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# The Market Value of Weighted Votes: An Alternative Approach to Voting Power

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Many committees are intended to represent groups of uneven size. One of the obvious ways to represent groups of uneven size is to assign the representatives different numbers of votes. However, there is widespread concern that power may not be proportional to the weights, and that the implications of weighted voting systems are difficult to understand, predict, or control. These complications often discourage the use of weighted voting rules altogether. Nevertheless, the adoption of a weighted voting rule by the Council of Ministers, the major legislative body of the European Community, has reinvigorated interest in making weighted voting work.

Analysts have long calculated power indices to indicate the relative strategic advantage of some actors relative to others. For example, among five actors with one vote each in a simple majority system, each one has equal power, and any power index gives each .20 of the power. However, with the same five actors, where one of those actors is given 3 votes to cast compared with one for each of the four others, it is apparent that the one actor has more power. The question is "how much more?"

How best to conceptualize the amount of power inevitably depends upon the purpose of the analyst. Social scientists have been interested in the power analysis for two distinct purposes: 1)To predict outcomes, and 2)To evaluate the "fairness" of the system. In this paper, we conceptualize power based upon usefulness for predicting outcomes, but then apply those notions to evaluate the fairness of particular systems.

Social scientists are interested in predicting both specific collective choices and the distribution of benefits among actors. One might initially expect that if one knew the power of each actor and the interests of

each actor, one would be able to deterministically predict the outcome of a collective decision. However, committee voting does not take place in a vacuum. Social actors rarely act in their narrow immediate self-interest, and that is very fortuitous for society. Rather actors take account of the interests and actions of others and the contingencies among them. Consequently, it is very difficult to predict specific outcomes of collective decisions that interact with other decisions that vary in participants and issues distributed over both space and time. For example, a decision taken today by the European Council of Ministers reflects not only interests of the participants on this issue, but also the interests of these same actors in other issues considered by the Council, the interests of these same actors in issues resolved outside of the Council, and the interests of other actors (outside the Council) who control outcomes of interest to Council participants, in the past, present, and future. The power of an actor within a context should be understood in such a way that it can be combined with understanding about the whole constellation of actors, interests, and other decision making contexts that can interact.

Social engineers are interested in determining the power distribution in a system in order to design systems that meet particular systemic goals. In particular, a voting system should be "fair." In terms of the European Council of Ministers, one principle is to give each citizen of the member countries equal power over the collective choices. If different countries have different populations, then the only way for the citizens of the larger countries to have equal power to citizens of small countries is for the larger countries to have more power in the decision-making body. However, there are other somewhat conflicting normative principles as well. One might not want a majority of countries to be outvoted by a minority of countries. One might not want a majority of citizens of countries to be outvoted by a minority of the citizens. Yet, even as one wants to protect the sovereignty of a majority of nations and the interests of a majority of the citizens, one also does not want a

decision rule that does not allow a majority of countries and a majority of citizens to pass legislation that they favor. Finally, one might well want a simple decision rule whole underlying principles can be understood by the great bulk of interested parties. A simple and transparent system that meets other criteria is one that is most likely to engender trust and legitimacy. So, we examine power for the purposes of devising a "fair" system, but also consider modifications to take account of these other considerations.

In the following sections, we review the bases of the most common power indices, suggest important inadequacies of these indices for both prediction and evaluation of fairness, suggest an alternative approach to power that addresses these problems, show how this other power indices fit in a larger system of prediction of systemic outcomes, and consider the implications of power and other considerations for the design of a particular weighted voting system.

*Common Power Indices*

The two main approaches to power indices were developed by a) Penrose, later followed up by Banzhaf and Coleman, and b)Shapley and Shubik. The Penrose-Banzhaf-Coleman approach bases its power scores on the likelihood that a particular actor will be in a position to change a collective choice, assuming that all combinations of voter preferences are equally likely. For example, in the previously described situation where actor A has three votes, and four other actors each have one vote apiece with a majority rule, the big actor is decisive whenever the four ordinary actors are split evenly two to two or are split three to one or one to three. Of the thirty-two possible combinations of pro-con preferences among those five voters, 12 are split 2/2, another 8 are split 3/1, and still another 8 are split 1/3. .So actor A is decisive in 28 combinations. Actor E is only decisive when the other actors are split all against the big actor or the big actor against all of them; this occurs in only 4 of the 32 combinations. The situations of B, C, and D are abstractly identical to that of E. So,

the power scores are proportional to 28, 4 ,4 ,4 ,4 and normalized are approximately .64, .09,.09,.09,.09.

The Shapley-Shubik approach bases its power scores on the likelihood that a particular actor will be pivotal in putting an ordered coalition over the threshold to get a majority, based upon the assumption that coalitions form in random order. There are 120 permutations of the 5 actors. A is pivotal in 72 of them. Each of the other actors are pivotal in 12. So, the normalized power scores are .60, .10,.10,.10,.10. Note that these normalized power scores are very similar to the Banzhaf scores for the same situation calculated above.

However, both of these types of power indices have been criticized for being acontextual, in that they ignore any information about the relationships among the actors, or the likelihoods that they would ever prefer to be on the same side of an issue. Defenders suggest that these measures are deliberately "a priori" power scores, upon which preferences and personal relationships may be superimposed. The concept of a priori power is appealing to the extent that one can then use those a priori power scores as building blocks in a larger model of the social context. However, we suggest that these scores cannot serve as building blocks, because these power scores cannot be usefully input into larger models..

Consider contextual information on the likelihood of agreement between actors and/or their likelihoods of coalescing. If one knew that actor A and actor B agreed 75% of the time, instead of the 50% assumed in the Banzhaf calculations, then one would need to recalculate power scores from scratch based upon the new assumptions. There is no way to take the original power scores, combine them with the contextual information, and output a posteriori power scores. Similarly for the Shapley-Shubik measures. If one had information about the likely ordering of coalitions, one could use that information to recalculate modified SS power scores, but could not use the original power scores as an intermediate step.

Alternatively, consider contextual information in the form of policy preferences in a spatial model

organized around voter ideal points in multi-dimensional space. If one makes the common assumption that all plausible coalitions would be connected and compact, then the plausible minimal winning coalitions are changed from the supposedly acontextual model. There is no general way to use either the Banhaf or Shapley-Shubik power scores as intermediate steps in computing power scores in this context. Rather, one would need to return to the original weights and calculate the likelihood that each actor would be pivotal or decisive in one respect or the other, given the contextual possibilities of particular combinations and permutations.

We suggest that Banhaf and Shapley-Shubik measures are not properly conceptualized as acontextual at all; rather, they are based upon a particular context that is very unlikely to be approximated in any naturally occurring situations. Consequently, these indices do not provide very useful information about which outcomes will occur or which actors will benefit in any conditions other than those implausible conditions that are assumed. Furthermore, since they only apply to these peculiar contexts, they do not provide useful information about the fairness of the rules.

### *Conceptualizing Fungible Power Scores*

We suggest an alternative approach to power scores that is more adaptive to use across a broad range of contexts. Our model makes no assumptions whatsoever about the likelihoods of particular coalitions, or the likelihoods of common interests. Rather, our model assumes that the primary value of vote shares is their value in exchange with others whose interests will determine their usage. The question then becomes how much each vote share is worth to an outsider who would purchase a collective decision. Purchasing a collective decision requires purchasing a majority vote. We assume that the purchaser will not pay any more than necessary, and therefore will purchase no more than a minimal winning coalition. Furthermore, since purchasers are willing to

pay the same amount for any minimal coalition, the value of all minimal winning coalitions converge towards having the same total price.

This leads to our idea of "fungible power." If the weights of the actors are defined such that all minimal winning coalitions have the same total, then power is proportional to those weights. So, in the example previously used with A with three votes and each of the other four voters with a single vote each, all minimal coalitions have exactly four votes. The actor whose share is three votes has power equal to the total of any other three voters. If these scores are normalized, then the power scores are .430, ..143,.143,.143,.143. These scores are considerably different from the .60 and .10 scores from the other power indices. We focus attention on the fact that the three votes of actor A are directly substitutable for the votes of any three other actors, and that should be reflected in their power scores.

Now, we need to consider the well known fact that any given set of weights is functionally equivalent to many other sets of weights. For example, our sample rule could be expressed as either as weights of a 9 and four 4s, or as weights of a 15 and four 4s. These rules all require the big guy and one little guy or all four little guys to make a decision. The relative weights are very different from one another in these two sets of weights, and different from our original representation (the relative weight of the big actor to each little actor is 9/4=2.25 in one set, 15/4=3.75 in another set, and 3/1=3.0 in the original representation.) However, these two later representations do not have the property that all minimal winning coalitions have the same total value. With 9 and four 4s, the big guy coalition adds up to 13, while the little guy coalition adds to 16! With 15 and four 4s, the big guy coalition adds to 19, and the little guy coalition adds to 16! So, these weights do not meet our condition. Weights that meet the condition that all MWCs have equal total value have been called homogeneous weights, and it has been proven that any homogeneous representation of a rule is unique up to a

multiple; i.e. the only sets of weights meeting the conditions are multiples of one another--- e.g. a 3 and four 1s, or a 6 and four 2s. . (Note that this conclusion assumes that there are no dummies, or that dummies are assigned weights of zero.) For our present purposes, this implies that when there is a homogeneous set of weights, there is a unique normalized set of power scores.

Thus, our primary suggestion is that whenever there is a homogeneous set of weights that is functionally equivalent to a given set of weights, then the homogeneous weights should be taken to indicate the relative power of the actors. We suggest that the "best" representation among all the functionally equivalent sets of weights is the homogeneous set of weights, because it is a "what you see is what you get" situation. Any other set of weights misrepresents the relative positions of the actors in one way or the other.

Unfortunately, there are sets of weights for which there is no functionally equivalent homogeneous set of weights. For example, consider that A, B, C, D, E, F have weights 6, 5, 4, 3, 2, 1 totaling 21 and requiring a majority of 11. It can be readily determined that these weights are not homogeneous; ACD is a minimal winning coalition (MWC) with 13 votes, and AB is another MWC, but it has 11 votes. Since all MWCs do not have the same total votes, the set of weights is not homogeneous. With further analysis, it is also apparent that there is no homogeneous set of weights that is functionally equivalent to these. If all minimal winning coalitions would have the same totals, then among others the weight of ACE would have to equal that of ADE, and therefore the weight of C would have to equal that of D. On the hand, BCE is a minimal winning coalition, but BDE is not-- so C and D can never have the same weights under this decision rule . This is sufficient to show that there can be no homogeneous set of weights equivalent to these. So, our initial definition of power scores cannot be applied to this situation.

The fact that only a small minority of sets of weights are equivalent to some homogeneous set of scores

is a severe limitation of our fungible power scores. Nevertheless, we suggest that this approach is useful in two ways: 1) We can create sets of weights that are homogeneous, and suggest that such homogenous "what you see is what you get" weights be adopted wherever feasible. 2) We can extend our notion of fungible power scores by considering the ***best*** approximation to homogeneous weights, for all situations where there are no equivalent homogeneous weights.

### *Some Sufficient Conditions for Homogeneous Weights*

Note that our discussion above applies to "majority rule;" i.e. where either a coalition is winning or its complement is winning, but not both. In more general terms, majority voting is a special case where the quota is ½ of the total votes, and winning requires surpassing the quota. We proceed to discuss ways to create homogeneous weights. From our present perspective on weighted voting power, the practical advantage of homogeneous weights is "what you see is what you get." The weights in a homogeneous rule ARE the relative powers of the actors. The following can be extended to "qualified majority rule" involving higher quotas, but that is left for later consideration.

We consider integer weights, with n actors and m (odd) total weight, and minimal winning coalitions adding to (m+1)/2. We can prove the following theorems.

Theorems:

Theorem 1) A set of weights containing at least (m-1)/2 weights of 1 is homogeneous. Call this a ONE-SET

> Example: Weights of 5, 3, 2 and nine actors with weight 1. The total is 19; a majority is 10, and all MWCs have total of 10..

Corollary 1) to Theorem 1) A one-set can be created from another one-set by including an additional

group of actors such that the total weights of those weighted 1 each is at least as great as the total weights of the others in the group.(e.g. a 2 and two 1s; a 3, 2 and five 1s; etc.)..

Theorem 2) A set of weights such that :the effective quota (m+1)/2 is divisible by the largest weight, which is divisible by the next largest weight, etc. and such that the cumulative weights less than any weight exceeds that weight, is homogeneous. Call this a MULTIPLE SET

Examples: a) Weights of 1,1,1,2,2,2,2,6,6,12,12. The total is 47; a majority is 24, and all MWCs have a total of 24. b) Any set of weights of all 1s and 2s with at least three 1s and total weights minus one divisible by four.

Corollary 1) to Theorem 2) A multiple set can be created from another multiple set by including additional actors with weights equal to some existing weights in a group with total weight equal to twice the largest weight.

Corollary 2) to Theorem 2) A multiple set can be created from another multiple set by doubling the number of actors by cloning all of the actors, and then adding one more actor with weight 1..

Our corollaries provide ways to expand a system with homogeneous weights while preserving the homogeneity of the weights. It may not be practical to require that additions to a voting body join in certain types of restricted groups, but where that is possible, it can facilitate the transition. There are many homogeneous set of weights where adding new members, even with existing weights, disrupts the homogeneity of the weights. We believe that having some possibility of systematic expandability is a desirable property of a set of weights..

*Fungible Power For Non-Homogeneous Rules*

Considering that most sets of weights are neither homogeneous nor equivalent to a set of homogeneous weights, and policy makers do not and cannot always choose a homogeneous set of weights, it would be very useful to develop a notion of fungible power that applies to non-homogenous rules. We previously suggested that all minimal winning coalitions should have the same total value, because an outsider would not pay more for any one minimal winning coalition than for any other. We suggest that the totals will be equal where that is possible; but we now suggest that if there is no way to make them equal, then there will be a tendency towards making the values of the minimal winning coalitions as similar as possible. Specifically, we suggest that the set of weights that minimizes the range of values (i.e. highest minus lowest) will determine the fungible power scores.

Our investigation of these minimal deviations from homogeneity for non-homogeneous weights suggests that these scores are probably NOT unique, but that the different sets of scores with identical minimal ranges are likely to be very close together. More work needs to be done to prove this "near uniqueness" result. In the meanwhile, we know that the set of scores with minimal discrepancies must form a compact connected convex region in the space. It will be useful for future work to set limits on how small this convex region will be. If it is very small, the weights may practically be treated as if it they fall at a unique point.

Determining precise bounds for the set of weights with minimal discrepancies is a complex linear programming problem, but we have been able to estimate these power scores by using successive approximations. This process seems to work well. Starting with an arbitrary set of weights, we can generally find weights that reduce the discrepancy, but "better" weights become more elusive as the discrepancy is reduced.

*Fungible Power in Qualified Majority Rule*

Our discussion above concerns majority rule, but we have suggested that these ideas can readily be extended to "qualified majority rule" (i.e. involving higher than majority quotas). A qualified majority rule is homogeneous when all minimal winning coalitions have the same values as one another *and* all minimal blocking coalitions have the same value as one another. We believe that such sets of weights unique, just as for majority rule. When a set of weights is not homogeneous, then there is a discrepancy between the largest and smallest MWC and another discrepancy between the smallest and largest MBC. We suggest that fungible power be defined as the weights that minimize the larger of these two discrepancies. Just as with majority rule, we do not expect such weights to be unique, but expect that weights with equivalent discrepancies will be very similar to one another. Our theorems and corollaries assuring homogeneous weights for majority rules are readily extended to qualified majority rules. We continue our discussion of the usefulness of fungible power assuming application to either majority or qualified majority rules.

*Using Fungible Power Scores for Prediction in Larger Systems*

We suggest that these fungibility power scores are especially useful as intermediary values for understanding collective decision making systems. We envision the decision on any particular issue as part of a larger system, where actors are not necessarily voting for their preferred outcomes as much as having their votes used by others who particularly care about these outcomes. Coleman's *Mathematics of Collective Action* developed this type of model, where there are multiple issues, and actors may have different amounts of control over different issues. At the same time, actors can have different proportions of their interests in different issues. Coleman assumed that actors would trade their control over issues for control over the issues

about which they cared the most.

In actual application, there could be a "free rider problem" as actors perceived that others with the same interests would be willing to purchase a majority of votes without their having to contribute. In Coleman's original formulation, he tried to deal with this problem by suggesting that his model only truly applied to a probabilistic decision rule, where an amount of control indicated the extent to which that actor could change the probability of the outcome. However, he then admitted that there were no approximations to such a rule in any real situations of interest.

In the absence of probabilistic decision rules where each actor could control a certain part of the probability, he suggested that a power index could indicate the proportion of control of each actor. He specifically suggested using his own power index, essentially the Banzhaf index. That index does indicate the probability that an actor will be decisive if all actors decide randomly and independently of one another. However, we have already suggested that this is a highly implausible condition. It does not seem reasonable to suppose that actors will pay other actors for their votes according to the chances that those other actors would be decisive in some random culture, when everyone can readily observe that the situation is very different from that.

Rather, we suggest that it is a better approximation to suppose that actors will purchase votes on issues proportional to their value towards reaching a minimal winning coalition to produce a particular outcome. There is always the possibility that actors will try to act as free riders, but we suggest that there are sociological forces that encourage actors to act sincerely on their interests, and that assuming that actors will purchase votes in proportion to their interests in issues often provides a reasonable approximation. There is no reason for actors to purchase more than a minimal winning coalition, but since votes beyond minimal winning coalitions make no

difference, actors can vote their "free" votes any way they want for any reason. Consequently, even though actors are only willing to pay for "minimal" winning coalitions, the "free" votes may often make the actual winning coalitions considerably larger than minimal.

If a system was confined to decisions made within a particular decision making body, we expect actors to have power indicated by our power index. Actors would use their power to purchase control over issues according to their individual interests. Then, purchase and use of this control would determine the collective decisions with respect to those issues. Thus, our power scores would be inputs into the Coleman Collective Choice model that would then be used to determine the outcomes for any particular set of issues. According to the model, if an actor's power within a system is increased (while the actor maintains the same interests), that actor will receive proportionally more of her desired outcomes. However, it is difficult to compare across actors; whether an actor gets much of what she wants depends not only upon her power, but also upon the expensiveness of her tastes, and upon the power and tastes of others.

This approach also allows us to build systems that go beyond a particular legislative body. For example, in the European Community, a Minister in the Council of Ministers could trade a vote on an issue in the Council for support on different types of issues in the European Council, or for a particular concession of another government outside of this legislative arena altogether. The systems within which decisions are made are neither stable nor well bounded. Nevertheless, the more power an actor has within a body that makes decisions that are consequential for others, the more power that actor has over a full range of issues within and outside of the body itself.

*“Fair” Distribution of Power*

Our discussion above suggests that the power an actor is given within a legislative body can have an important impact on the overall balance of power among actors outside as well as inside of the body itself. This makes it particularly important to allocate “fair” vote shares to the various actors involved, and our present approach has importantly different implications for fair vote shares.

In analyzing the situation of the Council of Ministers in particular, we start with the assumption used by others that “fair” vote shares are those that give each citizen of each country equal power. Differences in implications arise from different notions of power.

Supporters of Banzhaf and Shapley-Shubik power index approaches have argued that equal power is equal probability that an individual citizen will be decisive in a decision. They then determine that a citizen’s chance of being decisive in the European Council of ministers is the product of the citizen’s probability of being decisive in her home country times the probability that the country will be decisive in the Council of Ministers. Based upon the same assumption that we have repeatedly criticized above, that actors act randomly and independently of one another, the probability of an individual being decisive in a country is inversely proportional to the square root of the population size. It then follows that to keep the overall probability of being decisive constant for citizens of all countries, each country needs to be assigned vote shares giving them power scores that are proportional to the square root of their population sizes. Larger countries get more power, but not by as much as their populations would dictate-- only by as much as the square roots of their populations would dictate.

In contrast, fairness in our fungibility perspective implies that a purchaser of votes should be willing to pay citizens of one country the same as citizens of another country as parts of accumulating a minimal winning coalition in the Council of Ministers. Purchasing control over the vote of a country requires purchasing the

votes of a majority of its citizens. That is proportional to the population size of countries. For the value of purchasing the votes of each citizen to be equal, the power of the country's votes must then be proportional to the population of the country (not to the square roots). Then, it would cost the same to gain control over a majority of votes in the Council by either purchasing majorities of the voters in a few large countries that had a combined population of more than half the total, or by purchasing majorities in several smaller countries that had a combined population of more than half the total; one would purchase the same total number of citizen's votes either way.

Analysis of the actual weights for the countries in the EC before 1995 indicates that the power scores, whether calculated by Banzhaf, Shapley-Shubik, or us, tend to be very highly correlated with the actual weights assigned to the actors. However, it is apparent that the weights have actually been assigned to be closer to proportional to the square roots of the population sizes than to the population sizes themselves. We suggest that this is a normative error if one wants to give all of the citizens equal power.

*Other Normative Considerations*

There are undoubtedly many reasons for the European Community treaty weights beyond fairness to citizens of different countries. Countries with a great stake in the development of the union might well be willing to give away more power to other countries than they would otherwise deserve. Also large wealthy countries may have control elsewhere within and outside the EC government system that more than compensate for any power given up in the Council of Ministers For example, the Treaty of Nice gives the large countries increased weights in the Council of Ministers compared with previous treaties, partially in exchange for reducing the numbers of representatives of those same large countries on the European Commission. The effect of the

distribution of power within the Council of Ministers on the overall power distribution depends upon the portion of overall decision making authority that is given to the Council of Ministers.  As long as the Council of Ministers itself has severely constrained jurisdiction, then any power discrepancies that manifest themselves there are unlikely to have much impact on the overall power distribution among these countries.

Besides equity among citizens of different countries, there are other normative issues affecting decisions about relative weights .  In particular, they want to protect the integrity and sovereignty of countries.  If countries are represented proportional to their populations, then a very small proportion of countries can have a majority of votes.  To protect the majority of countries from being outvoted by a small minority of countries, one would have to increase the quota required for passage to a very high level.  With the weights provided in the various recent treaties, that level would have to be as high as 88% of the votes.  Having a threshold that is not too initimidatingly high would seem to be another normative consideration.

One way to protect the majority of countries, without raising the threshold too high, is to reduce the relative proportions of votes controlled by the large countries.   However, even if the large countries are willing to accept representation less than proportional to their populations, they would presumably resist a rule that allowed countries with a minority of the overall population to override countries with a majority of the population.  If the representation of large countries was reduced too much, and the quota was lowered too far, then a majority of countries that had a small proportion of the population would be able to outvote countries with a majority.

So, the various normative criteria seem to lead to weights that increase with population but less than proportionally, combined with a threshold that is as low as possible while still being high enough to protect any majority of countries and any set of countries containing a majority of the population from being overridden.

The weights and quotas used throughout the history of the EC seem to have that property, the consistency of the pattern seems to suggest that these properties are more likely to be deliberate than accidental.

Representing countries proportional to the square root of their populations may sometimes serve these purposes, but there is no general reason to expect that it would generally serve these purposes, nor that it would be optimal for these purposes when it does. For normative purposes, it might be better to devise a prescription for assigning weights that assures that a majority of countries and a set of countries with a majority of votes cannot be overridden, even while keeping the required quota as low as possible. (Note that one could replace the entire weighted voting system by a dual requirement of support by representatives of a majority of the countries representing a majority of the population, but we suppose that the original intent of the qualified majority rule was to provide a simpler substitute for such complex compound rules. (Unfortunately, the Treaty of Nice seems to create the worst situation in this respect by supplementing a qualified majority rule with requirements for both numbers of countries and proportions of the overall population. There is often immediate political appeal of such ad hoc political compromises, but the ad hoc complexity itself is undesirable and may undermine the legitimacy of the system.)

*Summary*

In conclusion, we have suggested a power index that is based upon the market value of vote shares, such that combinations of vote shares that contribute equally to winning coalitions will have equal market value. Specifically, when there is a weighted voting rule such that all minimal winning coalitions have the same total, then the weights in that rule ARE the power scores. Such a set of weights is called a homogeneous set of weights, and such a rule is a homogeneous rule. Any set of weights that is equivalent to a homogeneous set of weights has its power scores determined by those homogeneous weights. However, for non-homogeneous

rules, we suggest that the power scores are determined by the set of weights that minimizes the discrepancies between the highest total value of a MWC and the lowest value of a MWC. While those power scores are generally not strictly unique, we expect that they vary within narrow bounds. We have devised a computer program that finds approximations to these power scores incrementally by varying the weights, and determining the discrepancies among the total values of the MWCs. We suggest that work needs to be done to determine bounds upon the set of equivalent weights that have the same minimal MWC discrepancies. It will also be useful to investigate how widely these discrepancies vary with small variations among the weights.

While our presentation focuses upon weighted simple majority rules, we suggest that there are straightforward extensions to qualified majority rules (i.e. those requiring more than a bare majority of votes). Specifically, we suggest that homogeneous weights in a qualified majority rule must have the property that all minimal winning coalitions have the same total weights as one another, *and* all minimal blocking coalitions must have the same total weights as one another. Nevertheless, sets of homogeneous weights can be created for qualified majority rules in similar ways to those for simple majority rule. Also, where a rule is not functionally equivalent to any homogeneous set of weights, power scores can be defined to be weights that minimize the discrepancies among both minimal winning and minimal blocking coalitions

We have suggested that our power scores are more appropriate bases for inputs to Coleman's market model of collective decision making than Coleman's own suggested Banzhaf scores. This model allows consideration of multiple issues in multiple domains simultaneously. Obviously, the effectiveness of prediction will depend upon the specification of the components and boundaries of the set of issues as well as the assumptions of the model. Further work might investigate the sensitivity of the model predictions to friction in the system and various sources of transactions costs.

Finally, we have criticized the normative approach taken by power analysts to the evaluation of the weighting within the Council of Ministers. We suggest adopting the more intuitive approach of determining weights as proportional to population rather than proportional to the square roots of population. Using population numbers themselves as weights is likely to provide fairer power scores. However, we also discuss normative considerations other than fairness that lead one to prefer departures from proportional weights for other reasons.

We hope that these types of analyses might make policy makers more willing to consider implementing weighted voting rules in further contexts. We suggest that homogeneous sets of weights have the advantage of "what you see is what you get," that may be more acceptable to people than systems with less obvious implications. We have described some specific conditions that assure homogeneous sets of weights (i.e. One-Sets and Multiple Sets). Further research may find other practical ways of assuring homogeneous weights.