (for this committee) except in Example

5.4, where it is greater than for the players belonging to either t or $(1–t)$. This makes intuitive sense because the two 'groups' (or more accurately, the collection of players conforming to a given standard) are of equal size and 'antagonistic' which leaves the neutral party in a more powerful position.[27] The reason is simple, each of the players in t and $(1–t)$ is more likely to form a coalition with the independent than with players from the antagonistic standard. In a sense we could say that the antagonism 'depletes' the resources (i.e. weights) of the members of these groups, and as a consequence neutrality increases the value of the independent voter.

The outcome for Example 5.3 where Ef_i for the players in t is greater than the players in $(1–t)$ also makes sense. Here we again have two opposing 'groups' and the members of the largest 'group' have a greater probability to be decisive than the smaller 'group'. Clearly – and obviously – Ef_i depends on the size of the group. This certainly makes sense; it confirms the idea that under certain circumstances there is power in numbers. We see this again in Examples 4–6. This merely reflects Hannah Arendt's (1970: 44) concept of power. '*Power*', she writes,

> corresponds to the human ability not just to act but to act in concert. Power is never the property of an individual; it belongs to a group and remains in existence only so ling as the group keeps together. When we say of somebody that he is 'in power' we actually refer to his being empowered by a certain number of people to act in their name. The moment the group, from which the power originated to begin with (*potestas in populo*, without people or group there is no power), disappears, 'his power' also vanishes.

Thus the underlying data of a voting game needs to be clearly specified before we can calculate the resources or the 'base of power'. The underlying data includes the partition structure, $\mathcal{P}$. This is the reason why we say that it is mistaken to believe that a violation of LM defined only by voting weights implies that power is not necessarily locally monotone in resources. It is beyond the scope of this paper, but we posit that a reasonable method for calculating a quantitative value of a player's resources put altogether in a voting game would probably produce a resulting measure of voting power that is locally monotonic in this quantity. Hence, the violation of LM under Straffin's approach is entirely

[27] One could say that it is a form of a quarrel, although our examples in no way display the so-called paradox of quarrelling members. See Brams (1975: 314).

reasonable because it can be easily explained and made consistent with a much more general concept of local montonicity.

To grasp this more general concept we need to get behind the intuition of the desirability relation and LM by considering a definition of power *simpliciter*, which we take to be '*i* has power to do *x* if *i* can do *x*'.[28] It is not hard to see that this definition entails locally monotonicity in a very general form. Consider that *x* means 'forcing a social outcome'. Then *i* has more power than *j* if *i* can force an *x* that *j* cannot, i.e. *i* has a *means* to achieving *x* that *j* does not posses. In other words, power is by definition resource based in much the same way that the production of goods is resource based: people are able to produce goods only if they have appropriate materials and tools which help them transform materials into goods; two people endowed with the same intelligence, skills, tools, and materials should be able to produce the same quantity of goods; and if one of the two are strictly more intelligent or skilful than the other or has more tools or materials at his disposal than the other then, *ceteris paribus*, he should still be *able* to produce as much as the other. Obviously LM can be readily taken to be a special case of this more general resources-to-power relationship. Denial of the local montonicity of power would seem difficult to stomach.

Strange as it may seem, when it comes to the violation of LM by measures of voting power both sides of the debate turn out to be wrong: the one side believing that the violation of LM by MWC measures are another instance of the fact of the sociological and political fact that power is not monotonic resources – see Brams and Fishburn (1995) for empirical examples of 'when size is a liability'; the other side believing that such a violation is pathological because it is contradicts a perfectly reasonable intuition. Both sides of the debate err in the same way by focusing *exclusively* on the vector of voting weights as given by $(w_1, w_2, \dots, w_n)$ as the resources describing the underlying data of the game. As we have argued above this is mistaken in the same way it is mistaken to say that a superbly outfitted army that is defeated by a band of poorly equipped guerrillas is evidence that power is not locally monotonic in military resources. True, military power is

[28] Although there may be major disagreements among political scientists and philosophers in particular about the nature of power, there seems to be a fair amount of agreement upon its formal definition. Nearly all defintions of power can be traced back to this form. See, for, among others, Bachrach and Baraz (1962), Dahl (1957), Goldman (1972), Ledyaev (1997), Lukes (1974), Morriss (1987), Oppenheim (1961, 1981), and Wrong (1979).

not necessarily monotone in guns; but guns do not fully describe the underlying data of the situation, which includes military intelligence, knowledge of local geography, and even physical acclimatisation to the theatre of operations. We would argue that such a ill-equipped band of guerrillas probably does have more resources than its well-equipped enemy.

We cannot pursue here, but we are sure that the same line of thought can be applied to the measures of voting power based on MWCs, i.e. the D-P and the PGI. In both these cases we need to take into account the rationale or 'story' of the indices as part of the underlying data: in the first case a bargaining structure; in the second the public good character of the outcome. And in both cases we have to take into account the rationality of the players and the incentives for only MWCs to form. It is only when we have exhausted all possibilities of accounting for the quantity that a 'smaller' player has over and above a 'larger' player which gives the smaller player more power than the larger one can we safely classify the violation of LM by these measures as 'pathological' or counter-intuitive to say the least.[29] This has still to be done, so as far as we are concerned the jury is still out as regards the acceptability of the D-P and PGI. In the case of a private good (e.g. creditors voting on the division of an insolvent firm) the bargaining situation may be such that the 'formal' weights are in fact not the ones that would be used in a game. Rather, the game may be ill defined because rational players may decide to 'shave' their weights in order to participate in a winning coalition in order to receive some payoff from the game.[30] This line of thought also suggests that we need to look very carefully at the 'type' of game to which voting rules are being applied. Is it economic and strategic, in the sense of the division of a fixed purse; or is it political and ideological in the sense of the definition of policy? It is also important to note from where the proposal to be voted upon originates. Is it

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[29] It should be noted that not all MWC measures violate LM. Levínský and Silársky (2001) define a measure based on 'least-member' MWC – an idea that actually can be found in Leiserson (1968).

[30] See Riker (1967) for experimental evidence on the strategy of 'shaving'. Also note that it is not clear which measure to use in such situations. Although Holler and Packel (Holler and Packel 1983) defined the PGI for public good contexts, the a measure with a similar structure can be motivated and derived for private good ones as well. See Brams and Fishburn (1995), Holler (1998) and Widgrén (2001). There are good reasons to prefer the PGI structure to that of the D-P because the latter tends to conflate 'power' with 'payoffs', which, while a notion that is held by many, is not necessarily correct. The PGI structure avoids this.

endogenous to the set of players or is it exogenous? This information is all part of the underlying data of the game and will have bearing on the resources-to-power relationship.

Although it could be argued that by expanding the definition of LM to include the missing 'quantities' in order to restore the expected resources–power relationship reduces LM to vacuity, this is in fact a usual step in economics. The axioms of revealed preference say that all behaviour is maximizing behaviour. The usefulness of this approach is that it focuses our attention: if we observe an apparently irrational choice, the revealed preference theory asks us to look again before deeming the individual to be irrational. An expanded definition of LM does likewise: it asks us to examine the voting situation again and find an explanation for an apparent paradox: that a player with less resources than another can exhibit more power than the other. This is a useful heuristic device. It can reveal interesting facts about power.

8. Conclusion

The main result of this paper is in one sense depressing. By showing that neither iso-invariance as defined by (P1) nor local monotonicity as defined by (P5) are necessarily compelling axioms or postulates of *a priori* voting power we have whittled down the remaining set to the somewhat trivial and related axioms or postulates of *ignoring dummies* (P2) and *vanishing for dummies* (P3). Namely, a reasonable measure of voting power does not assign any power to players that can never make any difference whatsoever to outcome of a vote.

For those familiar with the axiomatic approach to social choice problems, this result should come as no surprise. The related field of the measurement of freedom suffers from similar problems. Pattanaik and Xu (1990, 1998), for instance, have shown that an apparently harmless and compelling set of axioms that yield and uniquely characterize a 'naive' counting rule that measures an agent's total freedom by simply counting the number of options open to that agent. This is not the place to go into this debate, but suffice to say Pattanaik and Xu's axiomatic structure has been dissected and severely criticized and in one important contribution to the debate (Carter 2001) the axioms have been shown to be inconsistent with the very basic and widely agreed upon definition of freedom itself.

A more constructive way of casting our result is to say that *if* (a) we accept the logic of power *simpliciter* being monotonic in resources and (b) *if* it is at all justifiable that a measure of *a priori* voting power ignores (i) any information pertaining to the institutional structure of voting, (ii) the nature of the social outcome being voted upon, (iii) the rationality of the players, *then* such a measure ought to (must?) be locally monotone in voting weights. While we have suggested in the previous section that (a) more or less self evident, it is not at all self evident when we should accept any of the three conditions in (b). Given that in any concrete voting body that is under the microscope will provide some information on (i) and (ii) it seems entirely unreasonable to blank it out in the calculation of voting power. Only (iii) may be considered by some as a problem, unless one is an economist, in which case it is taken for granted and would not be considered as violating *a prioricity* in any real way. On the other hand if we reformulate (iii) as a model of a generic voter – whatever that may be – then it is doubtful that any social scientist would be ready to eschew it.

In fact, what this whole analysis seems to be pointing to is that *homo oeconomicus* may be the watershed property in this debate. In absence of rational actors, the measurement of voting power *is* but a mathematical structure. If that is all we are concerned with, the definition of LM only in terms of voting weights makes full sense. If, however, we believe that voting power belongs to the class of social power, then we must introduce actors of some form or another. If this results in a violation of an attractive mathematical property of voting power, then it may mean that the mathematical property is inappropriate in this context, and not that the measure is 'broken', 'pathological' or 'unacceptable'. If we are convinced that power is locally monotone in resources this only implies that an *a priori* measure of voting power must be locally monotone in voting weights if, and only if, the voting weights are the *only* resource in the voting game.

Finally, and to wind up, we must stress that we are not saying that LM in its present form is irrelevant to the study *a priori* voting power. Far from it. It has an important place to play as a *normative* criterion in institutional design. That is, *if* we desire to preserve the ranking of influence over social outcomes with that of the ranking of voting weights, *then* (i) we are compelled to create a voting structure in which there are no incentives such that players will *a priori* correlate their behaviour in one way or the other; and (ii) create conditions such that *a priori* not only MWCs will form. This perspective points to the possibility that the

debate about LM has also been confused by the ethical appeal of this postulate – it seems to reflect a requirement of fairness that goes back to Aristotle and the ethical appeal of treating players symmetrically. We have not examined it, but it seems straightforward to assume that behind a veil of ignorance rational players would choose a voting system that respects LM in those situations where the players have an *interest* in the outcome of a vote. It would seem inappropriate however to characterize (describe) voting power by axioms or postulates that capture, directly, our moral sensibilities.[31]

References

Allingham, M.G. (1975), Economic Power and Values of Games, *Zeitschrift für Nationalökonomie* 35: 293–299.

Arendt, H. (1970), *On Violence*, London: Penguin.

Aumann, R.J. and Dreze, J.H. (1974), Cooperative Games with Coalition Structures, *International Journal of Game Theory* 3: 217–237.

Bachrach, P. and Baratz, M. (1962), Two Faces of Power, *American Political Science Review* 56: 947.

Banzhaf, J.F. (1965), Weighted Voting Doesn't Work: A Mathematical Analysis, *Rutgers Law Review* 19: 317–341.

Barry, B. (1980a), Is it Better to be Powerful or Lucky? Part 1, *Political Studies* 28: 183–194.

Barry, B. (1980b), Is it Better to be Powerful or Lucky? Part 2, *Political Studies* 28: 338–352.

Braham, M. and Steffen, F. (2001), Notes on Power in Hierarchical Structures, *Sozialökonomisches Seminar der Universität Hamburg* University of Hamburg.

Braham, M. and Steffen, F. (Forthcoming 2002), Voting Power in Games with Abstentions, in: M.J. Holler, H. Kliemt, D. Schmidtchen and M.E. Streit (eds), *Jahrbuch für Neue Politische Ökonomie*, Tübingen: Mohr Siebeck.

Brams, S.J. (1975), *Game Theory and Politics*, New York: The Free Press.

Brams, S.J. and Fishburn, P.C. (1995), When is Size a Liability? Bargaining Power in Minimal Winning Coalitions, *Journal of Theoretical Politics* 7: 301–316.

Brueckner, M. (2001), Extended Probabalistic Characterization of Power Indices, in: M.J. Holler and G. Owen (eds), *Power Indices and Coalition Formation*, Dordrecht: Kluwer, 29–43.

Carter, I. (2001), Choice, Freedom, and Freedom of Choice, University of Pavia, Italy, Mimeo.

[31] There is an interesting parallel of this problem related to the monotonicity axiom in baragining theory. See Roemer (1986).

Coleman, J.S. (1971), Control of Collectivities and the Power of a Collectivity to Act, in: B. Lieberman (ed.) *Social Choice*, New York: Gordon and Breach, 269–300.

Dahl, R.A. (1957), The Concept of Power, *Behavioral Science* 2: 201–215.

Deegan, J. and Packel, E.W. (1978), A New Index for Voting Games, *International Journal of Game Theory* 7: 113–123.

Deegan, J. and Packel, E.W. (1983), To the (Minimal Winning) Victors Go the (Equally Divided) Spoils: A New Power Index for Simple n-Person Games, in: S.J. Brams, W.F. Lucas and P.D. Straffin (eds), *Political and Related Models*, Berlin: Springer, 239–255.

Dubey, P. and Shapley, L.S. (1979), Mathematical Properties of the Banzhaf Power Index, *Mathematical Operations Research* 4: 99–131.

Felsenthal, D.S. and Machover, M. (1995), Postulates and Paradoxes of Relative Voting Power – A Critical Re-appraisal, *Theory and Decision* 38: 195–229.

Felsenthal, D.S. and Machover, M. (1997), Ternary Voting Games, *International Journal of Game Theory* 26: 335–351.

Felsenthal, D.S. and Machover, M. (1998), *The Measurement of Voting Power*, Cheltenham: Edward Elgar.

Felsenthal, D.S. and Machover, M. (2000), Myths and Meanings of Voting Power: Comments on a Symposium, *Journal of Theoretical Politics* 13: 81–97.

Felsenthal, D.S., Machover, M. and Zwicker, W. (1998), The Bicameral Postulates and Indices of A Priori Voting Power, *Theory and Decision* 44: 83–116.

Freixas, J. and Gambarelli, G. (1997), Common Internal Properties Among Power Indices, *Control and Cybernetics* 26: 591–603.

Garrett, G. and Tsebelis, G. (1999), Why Resist the Temptation of Power Indicies in the European Union?, *Journal of Theoretical Politics* 11: 291–308.

Goldman, A.I. (1972), Toward a Theory of Social Power, *Philosophical Studies* 23: 221–268.

Harsanyi, J.C. (1955), Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility, *Journal of Political Economy* 61: 309–321.

Holler, M.J. (1982), Forming Coalitions and Measuring Voting Power, *Political Studies* 30: 262–271.

Holler, M.J. (1985), Strict Proportional Power in Voting Bodies, *Theory and Decision* 19: 249–258.

Holler, M.J. (1997), Power, Monotonicity, and Expectations, *Control and Cybernetics* 26: 605–607.

Holler, M.J. (1998), Two Stories, One Power Index, *Journal of Theoretical Politics* 10: 179–190.

Holler, M.J. and Packel, E.W. (1983), Power, Luck and the Right Index, *Zeitschrift für Nationalökonomie* 43: 21–29.

Holler, M.J. and Widgrén, M. (1999), Why Power Indices for Assessing European Union Decision-Making?, *Journal of Theoretical Politics* 11: 321–330.

Isbell, J.R. (1958), A Class of Simple Games, *Duke Mathematical Journal* 25: 423–439.

Jones, P. and Sugden, R. (1982), Evaluating Choice, *International Review of Law and Economics* 2: 47–65.

Lane, J.-E. and Berg, S. (1999), Relevance of Voting Power, *Journal of Theoretical Politics* 11: 309–320.

Laruelle, A. (1999), On the Choice of a Power Index, *Discussion Papers, Department of Applied Economics* Basque Country University, Spain.

Laruelle, A. and Valenciano, F. (1999), Why Power Indices Should be Efficient?, *Discussion Papers, Department of Applied Economics* Basque Country University, Spain.

Laruelle, A. and Valenciano, F. (2001a), A Probabalistic Reformulation of Power Measures, *Discussion Papers, Department of Applied Economics* Basque Country University, Spain.

Laruelle, A. and Valenciano, F. (2001b), Shapely-Shubik and Banzhaf Indices Revisted, *Mathematics of Operations Research* 26: 89–104.

Laruelle, A. and Valenciano, F. (2002), Positive and Normative Assessment of Voting Situations, Universidad de Alicante and Universidad del País Vasco, Mimeo.

Ledyaev, V.G. (1997), *Power: A Conceptual Analysis*, Commack, NY: Nova Science Publishers.

Leech, D. (1990), Power Indices and Probabalistic Voting Assumptions, *Public Choice* 66: 293–299.

Leiserson, M. (1968), Factions and Coalitions in One-Party Japan: An Interpretation Based on the Theory of Games, *American Political Science Review* 62: 770–787.

Levínský, R. and Silársky, P. (2001), Global Monotonicity of Values of Cooperative Games: An Argument Supporting the Explanatory Power of Shapley's Approach, in: M.J. Holler and G.O. Owen (eds), *Power Indices and Coalition Formation*, Dordrecht: Kluwer, 105–125.

Lévi-Strauss, C. (1978), *Structural Anthropology***2**), London: Penguin.

Lukes, S. (1974), *Power: A Radical View*, London: Macmillan.

Martens, B.*, et al.* (2002), *The Institutional Economics of Foreign Aid*, Cambridge: Cambridge University Press.

Maschler, M. and Peleg, B. (1966), A Characterization, Existence Proof and the Dimension Bounds for the Kernel of a Game, *Pacific Journal of Mathematics* 18: 289–323.

Morgenstern, O. (1951), Prolegomena to a Theory of Organization, *Rand Research Memorandum* RM-5438-PR.

Morriss, P. (1987), *Power: A Philosophical Analysis*, Manchester: Manchester University Press.

Nagel, J. (1973), The Descriptive Analysis of Power, Ph.D: Yale University.

Oppenheim, F.E. (1961), *Dimensions of Freedom*, New York: St. Martins Press.

Oppenheim, F.E. (1981), *Political Concepts: A Reconstruction*, Chicago: University of Chicago Press.

Owen, G. (1972), Multilinear Extensions of Games, *Management Science* 18: 64–79.

Owen, G. (1975), Multilinear Extensions and the Banzhaf Value, *Navel Research Logistics Quarterly* 22: 741–750.

Pattanaik, P. and Xu, Y. (1990), On Ranking Opportunity Sets in Terms of Freedom of Choice, *Recherches Economiques de Louvain* 56: 383–390.

Pattanaik, P.K. and Xu, Y. (1998), On Preference and Freedom, *Theory and Decision* 44: 173–198.

Penrose, L.S. (1946), The Elementary Statistics of Majority Voting, *Journal of the Royal Statistical Society* 109: 53–57.

Rae, D.W. (1969), Decision-Rules and Individual Values in Constitutional Choice, *American Political Science Review* 63: 40–63.

Rawls, J. (1971), *A Theory of Justice*, Oxford: Oxford University Press.

Raz, J. (1986), *The Morality of Freedom*, Oxforf: Oxford University Press.

Riker, W.H. (1962), *The Theory of Political Coalitions*, New Haven: Yale University Press.

Riker, W.H. (1967), Bargaining in a Three-Person Game, *American Political Science Review* 61: 642–656.

Roemer, J. (1986), The Mismarriage of Bargaining Theory and Distributive Justice, *Ethics* 97: 88–110.

Roth, A.E. (1988), Introduction to the Shapley Value, in: A.E. Roth (ed.) *The Shapley Value: Essays in Honor of Lloyd S. Shapley*, Cambridge: Cambridge University Press, 1–25.

Russell, B. (1938), *Power: A New Social Analysis*, London: George Allen and Unwin.

Scheffler, H., W (1970), Structuralism in Anthropology, in: J. Ehrmann (ed.) *Structuralism*, Garden City: Anchor, 58–79.

Sen, A.K. (1991), Welfare, Preference and Freedom, *Journal of Econometrics* 50: 15–29.

Sewell, W.H. (1992), A Theory of Structure: Duality, Agency, and Transformation, *American Journal of Sociology* 98: 1–29.

Shapley, L.S. (1953), A Value for n-Person Games, in: H. Kuhn and A.W. Tucker (eds), *Contributions to the Theory of Games*, Princeton: Pinceton University Press, 307–317.

Shapley, L.S. (1962), Simple Games: An Outline of the Descriptive Theory, *Behavioral Science* 7: 59–66.

Shapley, L.S. (1977), A Comparison of Power Indices and a Nonsymmetric Generalization, *The Rand Paper Series* P-5872.

Shapley, L.S. and Shubik, M. (1954), A Method for Evaluating the Distribution of Power in a Committee System, *American Political Science Review* 48: 787–792.

Shubik, M. (1962), Incentives, Decentralized Control, The Assignment of Joint Costs and Internal Pricing, *Management Science* 8: 325–343.

Steffen, F. (2000), When is A Priori Voting Power Really A Priori, *Homo Oeconomicus* 16: 405–416.

Steffen, F. (2002), Power and the Internal Organization of the Firm, University of Hamburg.

Straffin, P.D. (1977), Homogeneity, Independence, and Power Indices, *Public Choice* 30: 107–118.

Straffin, P.D. (1978), Probability Models for Power Indices, in: P.C. Ordeshook (ed.) *Game Theory and Political Science*, New York: New York University Press, 477–509

Straffin, P.D. (1983), Power Indices in Politics, in: S.J. Brams, W.F. Lucas and P.D. Straffin (eds), *Political and Related Models*, Berlin: Springer, 256–321.

Straffin, P.D. (1988), The Shapely-Shubik and Banzhaf Power Indices as Probabilities, in: A.E. Roth (ed.) *The Shapley Value: Essays in Honor of Lloyd S. Shapley*, Cambridge: Cambridge University Press, 71–81.

Straffin, P.D. (1994), Power and Stability in Politics, in: R.J. Aumann and H. S (eds), *Handbook of Game Theory*, Amsterdam: Elselvier Science, 1127–1151.

Sugden, R. (1985), Liberty, Preference, and Choice, *Economics and Philosophy* 1: 213–229.

Taylor, A.D. and Zwicker, W.S. (1999), *Simple Games: Desirability Relations, Trading, Pseudoweightings*, Princeton: Princeton University Press.

van Hees, M. (2000), *Legal Reductionism and Freedom*, Dordrecht: Kluwer.

Widgrén, M. (2001), On the Probabalistic Relationship Between the Public Hood Indey and The Normalized Banzhaf Index, in: M.J. Holler and G.O. Owen (eds), *Power Indices and Coalition Formation*, Dordrecht: Kluwer, 127–142.

Wrong, D.H. (1979), *Power: Its Forms, Bases and Uses*, Oxford: Basil Blackwell.