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The Strategic Power Index

– Responses –

Abstract

The *strategic power index* (SPI) reflects a method of measuring 'power' that is not based on the notion that players need to form some kind of majority coalition. Our notion of 'power' is based on the average distance between players' ideal points and the equilibrium outcome in policy games in which players have different abilities to affect the final outcome of the decision-making procedure. The method proposed in this paper employs the analytical tools of non-cooperative game theory. Actor preferences, the policy space, as well as the rules of the decision-making process, are fully integrated into the analysis. Since it allows players to act strategically, the index derived in the paper is labeled the *strategic power index*. The *strategic power index* gave rise to several comments in the literature. Garrett and Tsebelis (1999) argue that the *strategic power index* – although an improvement compared to conventional indices – nevertheless suffers from a drawback generated by the statistics used in it. Felsenthal and Machover (2000) proved a theorem stating that the strategic power index is a modified Banzhaf index. Widgrén and Napel (2001) point to a potential for confusing power with luck. Moreover, whereas our approach uses the notion of a dummy player for an external normalization of the index, Widgrén and Napel follow the path of internal normalization. As they claim, this allows for different informational considerations and makes the analysis more procedural than in the case of the strategic power index. This paper contains our responses.

Key words: agenda setting, power indices, non-cooperative game theory, voting games, spatial voting

I. Introduction

The purpose of this paper is to respond to some comments on the new method of evaluating the distribution of power in policy games that we proposed several years ago. Conventional methods, such as the Banzhaf, Shapley-Shubik and other indices take the set of players as their domain and measure the voting power of players by the extent to which a player in any collective body that makes yes-or-no decisions by vote may turn a losing coalition into a 'winning' coalition for all mathematically possible permutations of players (Shapley-Shubik

index), or the relative number of times a player is decisive in a vote (Banzhaf index).[1] What is measured by these indices is *a priori* voting power, determined without taking into consideration voters' preferences ("prior bias regarding the bill voted upon"), or the degree of affinity (for example, ideological proximity) between voters (see Felsenthal and Machover 1998: 2). Critiques also point to the limited capability of traditional power indices to model both players' strategic interaction and a complicated institutional structure typical for real world decision making (see Schmidtchen and Steunenberg 2002). As mentioned in Felsenthal and Machover (1998), conventional voting power analyses are either based on cooperative games (see the Shapley-Shubik index measuring what Felsenthal and Machover call P-power, which posits an office-seeking motivation of voting behavior (see Felsenthal and Machover 1998: 171). Or they are entirely probabilistic measures (see Penrose (1946), Banzhaf (1965), Coleman (1971, 1986), which take a policy-seeking viewpoint focusing on the degree to which a member's vote is able to influence the outcome of a vote (I-power in the sense of Felsenthal and Machover 1998: 36)).

In this paper, we propose a different method of measuring 'power', one not based on the notion that players need to form some kind of majority coalition. Our notion of 'power' is based on the average distance between players' ideal points and the equilibrium outcome in policy games in which players have different abilities to affect the final outcome of the decision-making procedure. The method proposed in this paper employs the analytical tools of non-cooperative game theory. Actor preferences, the policy space, as well as the rules of the decision-making process, are fully integrated into the analysis. Since it allows players to act strategically, the index derived in the paper is labeled the *strategic power index*, *SPI* (see Steunenberg/Schmidtchen/Koboldt 1999).[2] The *strategic power index* refers to the ability of a player to make a difference in the outcome of a policy game. This ability depends on the rules of the game, which define the set of players, the sequence of moves and the set of available actions. Strategic power is of particular relevance in situations in which the decisions of different decisionmakers are interdependent. Interdependence means that the decision of a player directly affects at least one other player in the group. A player behaves strategically if

[1] These 'classical' indices have been supplemented with more recent power measures, such as the Johnston index (Johnston 1978), the Deegan-Packel index (Deegan and Packel 1979) and the Holler index (Holler 1982, 1984). The main differences between these indices are the ways in which coalition members share the benefits of their cooperation, and the kind of coalition players chose to form (see Colomer 1999). For a comparative investigation of traditional power indices see Felsenthal and Machover (1998) and Holler and Owen (2001).

he accounts for this interdependence in deciding what action to take (see Dutta 1999: 5). A rational player, while accounting for this interdependence, chooses her best action.

How to predict the behavior of rational people in a setting in which the optimal decision of a player depends on the decision of others? The only theory providing a consistent answer to this question is the theory of strategic or non-cooperative games. The strategic or non-cooperative approach requires one to specify, in close detail, what the players can and cannot do during the game and then searches for an optimal strategy for each player. Optimal being what maximizes (expected) utility of a player. However, in a strategic setting, what is best for one player depends on what he believes his fellow players will do, which in turn depends on what they believe the first player will do. Based on a complete description of the rules of a game (Who can do what and when; what are the consequences and who gets how much when the game is over?), the strategies of the players can be studied in detail in order to find the strategy profile determining the play of the game. This theory asserts that, if players behave rationally, Nash equilibria will be the outcome.

The *strategic power index* gave rise to several comments in the literature. Garrett and Tsebelis (1999b) argue that the *strategic power index* – although an improvement compared to conventional indices – nevertheless suffers from a drawback generated by the statistics used in it. Felsenthal/Machover (2000) proved a theorem stating that the strategic power index is a modified Banzhaf index. Widgrén and Napel (2001) point to a potential for confusing power with luck (see pp. 3-4). Moreover whereas our approach uses the notion of a dummy player for an external normalization of the index, Widgrén and Napel follow the path of internal normalization: "This means that whether a player is dummy or not depends on her capabilities in the game. Contrary to Steunenberg et. al. (1999), we assume that any player, dummy or not, is one of the players and not an external observer. We define power as one's ability to change the current state of affairs. This allows for different informational considerations and makes the analysis more procedural than in the case of StPI (=strategic power index, the authors). Our approach leads to a definition of power, which, in fact, corresponds to that of established power indices." (Widgrén and Napel 2001: 4) This paper contains our responses.

Section II is concerned with the logic of the strategic power index. In section III we address the critique put forward by Garrett and Tsebelis (1999, a, b). Section IV deals with the Felsenthal/Machover proposition that the strategic power index is nothing but a modified Banzhaf index. Section V addresses the Widgrén and Napel comments. Section VI concludes.

II. The Logic of Strategic Power

1. The Framework

Our method of evaluating the distribution of power is embedded in the recently developed theory of political institutions, which shows that institutions play an important role in shaping the process of political decision-making (see, for instance, Ostrom 1986; Riker 1980; Shepsle 1989; Shepsle and Weingast 1981). Institutions may determine the extent to which players are able to participate in the decision-making process. They affect the kind of actions players are allowed to take and the sequence in which players may act. By excluding specific player options, institutions may induce political stability and determine a game's equilibrium outcome. This notion of institutions forms the basis of the concept of structure-induced equilibrium developed by Shepsle (1979). A structure-induced equilibrium is a stable policy that cannot be defeated or changed by the players, given the decision-making rules of the game.

This concept has been widely used to analyze the outcomes of decision-making under various legislative institutions (see Moser 1997 for a review of this literature). These models generally point to unique equilibrium outcomes, which can be either the status quo, when one of the players objects to a proposed change, or a new policy, when all players prefer a change.

We approach power as a player's ability to affect the equilibrium outcome in a game which is defined by a decision-making procedure. The stronger a player's influence on the outcome under a specific decision-making procedure, the more powerful this player is. This ability to exert influence can take various forms. Some players, such as the European Commission may be able to propose a policy, while others, such as the members of the Council, only have the right to veto a proposal, thereby constraining the initial choice of another player. To distinguish 'power' from 'luck', we propose a measure that is independent of the specific preferences of players, which, together with the decision-making procedure, determines the (equilibrium) outcome of a game. This can be achieved by measuring a player's power under some decision-making procedure with reference to the *mean or expected distance between the equilibrium outcome and this player's ideal point* for all possible combinations of players' preferences and all possible combinations of the status quo. By focusing on the expected distance the measure will indicate a player's a priori prospects of playing a game without knowing the preference configuration of all players in the game and the location of the status quo.

More formally, let n N be the number of players in a game describing a decision-making procedure π. For an m-dimensional and finite outcome space X ê $\sum^{m}$, these players have Euclidean preferences which can be characterized by player i's ideal point $x_i = (x^1_{\mathrm{i}},\ x^2_{\mathrm{i}}, \ldots\ x^{\mathrm{m}}_{\mathrm{i}})$. Let q X denote the status quo, that is, the hypothetical state of affairs before the start of the decision-making process. This can be the current policy, or the situation without such a policy. We call a combination of a particular ideal point for each player and the status quo a 'state of the world', which will be denoted as $\xi = \left(x_1, x_2, \ldots x_n, q\right)$. Finally, let $x^{\pi}(\xi)$ be the unique equilibrium outcome of the game based on procedure π, given the state of the world ξ (that is, given players' preferences and a status quo).[3]

Our measure of power is based on the expected distance between the equilibrium outcome and the player's ideal point for all possible configurations of preferences and the status quo, or states of the world. In this context, each particular state of the world is assumed to be the instance of a random variable $\bar{\xi} = \left(\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n, \bar{q}\right)$. The expected or mean distance between the equilibrium outcomes for some decision-making procedure, π, and player i's ideal point is then given by

$$\Delta_i^{\pi} = \delta_i^{\pi} f\left(\bar{\xi}\right) d\left(\bar{\xi}\right)$$

where

$$\delta_{\mathrm{i}}^{\pi} = \sqrt{\sum_{\mathrm{k=1}}^{\mathrm{m}} \left(\mathrm{x}^{\pi}\left(\bar{\xi}\right)^{\mathrm{k}} - \bar{\mathrm{x}}_{\mathrm{i}}^{\mathrm{k}}\right)^{2}}$$

is the Euclidean distance between the equilibrium outcome of the game and the ideal point of player i in any particular state of the world, and $f\left(\bar{\xi}\right)$ is the density function if $\bar{\xi}$ is a continuous random variable. The mean distance as expressed by Δ_i^{π} allows us to assess the relative power of different players within a game: all other things being equal, a player is more powerful than another player if the expected distance between the equilibrium outcome and its ideal point is smaller than the expected distance for the other player.

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[3] At this point we focus on a unique equilibrium outcome only for expositional convenience. If the game does not have a unique equilibrium, but multiple equilibria, the simple Euclidean distance can be replaced by the average Euclidean distance, i.e. the sum of the Euclidean distances between each equilibrium outcome and the

2. Indices: Strategic Power and Inertia

A comparison of players' abilities over different games requires some standardization of the power measure. This can be done with reference to a dummy player, that is, a player whose preferences vary over the same range as the preferences of the actual players, but that has no decision-making rights in the game.[4] This player's preferences, therefore, do not matter for the outcome of the game. It only experiences some equilibrium outcome that is set by the other players. Sometimes the dummy player is 'lucky' in having an ideal point that is close to the equilibrium outcome. However, the dummy player may also be less fortunate and encounter a policy outcome that is quite different from its preferred option. Consequently, the mean distance found for this player represents a minimum value that can be associated with a 'powerless' player. The notion of a dummy player allows us to indicate the absolute positions of players in the game. Only players with shorter expected distances can be regarded as 'powerful'. All other players hold symbolic positions in the game in the sense that their decision-making rights generally do not affect the equilibrium outcome.

Treating the dummy player, d, in a similar way as the actual players (with an ideal point and the corresponding random variable), the expected distance between the dummy player's ideal point and the equilibrium outcome of a particular game based on procedure π can be defined as Δ_d^π. We then define the power of player i as

$$\Psi_i^\pi = \frac{\Delta_d^\pi - \Delta_i^\pi}{\Delta_d^\pi} = 1 - \frac{\Delta_i^\pi}{\Delta_d^\pi}$$

which is called the *strategic power index*. This index lies in the interval [0,1] and increases with the power of player i. The expected distance for a player that is 'powerful' enough to dictate the outcome of a game under any preference configuration would be zero, leading to a corresponding value for the index of one. By contrast, if a player has an effect on the outcome of a game, similar to that of the dummy player (which, by definition, is 'powerless'), the expected distance for this player is the same as for the dummy player, leading to a corresponding index value of zero.[5]

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[4] To "vary over the same range" means that the actual players' ideal points have identical supports, a reasonable assumption from an a priori point of view. Moreover, it is quite natural to assume dummy player's d's random ideal point being uniformly distributed on the feasible outcome space.

[5] Since the ideal points for each player are independent random variables, the equilibrium outcomes can never be systematically biased *against* the interest of a particular player, and, therefore, no player can fare worse than the dummy player. Thus, under our assumption the proposed index can never become negative. Of course, it is possible to construct examples, where the dummy player has lower expected difference than an actual player

Based on this index, there is a natural way to approach the status quo bias of the decision-making process, that is, the extent to which players are unable to act and to pull a new policy away from the current state of affairs. For a specific procedure, that status quo bias can be measured by the expected distance between the equilibrium outcome and the status quo, which is defined as Δ_q^π. Substituting this value for the expected distance found for a player in the strategic power index, we get

$$\Psi_q^\pi = \frac{\Delta_d^\pi - \Delta_q^\pi}{\Delta_d^\pi} = 1 - \frac{\Delta_q^\pi}{\Delta_d^\pi}$$

which is called the *inertia index*. A value of one for this index means that under some procedure the status quo always prevails. The smaller the value for the index, the more the players are able to move the equilibrium policy away from the status quo.[6]

3. Measuring Strategic Power

The strategic power and inertia indices are difficult to analytically manipulate, since they are based on the equilibrium outcomes of the underlying policy games. These outcomes may depend on both players' preferences and the status quo in a nonlinear way. However, it is relatively simple to calculate such indices numerically, given some assumption about the distribution of individual preferences. Assuming that the set of available policy options is finite, that is, the outcome space X contains a finite number of points, the number of states of the world will also be finite. More specifically, assume that there are a possible outcomes. With n actual players, a dummy player, and the consideration of the status quo, there will be $s = a^{n+2}$ possible states of the world $\xi_1..\xi_s$. It is then possible to generate a list of all possible states of the world for which equilibrium outcomes can be computed, based on a decision-making procedure. Subsequently, the expected distance between these outcomes and the ideal points of players can be determined, weighting each particular state of the world with the

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point of the dummy player always coincide with the ideal point of the most powerful actual player. Given this restriction, the expected distance of the dummy player necessarily coincides with the expected distance of the most powerful actual player. Using our formula would lead to index value zero for the most powerful player and to negative values for the other actual players. However, it seems to us that such a dummy player is *on average* as lucky as the Commission has been in the particular game represented by Fig. 1. Our formula has not been set up to be applied to those constellations of preferences, which in our view do not fit in the a priori framework.

[6] It is possible that the inertia index exceeds the power index of the most powerful actual player. If one would assume that the ideal point of the dummy player always coincides with the status quo one might conclude that the dummy player is the most powerful player according to our index. However, we should repeat (see footnote

probability of its occurrence.[7] We followed this approach in the application of both indices to legislative decision-making in the EU (see Steunenberg/Schmidtchen/Koboldt 1999).

III. The Veil of Ignorance: Vice or Virtue?

In two articles published in the Journal of Theoretical Politics 1999, Garrett and Tsebelis (1999a, 1999b) argue that conventional power indices are of limited value, since they do not take account of agenda-setting and other institutional features (see also Tsebelis and Garrett 1997). Holler and Widgrén (1999) share this view but not to the extent laid out in the first paper of Garrett and Tsebelis (1999a).

With regard to power index analyses of the European Union, Garrett and Tsebelis state that the absence of institutions in these analyses is a "sufficient condition for rendering this approach irrelevant to understanding both policy change on specific issues between treaty revisions and negotiations about treaty revisions themselves." (Garrett and Tsebelis 1999: 332.) Garrett and Tsebelis favor, as we do, the application of non-cooperative game theory to constitutional and institutional analyses. However, they restrict themselves to conducting what they call a deterministic institutional analysis; that is the analysis of a particular game (posteriori approach). Of course, one can undertake this kind of deterministic analysis but should realize that it fails to address issues, the analysis of which requires something like a veil of ignorance. In this respect we side with the position of Lane/Berg (1999) who argue that constitutional analysis explores modalities anticipated from behind a veil of ignorance. In fact, the strategic power index has to be considered as an attempt to combine the deterministic institutional analysis – as represented by the search for an equilibrium in any given state of the world – with the veil of ignorance by calculating the expected distance between the equilibrium outcome and each player's ideal point for all states of the world.

Garrett and Tsebelis concede that the new power measure addresses strategic considerations which are at the heart of institutional analysis and they state that "(i)t occupies an intermediate position between the probabilistic institution-free environment of conventional indices and the deterministic institutional analysis that we have conducted in our papers." (Garrett and Tsebelis 1999b: 334.) But, instead of viewing this as a virtue, they add "that the probabilistic

[7] For example, if we assume that player preferences are uniformly distributed and, hence, each state of the world has the same probability of occurrence, the expected distance will simply be

$$ {}^{s} \quad \delta_i^{\pi} $$

feature that SS&K (the strategic power index, the authors) shares with conventional power indices is a liability, not an advantage" (Garrett and Tsebelis 1999b: 334). We disagree because the kind of constitutional analysis we are interested in requires the recognition of probabilistic features. But then, only the specific way these probabilistic features are introduced in the model can be at issue rather than the logic of the strategic power index as such.

The Garrett and Tsebelis critique focuses on three points: (1) the assumption that all possible states of the world are equally likely; (2) the treatment of a collective body as a "unified actor"; and (3) the assumption of a one dimensional policy space (see Garrett and Tsebelis 1999b: 334 – 336).

We will discuss this criticism in turn. First, the assumption of a uniform distribution of the states of the world affects the computation of the strategic power scores. Although it remains to be proven that the distribution of power is actually affected by the assumption of a uniform distribution of the states of the world. There is still the possibility that the individual power scores change without changing the distribution. But even if the kind of probability distribution is relevant, the issue would be what is the right cumulative distribution function rather than what is the algorithm used for the computation. Consequently, the focus of the scientific debate should shift to the question of finding the right cumulative distribution function, which is basically an empirical question.

Second, when calculating the strategic power index for different legislative procedures in the European Union we considered the Parliament as a unified actor holding, with equal probability, a unique preferred position on any of the eight points in the policy space. If Parliament would be modeled as composed of several members, each of whom having the same probability of being located at any point in the policy space, then the position of the median matters (see Garrett and Tsebelis 1999b: 335): "But this median is no longer uniformly distributed. Rather it is distributed according to a beta distribution centered in the middle of the interval." (Garrett and Tsebelis 1999b: 335.) Again, this argument does not attack the logic of the strategic power index, rather it calls for a restriction on the set of the states of the world used for the computation of the index. Only the subset with the Parliament centered in the middle of the interval should be used.

This argument also applies to Garret and Tsebelis' critique in that we assumed that the ideal points of the members of the Council are uniformly (and independently) distributed on the

Tsebelis (1999b: 336) claim) are again distributed according to a beta distribution, then a further restriction on the set of the state of the world used to compute the index seems reasonable.

Third, in our computation we considered only a one-dimensional policy space. As Garrett and Tsebelis (1999b: 336) argue: "If they had considered two dimensions, there is no reason to believe that their results would have been the same, and this would also be true if one moved to a policy space of higher dimensionality." We agree. But again, this argument does not attack the logic of strategic power, rather it calls for its extension. However, problems arise if a game lacks an equilibrium or does not have a unique one. It is clear that the computation of the strategic power index is only possible on the basis of an equilibrium outcome identified for each specific game. However, we doubt whether games without any equilibrium are suitable candidates for constitutional analyses. The relevant games are, rather, games based on the concept of structure-induced equilibria (developed by Shepsle 1979). Rules, such as issue-by-issue voting, can induce a stable outcome (but not necessarily, see Ordeshook 1992: 286). If a game has multiple equilibria rather than a unique one, one can draw on the refinement literature (which, however, is rather disappointing) or one can replace the simple Euclidean distance by the average Euclidean distance, i.e., the sum of the Euclidean distances between each equilibrium outcome and the player's ideal point for all equilibria in a particular state of the world, divided by the number of equilibria.

Garret and Tsebelis try to make a case for resisting the temptation to apply power indices. They claim: "If power index analysis is to be analytically attractive for any reason, it is that the analysis generates numbers that summarize 'power' in some very general and robust sense." (Garrett and Tsebelis, 1999b: 336.) A look at the variety of results generated by different index calculations in the literature (see table 3 in Steunenberg/Schmidtchen/Koboldt 1999) reveals that power index calculations are sensitive to underlying assumptions. To be sure, in a priori analyses concerned with decision-making behind a veil of ignorance or with incomplete contracting (see Holler and Widgrén 1999), measuring power in a "very general and robust sense" turns out to be difficult. Nevertheless, we should resist the temptation to abandon the whole exercise on the ground that assumptions always matter. The task ahead simply is to find the reasonable assumptions.

IV. Strategic Power = Banzhaf Power?

1. The theorem

In their comment on the Symposium Power Indices and the European Union in the *Journal of Theoretical Politics* Felsenthal and Machover argue that strategic power is simply the Banzhaf power multiplied by a constant that depends on the shape of the state space (see Felsenthal and Machover 2000).

Consider a simple voting game *W*. Let S denote a state space which is totally symmetric.

X_i, ...X_n, Y, Z	are independent random variables, all of which take their values in the state space.
X_i, Y, Z	stand, respectively, for the ideal point of player i, state if a proposed bill will be passed and status quo (state continue to prevail if policy proposal is defeated).

Let R and r, respectively, denote the greater and smaller of the two distances ½X_i - Y½ and ½X_i - Z½ Then the distance can be defined as

$$D_i = (1-p) ? R + p ? r ,$$

with *p* the probability that i's voting decision agrees with the outcome of the vote.

Using Penrose's theorem, which state

$p = \frac{1+\beta'(W)}{2}$, with β'[*W*] the Banzhaf,

one can define the mean value of D_i, denoted $\Delta_i[W]$,

$\Delta_i[W] = \frac{1-\beta'[W]}{2} ? R + \frac{1-\beta'[W]}{2} ? r$, for player i, and

$\Delta_d[W] = \frac{R+r}{r}$ for the dummy player. This gives

$$\Psi_i[W] = \frac{R-r}{R+r} ? \beta_i^{'}[W] .$$

Felsenthal/Machover (2000): "Thus $\Psi_i[W]$ is simply the Bz power of i multiplied by a constant that depends on the shape of S. Note, in particular, that in the simplest possible case, where S consists of just two points, r is clearly 0, so in this case $\Psi_i[W] = \beta_i^{'}[W]$ exactly."

2. Evaluation

We welcome the Felsenthal and Machover approach. It forms a very interesting foundation of the approach presented here, which would allow the strategic power index to be fully characterized by the set of the axioms the Banzhaf is founded on. This axiomatic characterization would facilitate comparisons with other power measures. Although the theorem proved by Felsenthal and Machover provides for important insights into the logic of the strategic power index, two comments seem in order.

First, we agree that Felsenthal and Machover succeeded in reformulating the algorithm of the strategic power index as far as simple voting games are concerned. Simple voting games take the proposals to be voted upon as exogeneously given. Thus, they can be treated – as in Felsenthal and Machover – as a random variable. However, the most important feature of the strategic power index namely the strategic interaction is not taken account of. The bills proposed are not randomly chosen but are the result of strategic thinking along the subgame perfect equilibrium path. To illustrate. Consider a decision-making procedure used in the European Union. With regard to legislative decision-making, the EC Treaty initially provided only for the unanimity version of the consultation procedure. This procedure allowed the Commission to propose new regulations or directives, which are subjected to unanimous consent by the Council. The latter implies that, in fact, each Council member has the right to veto the Commission's proposal. The European Parliament only needs to be consulted in this procedure. Since the Council can adopt a proposal regardless of the position Parliament takes, Parliament does not play a significant role and thus will not be discussed further.

Now assume that policies can be represented by a one-dimensional (left-right) outcome space and players have Euclidean preferences. In addition, assume that players have perfect and complete information. The Commission selects a proposal, which is then decided upon by the Council members. We assume that Council members are not allowed to add new proposals to the agenda or to amend the Commission proposal. The interactions between the Commission and Council members now resemble the well-known agenda-setter model of Romer and Rosenthal (1978, 1979).

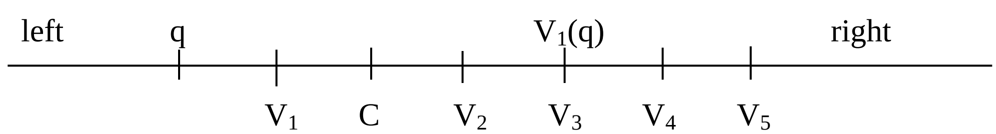

Figure 1: Preferences of the Commission and the Council Members

Figure 1 presents a preference configuration that may occur for the Commission, which is conceived as a unitary actor, and a five-member Council. In this Figure V_i and C denote the most preferred or ideal points of Council member i and the Commission, respectively, and $V_i(q)$ stands for member i's point of indifference to the status quo q. The Commission, C, has a more progressive preference than most Council members, V_i. Nevertheless, the leftmost Council member, V_i, holds an even more extreme position. Given a status quo to the left of these players, the Commission will propose a measure that is equivalent to its own most preferred point. Since all Council members prefer this point to the status quo, the proposal will not be vetoed. So, in equilibrium, the outcome of this game is a legislative policy x = C.[8]

Here, the outcome of a specific sequential game is partly due to the value of the random variables and partly the result of strategic thinking on the side of all players. It is natural to think about how to introduce this factor in the Felsenthal and Machover set up. One can take account of strategic thinking by restricting the domain of proposed bills in the state space. The question is whether we can find some reasonable equivalent to the equilibrium concept used in non-cooperative game theory.

Second, Felsenthal and Machover are of the opinion that the strategic measure "is a natural generalization of a priori I-power, which allows the incorporation of additional information, and thus the study of a posteriori voting power" (Felsenthal and Machover 2000: 16). They argue that the notion of I-power is not fundamentally game theoretic and that the voting power of a voter has nothing whatsoever to do with payoffs: "Rather, a voter's I-power depends only on the structure of the SVG itself, which contains no information about any payoffs. (So from the viewpoint of I-power, it is not really a game in the true game-theoretic sense, which requires payoffs to be specified.)" (Felsenthal and Machover 2000: 5.)

We do not want to discuss whether the strategic measure belongs to the I-power or the P-power camp. Rather, we want to stress that the strategic power index *is* fundamentally game theoretic. Note that is based on games, not on game forms. Furthermore, the reformulation of the strategic power index, as presented by Felsenthal and Machover, is based

[8] In this context, all players have to approve a measure, and no measure can be taken without the support of each one of them. Each (last) player has the same probability of being pivotal, and each player is necessary to form the (minimum) winning coalition of all players. The Shapley-Shubik, Banzhaf, Johnston and Holler indices therefore allocate power values of 1/6 to each player. Following Berg and Lane (1997) and Turnovec (1997), the aggregated score for the Council would be 5/6, which implies that the Council would be *more* powerful than the Commission. However, the abilities of these players to affect the equilibrium outcome differ. The Commission can take the initiative and draft a proposal, while Council members can only approve or reject this proposal. Council members may restrict the Commission's policy choice, but they cannot set the final proposal. The

on payoffs, since one cannot calculate differences without knowing the ideal points for all players. It is implicitly assumed that voters care about distances and that decisions are (rationally) determined by the distance of the ideal point from both the proposed bill and the status quo. These distances are utility measures.

V. Widgrén/Napel: The Power of a Spatially Inferior Player

Widgrén and Napel claim to take an alternative road to strategic power. Distinguishing between inferior and non inferior players, they have constructed a strategic power index – labeled strict power index – which has spatial preferences and strategic agenda setting as its main building blocks (see Widgrén and Napel 2001: 18). They recognize that in Steunenberg et. al. (1999) a different strategic power index is introduced. The latter, however, is judged inferior compared to the strict power index for three reasons:

1. The strategic power index suffers from a power-luck confusion: " This measure, contrary to what we propose here, defines power as proximity between one's ideal point and the outcome of the game. But, proximity may be due to luck and, indeed, in this paper we demonstrate that under strategic agenda setting players whose ideal points are located close to the outcome tend to have luck, not power." (Widgrén and Napel 2001: 18 – 19).
2. The strategic power index follows external rather than internal normalization.
3. Defining power "as one's ability to change the current state of affair", as proposed by Widgrén and Napel, allows for "different informational considerations and makes the analysis more procedural than in the case of StPI." (Widgrén and Napel 2001: 4).

We will discuss this criticism in turn. As for the power-luck confusion, we should mention that we, first, identified the problem and, second, presented a solution. Consider what we wrote in 1999:

Figure 1 also illustrates the importance of distinguishing 'power' from 'luck'. The equilibrium outcome of the game is x = C, that is, the most preferred position of the Commission. This outcome seems to be more favorable to Council member 2 than member 5, since the distance to V_2 is less than the distance to V_5. Is member 2 therefore also more powerful? Both players have the same abilities to affect the outcome, that is, to veto the Commission proposal. So, from this perspective, there is no difference in power. Nevertheless, the outcome is closer to member 2's preferences. This indicates that member 2 is more 'lucky' than member 5. Having a preference that lies close to the equilibrium outcome of a particular game does not

Council member 1 is more 'powerful' than the other Council members, since this player defines the boundary, $V_1(q)$, where the Commission can no longer select its ideal point, should this player move to the right. If the position of this member can also be occupied by any other player, or the status quo can be located at any other point along the policy dimension, Council member 1 is just more 'lucky' than the others. Following Barry (1980), we regard a player's *success*, which is defined as the extent to which the outcome of the decision-making process corresponds to its ideal point, as the composite effect of 'power' and 'luck'. Part of a player's success is therefore based on 'luck', the other part is due to the 'power' a player exerts.

While power can be associated with a player's ability to affect the final outcome (which is basically a matter of the rules of the game telling us *who* can do *what* and *when* and who gets *how much* when the game is over (see Binmore, 1992:25)), 'luck' is related to the preferences of the players and the location of the status quo, which are assumed to be exogeneously determined. The latter can be illustrated by the role of the Commission in our example of the consultation procedure. The fact that the outcome of the game coincides with the Commission's most preferred point does not imply that the other players in the game are 'powerless'. This result depends on the preferences of the Council members and the location of q. A shift of V_1 to the left may, for instance, force the Commission to propose a policy $x = V_1(q)$. Thus, given the preference configuration, the Commission is 'lucky' that Council members have preferences that allow for the equilibrium outcome $x = C$. This clearly indicates that the *success* of a player in a game is the combined result of *abilities* (defined by the rules of a game) and the specific *preference configuration*. To assess a player's power, a measure should be based on the former and not the latter. How to facilitate this?

As we wrote: "To distinguish 'power' from 'luck', we propose a measure that is independent of the specific preferences of players, which, together with the decision-making procedure, determines the outcome of the game. This can be achieved by measuring a player's power under some decision-making procedure with reference to the <u>mean</u> <u>or</u> expected distance between the equilibrium outcome and this player's ideal point for all possible combinations of players' preferences and all possible combinations of the status quo." In doing so, the power-luck confusion vanishes. The fact that our power scores turned out to be sensitive to a change of the decision-making procedures (all other things being equal) gives further support to this conclusion.

The strategic power index is normalized by the introduction of a dummy player. As Widgrén and Napel rightly mention this player is not a true player but rather an outside observer. The question is whether or not the type of dummynazation matters. We feel it does not. Consider a 3-player simple game where the only winning coalitions are the grand coalition ABC and the two coalitions AB and AC (this example is from Widgrén and Napel 2001: 1-2). Looking at this game as a coalitional form game the Banzhaf and Shapley-Shubik power vectors are $(\frac{3}{5}, \frac{1}{5}, \frac{1}{5})$ and $(\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$, respectively.

From the point of view of non-cooperative game theory – following Widgrén and Napel – the game can be looked at as a sequential game, in which A makes an ultimatum offer to B, asking for approval in return for an only marginal (and in the limit non-extent) concession to B's interest (see Widgrén and Napel 2001: 1). A rational player B would have to accept the proposal (the same holds for C). Thus, we would conclude that B and C are powerless in this game (which is supported by the fact that the core and the nucleolus of this game are both {(1, 0, 0)} (see Widgrén and Napel 2001: 2).

Application of the machinery of the strategic power index would lead to the same result. Consider a policy space with three possible outcomes and identical distance, denoted δ, between two neighboring outcomes; Player set {A, B, C} and D as dummy player. The ideal points are uniformly distributed on the policy space. Fig. 2 shows one of the feasible preference constellations.

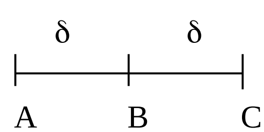

Fig. 2: Preference constellation

Translating the notion of an ultimatum game to our setting means that, whatever the distribution of ideal points (preference profile) of players A, B, C, the policy outcome always corresponds to A's ideal point. Thus, A's power score $\Psi_A = 1$. Since we assumed that the probability distribution of D's ideal points is the same as those of B and C, D's expected distance equals those of B and C: $\Delta_B = \Delta_C = \Delta_D$. Thus $\Psi_B = \Psi_C = 0$.

In our view strategic power refers to the ability of a player to make a difference in the outcome of a policy game. Widgrén and Napel define strategic power as "one's ability to

be either the status quo or the equilibrium outcome. In both cases, power is defined in regard to a particular game. Then, however, it is impossible to distinguish power and luck. As already mentioned, the term should be used in the sense of a priori power. But then, the ability to change "the current state of affair", if interpreted as equilibrium of a game, is perfectly captured by the strategic power index. We cannot see the improvement coming from the strict power index. If, however, "current state of affair" refers to the "status quo", the ability to change it is also implied by the strategic power index. At the same time, the basic logic of this index can be used to measure the status quo bias, see the inertia index. But note, the inertia index does <u>not</u> measure the power of a player, but rather a feature of the rules of a game.

It is not clear to us why the approach adopted by Widgrén and Napel allows for "different informational considerations and makes the analysis more procedural than in the case of StPI", i. e. our strategic power index. We applied the index to games of perfect and complete information. But the applicability is not restricted to those games. We could as well analyze games of imperfect and incomplete information, or repeated games or nested games (see Tsebelis). All we need to calculate power is the notion of ideal points and the identification of equilibrium in games.

We also cannot see why the Widgrén and Napel approach makes the analysis more procedural than our approach. On the contrary, we would maintain that our approach includes all procedural aspects of a game (by the way, what does it mean to make an analysis "more" procedural?) and, more important, is applicable to a much wider class of games. Widgrén and Napel are concerned with voting games. The strategic aspects of a simple voting game with an agenda setter, however, are rather weak. Basically, it is the agenda setter who decides on the policy proposal, taking the preference constellation and the status quo as given when doing the backward induction. He is in a position of a first mover, quite similar to a Stackelberg leader. Strategic aspects would play a more important role in voting games of imperfect or incomplete information. We feel that, in contrast to the Widgrén and Napel approach the domain of which is restricted to simple voting games, our approach can be applied without restriction to all kinds of games. To repeat: All we need is equilibrium.

VI. Conclusions

In this paper we have discussed a new method for evaluating the distribution of power in policy games. In contrast to voting power indices, which are based on the theory of simple games, the new approach is based on a modeling of decision-making processes as a non-

distance between the equilibrium outcome and the ideal points of players as a proxy for their power. Focusing on equilibrium outcomes, the proposed method avoids two drawbacks of the traditional approach, based on voting power indices, which neglects the preferences of players on concrete political issues and the more complex institutional structure of the decision-making process.

The strategic power index, as proposed in this paper, refers to the ability of a player to make a difference in the outcome of a policy game. This index has many desirable features. First, the proposed index can be based on a careful and detailed analysis of some decision-making process in which the preferences of all players and all relevant institutional complexities are taken into account. Second, like the voting power indices, the strategic power index measures *a priori* power. Third, the index levels out the effect of 'luck' or a particular preference configuration on the outcome of a game, since it is based on mean distances. Fourth, the index measures power in absolute terms, since it relates the position of a player in a game to a dummy player, who is by definition 'powerless'. Finally, the strategic power index provides a unified method to study the composite edifice of a priori and a posteriori power as a whole (a requirement put forward by Felsenthal and Machover). But note our interpretation of a posteriori power: It simply refers to the distance of an equilibrium outcome and the ideal point of a player in a fully specified game.

Constitutional analyses try to identify the power of the players anticipated from behind a veil of ignorance. In order to calculate the strategic power one must fix a probability distribution for the random variables of a model. Any critique directed at such a specific assumption does not hit the logic of strategic power, rather it suggests searching for reasonable assumptions.

In the case of simple voting games lacking a sequential structure, strategic power can be reformulated as modified Banzhaf power. It remains to be shown that such a reformulation is also possible for games with a more general structure. By incorporating preferences, beliefs, information, and institutions, the strategic power measure provides a unified method which allows the study of the interplay between *a priori* and *a posteriori* (actual) power.

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