Strategic Power Revisited*

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Abstract

Traditional axiomatic indices of decision power in political and economic institutions fail to take agents' preferences and strategic interaction adequately into account. Equilibrium analysis of particular procedures modelled as non-cooperative games is unsuitable for normative analysis and assumes information which is typically not available. These have been main arguments in a lingering debate about which approach to power analysis is the right one. A unified framework that works both sides of the street is developed here. It rests on a notion of a posteriori power which formalizes players' marginal impact or contribution to outcomes in cooperative and non-cooperative games, for sophisticated strategic interaction and purely random behaviour. Taking expectations with respect to preferences, actions, and procedures then defines a meaningful a priori measure. The framework can distinguish between different notions of objective and subjective power. Established indices turn out to be special cases.

Keywords: Power indices, spatial voting, equilibrium analysis, decision procedures

1 Introduction

Scientists who study decision power in political and economic institutions are divided into two disjoint methodological camps. The first one uses non-cooperative game theory to analyse the impact of explicit decision procedures and given preferences over a well-defined – usually Euclidean – policy space.[1] The second one stands in the tradition of cooperative game theory with much more abstractly defined voting bodies: The considered agents a priori have no preferences and form winning coalitions which a posteriori implement unspecified policies. Individual chances of being part of and influencing a winning coalition are then measured by a *power index*.[2]

Proponents of either approach have recently intensified their debate.[3] The non-cooperative camp's verdict is that "power indices exclude variables that ought to be in a political analysis (institutions and strategies) and include variables that ought to be left out (computational formulas and hidden assumptions)" (Garrett and Tsebelis, 1999a, p. 337). The cooperative camp has responded by clarifying the supposedly hidden assumptions underlying power formulas and giving reasons for not making institutions and strategies – corresponding to decision procedures and rational, preference-driven agents – explicit.[4]

Several authors, including ourselves, have concluded that it is time to develop a *unified framework* for measuring decision power (cf. Steunenberg et al., 1999, and Felsenthal and Machover, 2001a).[5] On the one hand, such a framework should allow for predictions and

[1]See e.g. Steunenberg (1994), Tsebelis (1994, 1996), Crombez (1996, 1998), Moser (1996, 1997).

[2]See e.g. Brams and Affuso (1985a, 1985b), Widgrén (1994), Hosli (1993), Laruelle and Widgrén (1998), Baldwin et al. (2000, 2001), Felsenthal and Machover (2001b), Leech (2002) for applications of traditional power indices to EU decision making. Felsenthal and Machover (1998) and Nurmi (1998) contain a more general discussion regarding the cooperative analysis of the EU.

[3]Cf. the contributions to the symposium in *Journal of Theoretical Politics* 11(3), 1999, together with Tsebelis and Garrett (1997), Garrett and Tsebelis (2001), and Felsenthal and Machover (2001a).

[4]See, in particular, Holler and Widgrén (1999), Berg and Lane (1999), and Felsenthal and Machover (2001a).

[5]Gul (1989) and Hart and MasColell (1996) give non-cooperative foundations for the Shapley value and thus the prominent Shapley-Shubik index.

a posteriori analysis of decisions based on knowledge of procedures and preferences. On the other hand, it must be open to a priori analysis when detailed information is either not available or should be ignored for normative reasons. Unfortunately, the first attempt to provide such a framework (Steunenberg et al., 1999) is problematic. It confuses cause and effect, confounding power and the success that may, but need not, result from it. This paper reviews their attempt and proposes an alternative framework.

In particular, we generalize the concept of a player's *marginal impact* or *marginal contribution* to a collective decision in order to establish a common (a posteriori) primitive of power for cooperative and non-cooperative analysis. Its evaluation amounts to the comparison of an actual outcome with a *shadow outcome* which alternatively could have been brought about by the considered player. In our view, this goes a long way towards a reconciliation of equilibrium-based non-cooperative measurement and winning coalition-based traditional power indices. Namely, one can transparently measure a priori power as a player's expected a posteriori power or marginal impact. This is in line with the probabilistic interpretation of traditional power indices (cf. Owen 1972, 1995 and Straffin, 1988). Expectation is to be taken with respect to an appropriate probability measure on the power-relevant states of the world. The framework is flexible and allows for different degrees of a priori-ness, concerning players' either preference-based or purely random actions as well as decision procedures. Traditional a priori power indices, such as the Penrose index or Shapley-Shubik index, can be obtained as special cases.

The cooperative or index approach to power analysis has evolved significantly in the last 50 years. It has reached a point where its integration into a framework that also allows for explicit decision procedures and preference-driven agent behavior is but a natural step. To demonstrate this we first give a short overview of the index approach in section 2. The fundamental critique by Garrett and Tsebelis and the creative response by Steunenberg et al. are sketched in sections 3 and 4. The main section 5 then lays out our unified framework and section 6 concludes.

2 The Traditional Power Index Approach

The traditional object of studies of decision power has been a *weighted voting game* characterized by a set of players, $N = \{1, \ldots, n\}$, a voting weight for each player, $w_i \geq 0$ $(i \in N)$, and a minimal quota of weights, $k > 0$. Subsets of players, $S \subseteq N$, are called *coalitions*, and if a coalition S meets the quota, i. e. $\sum_{i \in S} w_i \geq k$, it is a *winning coalition.* Formation of a winning coalition is assumed to be desirable to its members, e. g. because they can jointly pass policy proposals that are in their interest. More generally, a winning coalition need not be determined by voting weights. One can conveniently describe an abstract decision body v by directly stating either the set $W(v)$ of all its winning coalitions or its subset of *minimal winning coalitions*, $M(v)$.[6] The latter contains only those winning coalitions which are turned into a losing coalition by the exit of any of its members. An equivalent representation is obtained by taking v to be a mapping from the set of all possible coalitions, $\wp(N)$, to $\{0, 1\}$, where $v(S) = 0$ (1) indicates that S is winning (losing). Function v is usually referred to as a *simple game.* The difference $v(S) - v(S - \{i\})$ is known as player i's *marginal contribution* to coalition S.

The most direct approach to measuring players' power is to state a mapping μ – called an index – from the space of simple games to $\mathbb{R}^n_+$ together with a verbal story of why $\mu_i(v)$ indicates player i's power in the considered class of decision bodies. The main drawback of this approach is that despite the plausibility of some "stories", their verbal form easily disguises incoherence or even inconsistency.

The *axiomatic* or *property-based approach*, in contrast, explicitly states a set of mathematical properties $\{A_1, \ldots, A_k\}$ that an index is supposed to have – together with an (in the ideal case: unique) index μ which actually satisfies them. The requirements A_j are usually referred to as *axioms*, unwarrantedly suggesting their general acceptance. A prominent example for the axiomatic approach to power indices is the Shapley-Shubik index ϕ (cf. Shapley, 1953, and Shapley and Shubik, 1954). Though this may not be im-

[6]Typically, one requires that the empty set is losing, the grand coalition N is winning, and any set containing a winning coalition is also winning.

mediately obvious, four properties A_1–A_4 imply that $\phi_i(v)$ must be player i's weighted marginal contribution to all coalitions S, where weights are proportional to the number of player orderings $(j_1, \ldots, i, \ldots, j_n)$ such that $S = \{j_1, \ldots, i\}$.[7] Axioms can give a clear reason of why ϕ and no other mapping is used – in particular, if a convincing story for them is provided. A main drawback of the axiomatic approach is, however, that axioms usually clarify the tool with which one measures,[8] but not what is measured based on which (behavioural and institutional) assumptions about players and the decision body.

In contrast, the *probabilistic approach* to the construction of power indices entails explicit assumptions about agents' behaviour together with an explicit definition of what is measured. Agent behaviour is specified as a probability distribution P for players' *acceptance rates*, denoting the probabilities of a 'yes'-vote by individual players. A given player's a priori power is then taken to be his probability of casting a *decisive vote*, i. e. to pass a proposal that would not have passed had he voted 'no' instead of 'yes'. Thus power is inferred from the *hypothetical consequences of an agent's behaviour.*

The object of analysis is with this approach no longer described only by the set of winning coalitions or the mapping v, but also an explicit model of (average) behaviour. For example, the widely applied Penrose index (Penrose, 1947) – also know as the non-normalized Banzhaf index – is based on the distribution assumption that each player independently votes 'yes' with probability 1/2 (on an unspecified proposal). The corresponding joint distribution of acceptance rates then defines the index β where $\beta_i(v)$ turns out to be, again, player i's weighted marginal contribution to all coalitions in v, where weight is this

[7] The Shapley-Shubik index is uniquely characterized by the requirements that a (dummy) player who makes no marginal contribution in v has index value 0 (A_1: dummy player axiom), that the labelling of the players does not matter (A_2: anonymity axiom), that players' index values add up to 1 (A_3: efficiency axiom), and that in the composition $u \vee v$ of two simple games u and v, having the union of $W(v)$ and $W(v)$ as its set of winning coalitions $W(u \vee v)$, each player's power equals the sum of his power in u and his power in v minus his power in the game $u \wedge v$ obtained by intersecting $W(v)$ and $W(u)$ (A_4: additivity axiom).

[8] Even this cannot be taken for granted. Axioms can be too general or mathematically complex to give much insight.

time equal to $1/2^{n-1}$ for every coalition $S \subseteq N$.[9]

The Penrose index allows for important conclusions related to the design of political institutions. For example, it can be derived that in order to give every citizen of the EU the same a priori chance to (indirectly) cast the decisive vote on Council decisions, weights in the Council have to be such that each member state's index value is proportional to the square root of its population size.[10]

Where the direct approach and the axiomatic approach required "stories" to justify the index μ itself and the set of axioms $\{A_1, \ldots, A_k\}$, respectively, the assumption of a particular distribution P of acceptance rates has to be motivated. This points towards the drawbacks of the probabilistic index approach. First, the described behaviour is usually not connected to any information on agent preferences or decision procedures.[11] Second, decisions by individual players are assumed to be stochastically independent. This will, in practice, only rarely be the case since it is incompatible with negotiated coalition formation and voting on issues based on stable player preferences.

3 The Critique by Garrett and Tsebelis

Probabilistically defined power indices are a flexible tool. Imposing stochastic or deterministic restrictions for coalitions containing particular players or sub-coalitions can alleviate several shortcomings of traditional indices (see van den Brink, 2001, or Napel and Widgrén, 2001). This does not impress the critics of traditional power indices. Concerning applications to decision making in the European Union (EU), Garrett and Tsebelis (1999a, 1999b, and 2001) have taken a particularly critical stance, pointing out indices' ignorance

[9] This means that $\beta_i(v)$ is the ratio of the number of swings that player i does have to the number of swings that i could have. Alternatively, one can also derive β from the assumption that players' acceptance rates are independently uniformly distributed on $[0, 1]$.

[10] Weights do, in fact, approximately respect this rule – except for the biggest and smallest member states, Germany and Luxemburg.

[11] If such information is not available, the principle of insufficient reason seems a valid argument for the assumptions behind the Penrose index.

of decision procedures and player strategies.

It is not our purpose to contribute much to the ongoing debate sparked by their critique itself (see Berg and Lane, 1999, Holler and Widgrén, 1999, Steunenberg, Schmidtchen, and Koboldt, 1999, and Felsenthal and Machover, 2001a). Let it suffice to state that, first, we agree that institutions and strategies have to be taken into account by political analysis. Second, in our view both the normative or constitutional a priori analysis of political institutions and the positive or practical political analysis of actual and expected decisions are valuable.

There is a fundamental difference between giving normative reasons for or against particular voting weights, veto rules, etc. in European decision making on the one hand, and providing decision support e. g. to Mrs Thatcher during negotiations of the Single European Act on the other hand.

It is a legitimate question to ask: Which voting weights in the Council would be equitable? For an answer, countries' special interests and their potentially unstable preferences in different policy dimensions should not matter. Hence they are best concealed behind a 'veil of ignorance' – as it is done by the Penrose index. Another case for the purely probabilistic assumptions of traditional index analysis is e. g. the study of the susceptibility of multi-stage indirect decision procedures to the 'Bush effect', i. e. one candidate's indirect election despite a nation-wide majority for a competitor. Minimization of the probability that upper-level decisions are taken against a majority at a lower level should not be conditioned to any reality-informed preference assumptions.

However, evaluation of the medium-run expected influence on EU policy from a particular country's point of view will be more accurate if available preference information is taken into account.[12] Garrett and Tsebelis's goal of "understanding . . . policy changes on specific issues . . . and negotiations about treaty revisions" (Garrett and Tsebelis, 1999b, p. 332)

[12]The 'medium run' can, however, be short-lived: A social democratic Council member can quickly turn into a right-wing conservative through elections. News about the first national or another foreign outbreak of mad-cow disease can quickly change voters' and a government's views on agricultural, health, or trade policy.

or, in general, “understanding decision-making in the EU” (Garrett and Tsebelis, 2001, p. 105) seems impossible to achieve by looking *only* at the voting weights of different EU members and the relevant qualified majority rule. The traditional power index approaches outlined in section 2 are not the right framework to discuss these positive questions related to power.

We disagree with Garrett and Tsebelis’ (2001) call for “a moratorium on the proliferation of index-based studies” (p. 100) and take seriously what they dismiss as “the mantra of index scholars about the importance of ‘a prioristic constitutional considerations’ ” (p. 103). As has already been pointed out by Felsenthal and Machover, it is a matter of taste whether one deems the pursuit of positive or normative analysis more worthwhile. We believe in both and agree with Garrett and Tsebelis that the strategic implications, which are hard to separate from players’ preferences, of particular institutional arrangements in the EU and elsewhere have received too little attention. We think it desirable to have a general unified framework which allows for positive and normative analysis, actual political and constitutional investigations.

4 The Strategic Power Index of Steunenberg et al.

Replying to the critique by Garrett and Tsebelis, Steunenberg et al. have proposed what they believe to be a framework which could reconcile traditional power index analysis and non-cooperative analysis of games which explicitly describe agents’ choices in a political procedure and (their beliefs about) agents’ preferences. Steunenberg et al.’s starting point is Barry’s (1980) distinction between ‘power’ and ‘luck’. Namely, a player who is particularly satisfied by the equilibrium outcome of a game – interpreted as success in the game – need not be powerful. Steunenberg et al. strive to isolate “the ability of a player to make a difference in the outcome” (p. 362).

They consider a spatial voting model with n players and an m-dimensional outcome space. In our notation, let $N = \{1, \ldots, n\}$ be the set of players and $X \subseteq \mathbb{R}^m$ be the outcome or policy space. Γ denotes the procedure or game form describing the decision-

making process and $q \in X$ describes the status quo before the start of decision-making. Players are assumed to have Euclidean preferences where $\lambda_i \in X$ $(i \in N)$ is player i's ideal point. A particular combination of all players' ideal points and the status quo point defines a 'state of the world' ξ. Assuming that it exists and is unique, let $x^*(\xi)$ denote the equilibrium outcome of the game based on Γ and ξ.[13] Steunenberg et al. note that "[h]aving a preference that lies close to the equilibrium outcome of a particular game does not necessarily mean that this player is also 'powerful'" (p. 345). Therefore, they suggest to consider not one particular state of the world but many.

In particular, one can consider each λ_i and the status quo q to be realizations of random variables $\tilde{\lambda}_i$ and $\tilde{q}$, respectively. If P denotes the joint distribution of random vector $\tilde{\xi} := (\tilde{q}, \tilde{\lambda}_1, \ldots, \tilde{\lambda}_n)$, then

$$\Delta_i^\Gamma := \int \left\| \tilde{\lambda}_i - x^*(\tilde{\xi}) \right\| dP \tag{1}$$

gives the expected distance between the equilibrium outcome for decision procedure Γ and player i's ideal outcome ($\|\cdot\|$ denotes the Euclidean norm). Steunenberg et al. believe that "all other things being equal, a player is more powerful than another player if the expected distance between the equilibrium outcome and its ideal point is smaller than the expected distance for the other player" (p. 348). In order to obtain not only a ranking of players but a meaningful measure of their power, they proceed by considering a dummy player d – either already one of the players or added to N – "whose preferences vary over the same range as the preferences of actual players." This leads to their definition of the *strategic power index (StPI)* as

$$\Psi_i^\Gamma := \frac{\Delta_d^\Gamma - \Delta_i^\Gamma}{\Delta_d^\Gamma}.$$

The remainder of Steunenberg et al.'s paper is then dedicated to the detailed investigation of particular game forms Γ which model the consultation and cooperation procedures of EU decision making. They derive the subgame perfect equilibrium of the respective policy game for any state of the world ξ, and aggregate the distance between players' ideal

[13]Non-uniqueness may be accounted for by either equilibrium selection criteria or, in a priori analysis, explicit assumptions about different equilibria's probability.

outcome and the equilibrium outcome assuming independent uniform distributions over a one-dimensional state space X for the ideal points and the status quo. The uniformity assumption is not as innocuous at it may seem. It is sensitive to the chosen mapping of reality to states, i. e. one's choice of X. Garrett and Tsebelis (2001, p. 101) claim that "the costs [of this simplifying assumption] are very high when we know ... that some Council members tend to hold extreme views (such as the UK) whereas others are generally more centrally located (such as Germany)." However, the calculations could quite easily be re-done with a more complex distribution assumption. The more important questions are: Is Ψ_i^Γ informative? Does it measure what it is claimed to do?

The answer to the second question is no. In particular, (1) defines Δ_i^Γ to be player i's *expected success*. Just like actual distance measures success (a function of luck and power), so does average distance measure average success. Unless one regards average success as the defining characteristic of power, considering expectations will only by coincidence achieve what Steunenberg et al. aim at, namely to "level out the effect of 'luck' or a particular preference configuration on the outcome of a game" (p. 362).

As a first example, consider three agents and outcome space $X = \{-1, 0, 1\}$. Assume that voter 1's random ideal point, $\tilde{\lambda}_1$, is degenerate and always equal to 0, while $\tilde{\lambda}_i$ is uniformly distributed on X for $i \in \{2, 3\}$.[14] The status quo is always $\tilde{q} = 0$. Let the procedure consist of pair-wise simple majority decisions between all alternatives, and then a final vote against the status quo. For example, let players start with a vote between -1 and 0, then there is a vote between the winning alternative and 1, and finally they vote between the winning alternative of the last stage and $q = 0$. Suppose that each player has equal voting weight. Then there are only two out of nine states of the world in which the status quo does not prevail (namely, $\tilde{\xi} = (\tilde{q}, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3)$with $\tilde{q} = 0$, $\tilde{\lambda}_1 = 0$, and either $\tilde{\lambda}_2 = \tilde{\lambda}_3 = -1$ or $\tilde{\lambda}_2 = \tilde{\lambda}_3 = 1$). Average distance from the equilibrium outcome is 2/9 for player 1 but 4/9 for players 2 and 3, i. e. player 1 turns out to be the most successful and,

[14]The point is not that $\tilde{\lambda}_1$ is a constant, but that its distribution is more concentrated around the mean of X than those of $\tilde{\lambda}_i$ $(i \neq 1)$. Alternatively, choose e. g. uniform distribution on [0,1] for all $\tilde{\lambda}_i$ with $i \neq 1$ and Beta-distribution with parameters $(n+1)/2$ and $(n+1)/2$ for $\tilde{\lambda}_1$.

according to the StPI, most powerful. This may match the intuition that centrist players are more powerful under simple majority rule than players with (sometimes) more extreme positions. However, exactly the same equilibrium outcomes prevail when player 1's voting weight is reduced to zero, i. e. if he becomes a dummy player.[15] According to Steunenberg et al.'s Strict Power Index, he is still the most powerful player.

To give a second, less theoretical example, consider a group of four boys. The oldest one is the leader and makes proposals of what to do in the afternoon (play football, watch a movie, etc.) which have to be accepted by simple majority of the remaining three. Given a particular state of the world – determined by weather, the boys' physical condition, pocket money, etc. – some group member will more enjoy their afternoon programme than another. As Steunenberg et al. would acknowledge, the former boy is more lucky, not more powerful than the latter. Assume that all boys' preferences have the same distribution, but that they are independent of each other (for simplicity, let them draw unrelated conclusions about the desirability of football when it is raining). As the agenda setter, the oldest boy will have smaller average distance to the equilibrium outcome than the others. Amongst the latter, expected distance and hence their StPI value is the same. Now, enter the little brother of the group's leader. He is not given a say in selecting the daily programme but – generously – allowed to participate. It is plausible to assume that he does not always agree with his elder brother's most desired outcome, but does so more often than with the others' ideal alternatives. Mathematically speaking, let the ideal points of the two brothers be positively correlated. Then, the mean distance between the group's equilibrium activity and its youngest member's most desired recreation will be smaller than that of those group members who actually have their vote on the outcome.

For a final, real-world example note that the European Economic Community's Treaty of Rome gave Luxemburg insufficient voting weight to ever be decisive.[16] Sharing very

[15]For an even number of players, let the status quo win unless defeated by a majority.

[16]This was changed in 1973, when the UK, Ireland and Denmark joined the EC. The relative voting weight of Luxemburg was decreased but it became more powerful when measured by traditional power indices (Brams & Affuso 1985a, 1985b).

similar views with the other two Benelux countries it, in fact, did not need power to be a successful EU member. Therefore, the StPI would have indicated great power of Luxemburg despite its formal impotence.

There are other points worth making about the analytical framework of Steunenberg et al. In particular, their definition of a dummy player is not always meaningful. For them, a dummy player is “a player whose preferences vary over the same range as the preferences of the actual players, but that has no decision-making rights in the game” (p. 348). What does it mean to “vary over the same range” if the so-called actual players' ideal points (to stay in a spatial voting framework) have different supports, e. g. $\tilde{\lambda}_i$ is uniformly distributed on $[0,1]$ and $\tilde{\lambda}_j$ has triangular distribution on $[1/2,4]$?

Or, suppose that the marginal distribution of $\tilde{\lambda}_i$ calculated from the joint distribution of $\tilde{\xi}$ is uniform on $[0,1]$ for every $i \in N$. The natural assumption for dummy player d's random ideal point $\tilde{\lambda}_d$ then is a uniform distribution on $[0,1]$. But what if player 1 to n's ideal points are dependent, in particular assume that $\tilde{\lambda}_1 = \ldots = \tilde{\lambda}_{n-1}$ and $\tilde{\lambda}_n = 1 - \tilde{\lambda}_{n-1}$. The equilibrium outcome will be biased against player n's preferences under almost every decision procedure Γ. If, in particular, we have pair-wise simple or qualified majority voting between all alternatives, the equilibrium outcome is $x^*(\xi) = \lambda_{n-1} = 1 - \lambda_n$. So expected distance Δ_i^Γ is zero for $i = 1, \ldots, n-1$ and $\Delta_n^\Gamma = 1/2$. In contrast, dummy player d with an independently uniformly distributed ideal point has $\Delta_d^\Gamma = 1/3$. Contrary to a remark by Steunenberg et al. (p. 349, fn. 7), equilibrium outcomes *can* be systematically biased against the interest of a particular player. Therefore, the StPI can become negative – in this example $\Psi_n^\Gamma = -1/2$. For a measure of average normalized success this makes sense. It simply indicates that player n, always having a position ‘opposite’ of his $n-1$ colleagues, is less successful on average than a neutral member of the decision body would be.

The StPI is not an irrelevant concept for the analysis of decision bodies. It takes into account strategic interaction in possibly complicated procedures. It is a meaningful and, in our view, informative measure of expected success, which is important in both economic and political institutions. But Steunenberg et al.'s framework leads to a *strategic success index*, not a strategic power index.

It is slightly surprising that Felsenthal and Machover regard the StPI as “a natural generalization of a priori I-power” (p. 95). *I-power*, in the context of traditional (non-spatial) weighted voting games, refers to a player’s influence or “ability to affect the outcome of a division of a voting body – whether the bill in question will be passed or defeated” (p. 84). We have difficulties to see how the StPI is related to a player’s influence in the above examples.

Felsenthal and Machover regard the StPI as a “promising” and “interesting instance of a unified method” which superimposes information on affinities, preferences, and a priori power (pp. 91 and 96). Their case in point is that for perfectly symmetric state space X and independent uniform distributions of ideal points, the StPI is just a re-scaling of the Penrose index. One is tempted to conclude that by taking strategic interaction and players’ preferences explicitly into account, Steunenberg et al. have discovered a complicated detour to where a short-cut has existed for so long.

However, it is known that a success measure can, in some cases, work like a re-scaled Penrose index. The Rae index r (1969) measures players’ success or satisfaction in the sense that it gives the probability of voting ‘yes’ when a proposal passes and ‘no’ when a proposal is rejected.[17] It thus measures how often a group decision corresponds with an individual’s preferences, i. e. what is the expexted distance between the outcome and an individual’s ideal point on a simple $X = \{0, 1\}$ policy space. The relationship between the Rae index of a player i and the Penrose index β_i can be written as[18]

$$r_i = \frac{1}{2} + \frac{1}{2}\beta_i.$$

The StPI measures something reminiscent to r_i in a general spatial context. This makes Felsenthal and Machover’s observation less surprising.[19]

[17]See also Straffin et al. (1982) who distinguish between ‘effect on outcome’ and ‘effect on group-individual agreement.’ The former is measured by power indices while the latter is measured by satisfaction indices like r.

[18]See Rae (1969) for details.

[19]Note that the relationship between the Shapley-Shubik index and the corresponding satisfaction index is more complicated (see Straffin, 1978, for details).

Moreover, Felsenthal and Machover's a prioristic version of the StPI departs significantly from Steunenberg et al.'s own approach – to the extent that it ignores the contribution the latter have made. Steunenberg et al. derive a measure which takes strategic interaction into account and is therefore suited for the analysis of procedural decision making. When Felsenthal and Machover take Steunenberg et al.'s model behind a veil of ignorance, they replace the equilibrium proposal – formally the random variable $x^*(\tilde{\xi})$ – by a random proposal $\tilde{x}$ which is independent of the random vector of players' ideal points $\tilde{\lambda} = (\tilde{\lambda}_1, \ldots, \tilde{\lambda}_n)$. Thus all strategic interaction and the procedural constraints, which make for a complicated dependence of $x^*(\tilde{\xi})$ and $\tilde{\lambda}$, are dropped. The StPI loses its 'St', and one is back to Garrett and Tsebelis's observation (Garrett and Tsebelis, 2001, p. 101): "Making a proposal (i. e. agenda-setting in game-theoretic terms) has nothing to do with the probabilities calculated in power indices."

5 An alternative approach

Steunenberg et al.'s framework is suited to study success both a posteriori and, by taking expectations, a priori. The chief reason why the StPI does not measure power is its reliance on information only about the *outcome* of strategic interaction. Power is about potential and thus refers to consequences of both actual and hypothetical actions. It means the ability to make a – subjectively valuable – difference to something. The key to isolating a player's power in the context of collective decision making is his *marginal impact* or his *marginal contribution* to the outcome x^*. As mentioned in section 2, this concept is well-established in the context of simple games and also general cooperative games, where it measures the implication of some player i entering a coalition S. The fundamental idea of comparing a given outcome with one or several other outcomes, taking the considered player's behaviour to be variable, can be generalized to a non-cooperative setting which explicitly describes a decision procedure.

5.1 An Example

For illustration, consider the player set $N \cup \{b\}$, the rather restricted policy space $X = \{0, 1\}$ embedded in $\mathbb{R}$, and status quo $\tilde{q} = 0$. Let the decision procedure Γ be such that, first, bureaucrat b sets the agenda, i. e. either proposes 1 or ends the game and thereby confirms the status quo. Formally, he chooses an action a_b from $A_b = \{1, q\}$. If a proposal is made, then all players $i \in N$ simultaneously vote either 'yes', denoted by $a_i = 1$, or 'no' $(a_i = 0)$. This makes $A_i = \{0, 1\}$ their respective set of actions. The proposal is accepted if the weighted number of 'yes'-votes meets a fixed quota k, where the vote by player i is weighted by $w_i \geq 0$. Otherwise, the status quo prevails. Formally, the function

$$x(a) = x(a_b, a_1, \ldots, a_n) = \begin{cases} 1; & a_b = 1 \quad \wedge \quad \sum\limits_{i \in N} a_i w_i \geq k \\ 0; & \text{otherwise} \end{cases}$$

maps all action profiles $a = (a_b, a_1, \ldots, a_n)$ to an outcome. Traditional power index analysis for players $i \in N$ can easily be mimicked with this setting: Take

$$D_i^0(a) := x(a_b, a_1, \ldots, a_i, \ldots, a_n) - x(a_b, a_1, \ldots, 0, \ldots, a_n) \tag{2}$$

as player i's marginal contribution for action profile a – corresponding to $v(S) - v(S \setminus \{i\})$ for coalition $S = \{j | a_j = 1\}$ – and make probabilistic assumptions over the set of all action profiles. The latter replaces the probability distribution over the set of all coalitions which is usually considered (via assumptions on acceptance rates). Assume, for example, that bureaucrat b always chooses $a_b = 1$ and let P denote the joint distribution over action profiles $a \in \prod_{i \in \{b\} \cup N} A_i$. Then,

$$\mu_i^\Gamma := \int D_i^0(a) dP(a) \tag{3}$$

corresponds exactly to the traditional probabilistic measures obtained via Owen's multilinear extension (1972, 1995) and Straffin's power polynomial (1977, 1988). For example,

$$P(a) = \begin{cases} 1/2^{n-1}; & a_b = 1 \\ 0; & a_b = 0, \end{cases}$$

makes μ^Γ the Penrose index. For

$$P(a) = \begin{cases} \dfrac{(\sum_{i\in N} a_i - 1)!\,(n - \sum_{i\in N} a_i)!}{n!}; & a_b = 1 \\ 0; & a_b = 0, \end{cases}$$

it is the Shapley-Shubik index.

Economic and political actions are in modern theoretical analysis regarded to be the consequence of rational and strategic reasoning based on explicit preferences. So directly considering (probability distributions over) players' action choices without recurring to the underlying preferences is methodologically somewhat unsatisfying.[20] But it is usually not difficult to find (probability distributions over) preferences which rationalize given behaviour. In above example, one may e. g. assume that players $i \in N$ have random spatial and procedural preferences with uniformly distributed ideal points $\tilde{\lambda}_i$ taking values in X and the procedural component that for given policy outcome $x \in X$ they prefer to have voted truthfully.[21] Let the bureaucrat have only a procedural preference, namely one for putting up a proposal if and only if it is accepted.

Now consider a particular realization of ideal points $\lambda = (\lambda_1, \ldots, \lambda_n)$. In the unique equilibrium of this game, first, the bureaucrat chooses $a_b = 1$, i. e. he proposes 1, if and only if the set $Y := \{i|\lambda_i = 1\}$ meets the quota, i. e. $\sum_{i\in Y} w_i \geq k$. Second, every voter votes truthfully according to his preference in case that a proposal is made. Hence, the function

$$x^*(\lambda) = x^*(\lambda_1, \ldots, \lambda_n) = \begin{cases} 1; & \sum\limits_{i\in N} \lambda_i w_i \geq k \\ 0; & \text{otherwise} \end{cases}$$

maps all preference profiles (as determined by the vector of players' ideal points) to a unique equilibrium outcome. Assumptions about the distribution P' of random vector $\tilde{\lambda} =$

[20] It is a very convenient short-cut, however. Also note that models of boundedly rational agents who do not optimize but apply heuristic rules of thumb receive more and more attention in the game-theoretic literature (see e. g. Samuelson, 1997, Fudenberg and Levine, 1998, and Young, 1998).

[21] See Hanson (1996) on the importance of procedural preferences in the context of collective decision making. Our procedural preference assumption can e. g. be motivated by regarding each player as the representative of a constituency to which he wants do demonstrate his active pursuit of its interests.

$(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_n)$ can be stated such that P' induces the same distribution P over action profiles a in equilibrium which has been directly assumed above. For example, $P'(\lambda_1, \ldots, \lambda_n) \equiv 1/2^{n-1}$ implies equilibrium behaviour which lets μ_i^Γ in (3) equal the Penrose index.

5.2 Measuring A Posteriori Power

We propose to extend above analysis from the simple coalition framework of a priori power measurement and the very basic voting game just considered to a more general framework. First, take a player's marginal contribution as the best available indicator of his potential or ability to make a difference, i. e. his a posteriori power. Second, if this is of normative interest or a necessity for lack of precise data, calculate a priori power as expected a posteriori power. Expectation can be with respect to several different aspects of a posteriori power such as actions, preferences, or procedure. This allows the (re-)foundation of a priori measures on a well-specified notion of a posteriori power.

There are many – to us, at this stage, all promising – directions in which the notion of 'power as marginal impact' can be made precise. The uniting theme is the identification of the potential to influence an outcome of group decision making. Influence can equally refer to the impact of a random 'yes' or 'no'-decision, as assumed by the traditional probabilistic index approach, and to the impact of a strategic 'yes' or 'no'-vote based on explicit preferences.

Crucially, 'impact' is always relative to a what-if scenario or what we would like to call the *shadow outcome*. The shadow outcome is the group's decision which would have resulted if the player whose power is under consideration had chosen differently than he a posteriori did, e. g. if he had stayed out of coalition S when he a posteriori belongs to it, or had ideal point 0 instead of 1. While in simple games the difference between shadow outcome and actual outcome is either 0 or 1, a richer decision framework allows for more finely graded a posteriori power. It also requires a choice between several candidates for the 'right' shadow outcome and, possibly, the subjective evaluation of differences.

A natural way to proceed in general is to measure player i's power as the difference

in outcome for given (equilibrium) actions of all players $a^* = (a_b^*, a_1^*, \ldots, a_i^*, \ldots, a_n^*)$, also denoted by (a_i^*, a_{-i}^*), and for $(a_i', a_{-i}^*) = (a_b^*, a_1^*, \ldots, a_i', \ldots, a_n^*)$ with $a_i^* \neq a_i'$, i. e. the case in which – for whatever reasons – player i chooses a different action. For preference-based actions with a unique equilibrium a^*, this defines a player's a posteriori power as the hypothetical impact of a *tremble* in the sense of Selten's (1975) perfectness concept, i. e. of irrational behaviour or imperfect implementation of his preferred action. More generally, a tremble can refer to just any deviation from reference behaviour or reference preferences.

If A_i consists of more than two elements, different degrees of irrationality or – if preferences are left out of the picture – potential deviations from the observed action profile can be considered. In particular, one may confine attention to the impact of a local action tremble. If $A_i = X = \{0, \delta, 2\delta, \ldots, 1\}$ for some $\delta > 0$ that uniformly divides $[0, 1]$, a suitable definition of player i's marginal contribution is

$$D_i^{1'}(a^*) := \begin{cases} \frac{x(a_i^*, a_{-i}^*) - x(a_i^* - \delta, a_{-i}^*)}{\delta}; & a_i^* - \delta > 0 \\ 0; & \text{otherwise.} \end{cases} \tag{4}$$

If $\delta = 1$, this corresponds exactly to $D_i^0(\cdot)$ and the marginal contribution defined in the traditional power index framework. As players' choice set approaches the unit interval, i. e. $\delta \to 0$, one obtains

$$D_i^1(a^*) := \lim_{\delta \to 0} \frac{x(a_i^*, a_{-i}^*) - x(a_i^* - \delta, a_{-i}^*)}{\delta} = \left. \frac{\partial x(a)}{\partial a_i} \right|_{a=a^*} \tag{5}$$

for $a_i^* \in (0, 1)$.

Both (4) and (5) measure player i's power in a given situation, described a posteriori by action vector a^*, as the (marginal) change of outcome which would be caused by a small (or marginal) change of i's action. It is, however, not necessary to take only small trembles into account; one may plausibly use

$$D_i^{1''}(a^*) := \max_{a_i \in X} \left[x(a_i^*, a_{-i}^*) - x(a_i, a_{-i}^*) \right] \tag{6}$$

to define an alternative measure of a posteriori power.[22]

Players' preferences may enter (4)–(6) to define $x^*(\lambda_1, \ldots, \lambda_n) \equiv x(a^*)$ as the reference point for action trembles, i. e. the point at which the derivative of outcome function $x(\cdot)$ w. r. t. player i's action is evaluated. If $x^*(\lambda_1, \ldots, \lambda_n)$ is the unique equilibrium outcome, any action deviation resulting in a distinct outcome is irrational. A meaningful alternative to studying the potential damage or good that a player's irrationality could cause is to instead consider the effect of variations in his preferences while maintaining rationality. This refers to the following two criteria for power:

- If a player wanted to, could he alter the outcome of collective decision making?
- Would the change of outcome in magnitude (and direction) match the considered change in preference?

As in the case of hypothetical action changes, one may consider either any conceivable change of preferences relative to some reference point or restrict attention to slight variations. The latter requires some metric on preferences. It is, however, naturally given for Euclidean preferences.

For illustration, consider players $N = \{1, 2, 3\}$ with Euclidean preferences on policy space $X = [0, 1]$, described by individual ideal points $\lambda_i \in X$, and simple majority voting on proposals made by the players. Let $\lambda_{(j)}$ denote the j-th smallest of players' ideal points, i. e. $\lambda_{(1)} \leq \lambda_{(2)} \leq \lambda_{(3)}$. Depending on the precise assumptions on the order of making proposals and voting, many equilibrium profiles of player strategies exist. However, they yield the median voter's ideal point, $\lambda_{(2)}$, as the unique equilibrium outcome $x^*(\lambda_1, \lambda_2, \lambda_3)$. One can then investigate player 1's power for given λ_2 and λ_3, where without loss of

[22]The total range $D_i^{\max} := \max_{a_{-i} \in X^{n-1}}[\max_{a_i \in X} x(a_i, a_{-i}) - \min_{a_i \in X} x(a_i, a_{-i})]$ of player i's possible impact on outcome lacks any a posteriori character. It seems a reasonable a priori measure which requires no distribution assumptions on $\tilde{a}^*$ or $\tilde{\lambda}$. However, it is a rather coarse concept and would only discriminate between dummy and non-dummy players in the context of simple games. $D_i^{\max'}(a^*_{-i}) := \max_{a_i \in X} x(a_i, a^*_{-i}) - \min_{a_i \in X} x(a_i, a^*_{-i})$ holds an intermediate ground.

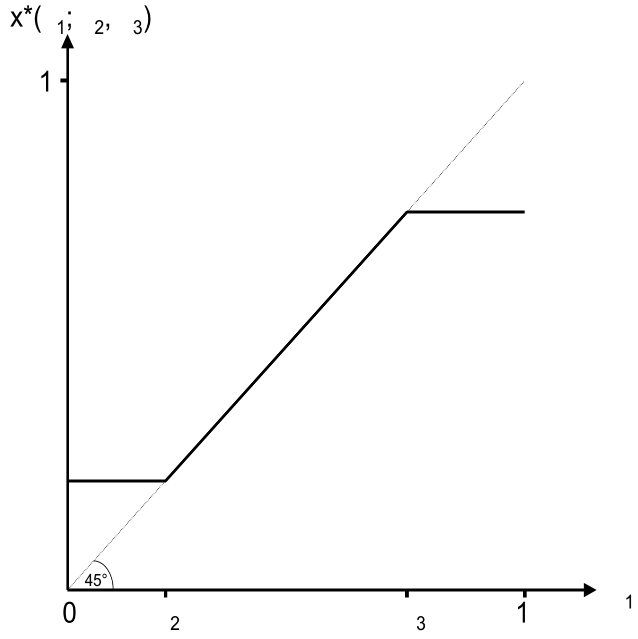

Figure 1: Equilibrium outcome of simple majority voting as λ_1 is varied

generality we assume $\lambda_2 \leq \lambda_3$. The mapping

$$x^*(\lambda_1, \lambda_2, \lambda_3) = \begin{cases} \lambda_2; & \lambda_1 < \lambda_2 \\ \lambda_1; & \lambda_2 \leq \lambda_1 \leq \lambda_3 \\ \lambda_3; & \lambda_3 < \lambda_1 \end{cases}$$

describes the equilibrium outcome (see figure 1), and

$$D_i^2(\lambda) = \frac{\partial x^*(\lambda)}{\partial \lambda_i} = \begin{cases} 0; & \lambda_1 < \lambda_2 \vee \lambda_3 < \lambda_1 \\ 1; & \lambda_2 < \lambda_1 < \lambda_3 \end{cases}$$

is a measure of player 1's power as a function of players' ideal points in X. According to $D_i^2(\cdot)$, player 1 is powerless (in the sense of not being able to influence collective choice although he would like to do so after a small change of preference) if he is not the median voter. For $\lambda_1 \in (\lambda_2, \lambda_3)$, he has maximal power in the sense that any (small) change in individual preference shifts the collective decision by exactly the desired amount.

So far, we have only considered ideal points in a one-dimensional policy space X. Both the derivation of a posteriori power and a possible formation of expectations will typically

be more complicated for higher-dimensional spaces.However, this is no obstacle to our two-step power analysis in principle.

To illustrate this, let $\Lambda = (\lambda_1, \ldots, \lambda_n)$ be the collection of n players' ideal points in $\mathbb{R}^m$ (an $m \times n$ matrix having as columns the λ_i-vectors representing individual players' ideal points). In a policy space $X \subseteq \mathbb{R}^m$, the opportunities even for only marginal changes of preference are manifold. A given ideal point λ_i can locally be shifted to $\lambda_i + h$ where h is an arbitrary vector in $\mathbb{R}^m$ with small norm. Which tremble directions it is reasonable to consider in applications will depend.[23] Multiples of the vector $(1, 1, \ldots, 1) \in \mathbb{R}^m$ seem reasonable if the m policy dimensions are independent of each other.

In any case, if the vector h that describes the direction of preference trembles has norm $\|h\|$ and so $\alpha = (\alpha_1, \ldots, \alpha_m) = \frac{h}{\|h\|}$ is its normalized version, one can define

$$D_i^2(\Lambda) := \lim_{t \to 0} \frac{x^*(\lambda_i + t\alpha, \lambda_{-i}) - x^*(\lambda_i, \lambda_{-i})}{t} = \frac{\partial_\alpha x^*(\lambda_i, \lambda_{-i})}{\partial \lambda_i} \tag{7}$$

as a reasonable measure of player i's a posteriori power provided that above limit exists. This is simply the directional derivative of the equilibrium outcome in the direction h or α. Other measures for the multidimensional case can be based on the gradient of $x^*(\lambda_i, \lambda_{-i})$ (holding λ_{-i} constant). In case of ideal points in a discrete policy space, a preference-based measure $D_i^{2'}(\lambda)$ can be defined by replacing the derivative in (7) with a difference quotient in analogy to (4) and (5). Another candidate for a meaningful a priori measure of player 1's power is – in analogy with $D_i^{1''}(a^*)$ –

$$D_i^{2''}(\lambda) := \max_{\lambda_i' \in X} \left[x^*(\lambda_i, \lambda_{-i}) - x^*(\lambda_i', \lambda_{-i}) \right]. \tag{8}$$

This is based on the consideration not only of small preference modifications but also of a complete relocation of the player's ideal outcome (see also fn. 22).

[23] One may also consider not a particular direction h but rather an entire neighbourhood of λ_i. This could be accomplished by taking the supremum of (7) for all possible directions α.

5.3 Calculating A Priori Power

Mappings D_i^1, D_i^2, and their discrete versions measure a posteriori power as the difference between distinct shadow outcomes and the observed equilibrium collective decision. We do not want to discuss at this point which is the most relevant shadow outcome and hence measure.[24] But let us stress that though these indices measure different types of power as determined by players' preferences and the decision procedure, they do not mix it up with the luck of a satisfying group decision as e. g. Steunenberg et al.'s proposal.

Having selected a meaningful measure of a posteriori power, it is straightforward to define a meaningful a priori measure. It has to be based on explicit informational assumptions concerning players' preferences or – if one does not want to assume preference-driven behaviour – actions. Denoting by $\tilde{\xi}$ the random state of the world as given either by preferences (and status quo) or players' actions, and by P its distribution,

$$\mu_i^\Gamma := \int D_i(\tilde{\xi})dP \tag{9}$$

is the a priori power index based on a posteriori measure $D_i(\cdot)$ and decision procedure or game form Γ.[25]

Traditional power indices, such as the Penrose or Shapley-Shubik index, consider the particularly simple decision procedure in which players $i \in N = \{1, \ldots, n\}$ choose an action $a_i \in A_i = \{0, 1\}$ and the outcome of decision making, $x(a)$, is 1 if set $Y := \{i|\lambda_i = 1\}$ is a winning coalition, i. e. if $v(Y) = 1$, and 0 otherwise. They use $D_i^0(a)$ (or $D_i^1(a)$ with $\delta = 1$). The StPI proposed by Steunenberg et al., too, is a linear transform of (9), albeit using the unreasonable a posteriori power measure $D_i(\tilde{\xi}) = \left\| \tilde{\lambda}_i - x^*(\tilde{\xi}) \right\|$.

Let us illustrate our aprioristic approach to measuring power more explicitly. As an

[24] Note that $D_i^{1''}(a^*)$ and $D_i^{2''}(\lambda)$ produce identical power indications if each action can be a player's most preferred one.

[25] We have omitted a sub- or superscript Γ in the definition of a posteriori measures for a concise notation. The procedure is, however, the central determinant of outcome functions $x(\cdot)$ or $x^*(\cdot)$. If several different game forms $\Gamma \in G$ are to be considered a priori, one has to take expectation over μ_i^Γ with the appropriate probability measure on G.

example assume a simple procedural spatial voting game where a fixed agenda setter makes a 'take it or leave it' offer to a group of 5 voters and needs 4 sequentially cast votes to pass it. The policy space is $X = [0,1]$, voters' ideal points are $\lambda_1, \ldots, \lambda_5$, that of the agenda setter is σ, and the status quo is 0. The unique equilibrium outcome of this game is

$$x^*(\lambda) = x^*(\lambda_1, \ldots, \lambda_5) = \begin{cases} 2\lambda_{(2)} & 0 \leq \lambda_{(2)} < \frac{1}{2}\sigma \\ \sigma & \frac{1}{2}\sigma \leq \lambda_{(2)} \leq 1 \end{cases}$$

where $\lambda_{(2)}$ is the second-smallest of the λ_i. For the agenda setter j we get the a posteriori power

$$D_j^2(\sigma, \lambda) := \begin{cases} 0; & 0 < \lambda_{(2)} < \frac{1}{2}\sigma \\ 1; & \frac{1}{2}\sigma < \lambda_{(2)} < 1 \end{cases}$$

and for voter i

$$D_i^2(\sigma, \lambda) := \begin{cases} 2; & 0 < \lambda_{(2)} < \frac{1}{2}\sigma \ \wedge \ \lambda_i = \lambda_{(2)} \\ 0; & \text{otherwise.} \end{cases}$$

Based on this, one can derive aprioristic strategic power measures μ_j^Γ and μ_i^Γ for the considered decision procedure. Suppose that all ideal points are uniformly distributed on $[0,1]$. We get

$$\mu_j^\Gamma = \int D_j^2(\sigma, \lambda) dP = \int_0^1 \int_0^1 D_j^2(x, y)\, f_\sigma(x)\, f_{\lambda_{(2)}}(y)\, dy\, dx$$

where f_σ and $f_{\lambda_{(2)}}$ are the densities of the independent random variables σ and $\lambda_{(2)}$, respectively. It follows that

$$\mu_j^\Gamma = \int_0^1 \int_{x/2}^1 1 \cdot 1 \cdot f_{\lambda_{(2)}}(y)\, dy\, dx = \int_0^1 \Big(1 - F_{\lambda_{(2)}}(x/2)\Big)\, dx = 0.625$$

with

$$F_{\lambda_{(2)}}(x) = \int_0^x 5\binom{4}{1} s\,[1-s]^3\, ds = 10x^2 - 20x^3 + 15x^4 - 4x^5$$

for $x \in [0,1]$ as the cumulative (Beta-) distribution function of $\lambda_{(2)}$. Analogously, one can compute

$$\mu_i^\Gamma = \int D_i^2(\sigma, \lambda) dP = 0.15,$$

which is the product of the probability 0.075 of player i having a swing that matters to the outcome and a posteriori power $D_i^2(\sigma, \lambda) = 2$ for these preference configurations. This means that ex ante a shift of the agenda setter's ideal point σ (voter i's ideal point λ_i) by one marginal unit will induce an expected shift of the outcome by 0.625 units (0.15 units). So the agenda setter's leverage on the outcome is ex ante more than four times larger than that of any given voter. One may want to compare agenda setting power to the power of the complete council consisting of all five voters. This can be done by ascribing the ideal point $\lambda_{(2)}$ to the latter. One obtains

$$\mu_{(2)}^{\Gamma} = \int D_{(2)}^2(\sigma, \lambda) dP = \int_0^1 \int_0^{x/2} 2\, f_\sigma(x)\, f_{\lambda_{(2)}}(y)\, dy\, dx = 0.75,$$

i. e. the council-of-five has a priori slightly more power in aggregate than the agenda setter.

6 Concluding Remarks

The uniform distribution of players' ideal points assumed in the previous section is convenient because its order statistics (the distribution e. g. $\lambda_{(2)}$) are easy to calculate. However, any other distribution can be studied if there are reasons to do so. Similarly, the decision procedure Γ need not be as simple as in above examples.

For more complex, possibly more realistic assumptions about preferences and procedures, the proposed two-step approach to a priori measurement of power remains valid. The only difference to our simple illustrations is that the calculations – i. e. determination of the equilibrium outcome as a function of preferences, its derivatives w. r. t. to players' ideal points, and aggregation via integration – will be more complicated. This is not a real problem if one does not insist on closed analytical solutions, but is primarily concerned with numerical values. In particular, Monte Carlo simulation even of complicated voting bodies is possible on a conventional desktop PC. It allows to approximate the desired probabilities and expectations with arbitrary precision.

We have defined a posteriori power as the *objective* marginal impact which a player's action or underlying preference has had on the outcome of collective decision making. It

is possible to go one step further. Namely, we have in passing hinted at the opportunity to understand – and measure – power as a *subjective* concept.

Consider a multi-dimensional policy space. Let the decision procedure give player i dictator power in some dimension d_i. The opportunity to define the collective decision in this dimension can be all that player i cares for. Judged in terms of his own preferences he has maximal power. The other players may be completely indifferent towards their joint decision's component in dimension d_i. Judged in terms of their preferences, i is a dummy who can never have an impact on their well-being. Alternatively, player i can hold dictator power on a dimension he does not care about (e. g. to pardon an unknown convict sentenced to death), but which is all-important to some, perhaps not all, other players. Player i is powerful depending on one's view-point, i. e. preferences.

Given the often very personal judgement of power in real life, it seems worthwhile to study the *subjective marginal impact* of players' actions or preferences. It is straightforward to replace the derivative of outcome function $x^*(\cdot)$ in above definitions by the derivative of players' utility of outcome, $u_i(x^*(\cdot))$, taken with respect to their own and other players' actions or parameterized preferences. A player's power is then not simply a real number, but a vector of subjective evaluations of it by all players (including himself). The corresponding index function is matrix-valued.

Subjective evaluation of players' power may be meaningless in the context of normative analysis of constitutional designs. However, it seems relevant for positive analysis, and is arguably the most relevant aspect to participants when decision procedures, e. g. in the EU or the WTO, are the object of multilateral negotiations.

Scholars of equilibrium-based a posteriori power and those favouring axiomatic a priori analysis will probably not see an urgent need to merge their fields. But our unified approach hopefully clarifies that they are not as far apart as it may seem. The axiomatic camp has been very little concerned with the notion of a posteriori power which is implicitly underlying their indices. Its members have jumped rather directly from an abstractly defined voting body to individual power values – without specifying how agents can and do act and investigating what intermediate a posteriori power is associated with this. The

non-cooperative camp has been interested only in the initial step of looking at equilibrium behaviour and its consequences for individual success. If the latter's attention is extended from success to power, which is in case of subjective evaluation the derivative of success, and if the former's large jump is decomposed into two smaller steps, both methodological approaches turn out to considerably complement each other.

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