Workshop on Voting Power Analysis

LSE, 9–11 August 2002

Organised by VPP project at CPNSS, London School of Economics

(Directors: Rudy Fara, Dan Felsenthal, Dennis Leech, Moshé Machover, Maurice Salles)

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PROGRAMME

ABSTRACTS OF PAPERS

PROGRAMME

(Provisional)

Friday, 9 August

16:00–17:45	***Registration***
18:00–19:30	***Opening session***: Address by Prof Sir Michael Dummett, member of VPP Panel of Advisers. (Title of address to be announced)
19:45–	***Dinner*** (Optional. Details about cost and arrangements will be circulated)

Saturday, 10 August

09:30–11:00	***Foundations of voting power*** (Chaired by Maurice Salles)
· Annick Laruelle:	*Positive and normative assessment of voting situations*
· Scott L Feld:	*Market value of weighted votes: An alternative approach to voting power*
	Coffee Break
11:30–13:00	***Strategic power*** (Chaired by Dennis Leech)
· Stefan Napel:	*Strategic power revisited*
· Dieter Schmidtchen:	*The strategic power index—responses*
	Lunch Break
14:00–15:30	***Paradoxes I*** (Chaired by Hannu Nurmi)
· Matthew Braham/Frank Steffen:	*Local monotonicity revisited*
· Federico Valenciano:	*Voting power paradoxes revisited*
	Coffee Break
16:00–17:30	***Round-table discussion on dissemination of Social Choice***

ideas (Chaired by Iain McLean)

Opening presentation by Manfred Holler: *How to sell power indices*

Free evening/Theatre

Sunday, 11 August

09:30–11:00 ***Generalizations of simple games*** (Chaired by Harrie de Swart)

· William Zwicker: *Further thoughts on weighted voting, abstention, and multiple levels of approval*

* Ines Lindner: *Computing power indices for voting games with several levels of approval*

Coffee Break

11:30–13:00 ***Paradoxes II*** (Chaired by Rudolf Fara)

· Agnieszka Rusinowska: *On some properties of the Hoede–Bakker index*

· Vincent Merlin: ***The probability of conflict in two-stage elections***

Lunch Break

14:00–14:45 ***Voting power on the Web*** (Chaired by Federico Valenciano)
· Antti Pajala: *Power index website: A voting power index calculation WWW-resource*
14:45–15:30 ***The inverse problem*** (Chaired by Federico Valenciano)
· Dennis Leech: *Power Indices as an Aid to Institutional Design*

Coffee Break

16:00–17:30 ***Round-table discussion on perspectives and prospects of voting power research*** (Chaired by Dan Felsenthal)
Opening presentation by Moshé Machover

19:15– ***Party***

ABSTRACTS

Annick Laruelle* and Federico Valenciano

Positive and normative assessment of voting situations

This paper is concerned with the analysis of voting situations. By a voting situation, we mean a situation in which a set of voters faces collective decision-making by means of a procedure that specifies when a proposal is to be accepted and when is to be rejected after a vote is cast. More specifically we are interested in the accurate formulation and quantification of several features involved in these situations. Traditionally voting situations have been modelled by simple superadditive games. In this context several measures or 'power indices' have been proposed in order to assess the voters' a priori 'decisiveness' or 'power'.

This classical approach can be criticized for several reasons. First, the game-theoretical framework and terminology is inadequate and misleading. There is no transferable utility involved, there are not really 'players' nor 'voters', for the voting rule only specifies when a proposal will pass: to describe a voting rule one should more properly speak just of 'seats'. There are no 'coalitions' – a term that suggests cooperation with a purpose where there is only coincidence of vote. There are no 'marginal contributions' where there is no cake to share. Moreover, as a rule the axiomatic characterizations of power indices either do not fit the specificity of the context, or lack compellingness. Second, the existence and misuse of several indices without a clear interpretation is confusing and does not contribute to their credit. Finally, power indices are often criticized on the basis that the only information they take into account is the voting rule, while the voters' preferences, which clearly influence their capacity of being successful, decisive or lucky, are ignored.

In this paper we propose a more general model which includes the two separate ingredients in any real-world voting situation: the voting rule and the voters. The voting rule, specifies for a given set of seats when a proposal is to be accepted or rejected depending on the resulting vote configuration. Voters, the second ingredient in a voting situation, are included via their voting behavior, which is summarized by a distribution of probability over the vote configurations. This distribution of probability depends on the preferences of the actual voters over the issues they will have to decide upon, the likelihood of these issues being proposed, etc. In each real-world voting situation these probabilities must be approximated from the available data. Thus this general model, unlike the traditional one, is apt for positive or descriptive purposes.

Within this general framework we re-examine the concepts of 'success', 'decisiveness' and 'luck' that in a more or less clear formulation can be traced a long way back in the literature. This more general setting allows a simple and precise reformulation of these concepts as probabilities which depend on the voting rule and the voters' voting behavior. Moreover, our formulations extend previous purely normative notions to more general positive/descriptive concepts, which can be particularized into some familiar but not always well-understood notions, shedding new light on their meaning and relations.

Scott L. Feld, Bernard Grofman, and Leonard Ray

Market Value of Weighted Votes: An Alternative Approach to Voting Power (Preliminary Abstract, March, 2002)

Measures of voting power are generally based upon the likelihood that an actor will be pivotal to coalitions with respect to their reaching a threshold quota of votes. Critics note that such analyses usually ignore structural, historical, sociological and psychological factors that greatly affect coalition formation. But, it is generally impractical to take those factors into account, because they are too complex, transient and/or manipulable to model effectively. In response to this problem, we suggest an alternative approach that depends upon the forces of a market to set prices of voting shares, irrespective of the likelihoods of particular coalitions.

We approach the problem from the perspective of someone who wants to purchase a group decision. The question becomes how much to pay each voter for his/her voting share .The payer should pay no more than s/he would pay for the same commodity elsewhere. Thus, the total price should be the same to buy k votes, whether one buys them from a single actor or from several actors. This approach leads the value of the votes of actors towards being proportional to their weights. With small numbers of actors, the relative values may not be fully determinate, but we explore increasingly constraining bounds on the relative values of vote shares as the number of actors increases.

We go on to consider normative implications of our approach to voting power, particularly for the problem of assigning voting weights in the European Council of Ministers. We consider that any weight distribution other than one proportional to population, operating with a weighted majority rule, creates the possibility that a coalition with a minority of the population could pass legislation over the objections of countries representing the majority. Increasing the required quota corrects that problem, but creates opposite problems—e.g., there are some coalitions that represent both a majority of the population *and* a majority of the countries that cannot meet the increased quota of votes. That has been true of all actual rules and quotas used by the European Council. Our approach suggests using weights proportional to whatever underlying values are intended to be represented. For multiple different values (e.g. population, GDP, individual countries), one can require a multi-stage voting process with each stage using weights proportional to the values represented in that stage.

Finally, our approach suggests that market values are fungible. The European Council of Ministers is only one of several quasi-legislative bodies within the European community, and those bodies themselves compose only a small part of the overall collective decision making processes among the member countries. One can consider how control over weighted votes in a chamber might be traded off both within the body and across political domains.

Stefan Napel* and Mika Widgrén

Strategic Power Revisited

A lively debate about the adequate tool for measuring decision power in real-life institutions such as the European Union has been sparked by Garret and Tsebelis (1999, G&T). Their verdict that "power indices exclude variables that ought to be in a political analysis (institutions and strategies) and include variables that ought to be left out (computational formulas and hidden assumptions)" has motivated many authors to defend traditional power indices by clarifying the supposedly hidden assumptions underlying power formulas, and giving reasons for not taking institutions and strategies—corresponding to explicit decision procedures and rational, preference-driven agents—into account.

Earlier attempts to take players' preferences into power index calculations include the Shapley–Owen modification of the Shapley–Shubik index (Shapley 1977, Owen and Shapley 1989). It, however, still neglects the procedural aspects of decision making. So, motivated by the recent debate, Steunenberg, Schmidtchen, and Koboldt (1999, SS&K) have proposed a new "strategic power index" (StPI) which explicitly takes both decision procedures and players' preferences into account.

We disagree with G&T's critique on many points and in particular see an important role for a priori power analysis that does not rely on detailed knowledge of preferences on this and that policy issue. But, like SS&K, we see a need to have adequate tools for measuring power if information on preferences and procedures is available. Unfortunately, SS&K's measure is not the right answer. It can yield extremely counterintuitive indications in rather basic examples and, for instance, give a player negative power despite normalization. The latter makes sense once the P for "power" in the StPI is replaced by an S for "success". Namely, on closer inspection, SS&K's StPI turns out to be a useful measure of average success, however not one of power.

We propose an alternative unified approach. It takes up G&K's critique but still allows for — sometimes merely convenient, sometimes desirable — a priori power measurement. Our approach rests on a player's marginal contribution or marginal impact on the collective decision taken a posteriori as the primitive of power, comparing an actual outcome with a shadow outcome which alternatively could have been brought about by the considered player. This generalizes the concept of swings or pivot positions from cooperative games to non-cooperative games, and yields a measure of a posteriori power which is suited to take decision procedures and preferences into account. Having defined a meaningful measure of a posteriori power, meaningful a priori measures can be constructed. A priori-ness of different degrees can be considered, concerning players' preference-based or unmotivated actions, decision procedures, or both. By making corresponding distribution assumptions, one obtains traditional a priori power indices such as the Penrose index or Shapley–Shubik index as a special case. More generally, our measure defines power as a player's expected marginal impact, where expectation is

taken with respect to an appropriate probability measure on (power-relevant) states of the world.

Dieter Schmidtchen and Bernard Steunenberg

The strategic power index — responses

The paper deals with the recent responses by Machover and Widgren/Napel to the strategic power index. It highlights the strategic nature of the index. This feature seems to be downplayed by Machover and Widgren/ Napel in order to get their axiomatic approach.

Local Monotonicity Revisited[*]

Matthew Braham and Frank Steffen

In a simple voting game $v = (N, W)$, where $N = \{1, 2, \ldots, n\}$ is the set of voters, $\wp(N) = \{0, 1\}^n$ is the set of feasible coalitions, and $W(v) \subseteq \wp(N)$ called *winning coalitions* satisfying $\varnothing \notin W(v)$, $N \in W(v)$, and if $S \in W(v)$ and $S \subseteq T$ then $T \in W(v)$, the power of a player is conventionally given by a mapping $\xi: G^N \rightarrow \mathbb{R}^n_+$ that assigns to each player $i \in N$ a number $\xi_i(v)$ indicating i's power in the game v, where G^N is the set of all such n-person simple voting games.

It is a widespread belief that any reasonable measure of voting power should satisfy at least the three postulates of:

(P1) *Iso-invariance:* If there is an isomorphism of v to v' that maps a player i to i', then $\xi_i(v) = \xi_{i'}(v')$.

(P2) *Ignoring dummies:* If v and v' have exactly the same minimal winning coalitions, which is a winning coalition in which no proper subset is also winning, then $\xi_i(v) = \xi_i(v')$ for any player i common to both.

(P3) *Vanishing for dummies:* $\xi_i(v) = 0$ if i is a dummy in v, i.e. a player who can never change a winning coalition to a losing coalition has no power.

In some cases a fourth postulate has been added:

(P4) *Normalization:* $\sum_{i \in N} \xi_i(v) = 1$.

And finally, a fifth has been added:

(P5) *Local Monotonicity:* If in a weighted voting game v, where v is described by vector of voting weights $(w_1, w_2, \ldots, w_n)$ and a quota q which is the quota of votes necessary to establish a winning coalition such that $0 < q \le \sum_{i \in N} w_i$, $w_i \ge w_j$ then $\xi_i(v) \ge \xi_j(v)$.

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[*] An early forerunner of this paper was presented at the IAW research seminar in June 2001. We thank the participants for helpful suggestions.

As is well known, the Shapley-Shubik index (Shapley and Shubik 1954) naturally satisfies all five postulates, while the Banzhaf measure (Banzhaf 1965) naturally satisfies all except (P4), i.e. it requires *a posteriori* normalization. The Deegan-Packel index (Deegan and Packel 1978) naturally satisfies all except (P5) and the Holler-Packel or Public Good Index (PGI) (Holler 1982, Holler and Packel 1983) naturally satisfies only (P1)–(P3) (i.e. also requires *a posteriori* normalization).

Postulate (P4) has been shown not to be necessarily meaningful (Dubey and Shapley 1979, Felsenthal and Machover 1998, Laruelle and Valenciano 1999) and therefore on the whole has been discarded, while (P5) has been the subject of a fair amount of dispute. Freixas and Gambarelli (1997) and Felsenthal and Machover (1998) have taken the position it is such an intuitively compelling postulate that any measure that violates cannot be used as a reasonable yardstick of voting power. This would mean that the Deegan-Packel index and PGI in a sense suffer 'pathological' defects.

On the other hand, Deegan-Packel (1978) and Holler (1997) as well as Brams and Fishburn (1995) take the position that if the rationale or story of a measure is reasonable and acceptable, then we have to accept that power is not locally monotone and that we must accept this given that power is a social concept (they cite empirical evidence to this effect). The underlying argument being that it is mistaken to take an axiomatic approach to the analysis of social interaction.

This debate has ignored the general violation of (P5) by another set of measures derived from Straffin (1977). This is a probabalistic interpretation of voting power based on Owen's (1972, 1975) *multilinear extension* (MLE) of a game. As is well known, the Shapley-Shubik and absolute Banzhaf indicies can be derived as special cases of the MLE given a probability model of voter behaviour. Straffin's partial homogeneity approach allows us to mix these probability models so that we can derive an infinite set of families of power measures.

The reason for the violation of (P5) by the family of power measures derived by Straffin's approach is wholly different to that of the violation by the Deegan-Packel index and the PGI. In the latter case the reason is due to the fact that the measures are based only on minimal winning coalitions. In the former case it is due to the fact that the partial homogeneity approach does not treat voters symmetrically with the result that coalitions are not equally weighted.

While it could be argued that because the partial homogeneity approach violates (P1) it is not *a priori* and therefore (P5) may not be relevant (i.e. it is *a posteriori* power) and thus its violation has no meaning for the understanding of *a priori* power, such an argument is mistaken. In this paper we demonstrate that in the same way that any measure of voting power requires the specification of a probability model of voter behaviour (i.e. a power measure is not based only on the decision rule), this probability model implies the specification of a particular structure in which a decision rule is embedded. Thus contrary to the general belief that violation of (P1) implies attributing names or preferences to voters this is not necessarily the case at all. It is possible to violate (P1) by taking into account the fact that a committee may be structured in accordance with an organizational hierarchy, which itself is independent of the personalities and predilection of individuals and that voters fill its positions and carry out its pre-defined functions. Thus (P1) implies an undifferentiated committee structure; while the partial homogeneity approach can imply a differentiated structure (Braham and Steffen 2001). Put differently, (P1) is a very special case of *a prioricity*.

Further, we argue that (P5) is a special case of a more general monotonicity condition. That is, voting weights are a special case of *resources*. Thus local montonicity concerns the relationship between resources and power and this implies we need to take into account the value of resources as they enter a game or are augmented by the structure of the decision-making environment which forms part of a game. Contrary to the position taken by Deegan-Packel (1978, 1983) and Holler (1997) and Brams and Fishburn (1995), a violation of (P5) does not allow us to conclude that power *is* non-monotone in resources because the resources that we take into a game may be modified by the structure of the decision-making environment. In fact, we argue that this line of reasoning enables us to 'rescue' local montonicity for the Deegan-Packel index (but not necessarily for the PGI) if we recognize that the bargaining environment leads to a 'shaving' of weights (Riker 1967). Thus, weights in a game where only MWCs form are not their 'face value'.

The main conclusion of this paper is that neither iso-invariance (P1) nor local monotonicity (P5) are necessarily compelling postulates of *a priori* power and therefore it is invalid to reject (classify as 'pathological') measures that violate one or both of these conditions. In particular, we demonstrate that we cannot even restrict local monotonicty in voting weights to what Felsenthal and Machover

(1998) call I-power or 'power as influence' (in contradistinction to P-power, or power as prize). Local monotonicity is fulfilled only for very particular decison-making structures and environments.

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Annick Laruelle and Federico Valenciano*

Voting power paradoxes revisited

Different power indices have been proposed to assess the a priori distribution of power in voting situations. That is, the distribution of power among the voters for a given decision rule. Since the only recently vindicated Penrose (1946) and the later but much more popular Shapley and Shubik's (1954) and Banzhaf's (1965, 1966) indices, some other power indices have been proposed: the Coleman (1971, 1986) indices, the Deegan and Packel (1978) index, the Johnston (1978) index, and the Holler and Packel (1983) index.

The results given by these indices may differ quite widely. In order to test and compare these indices, some authors have proposed 'natural' properties that a power measure 'should' satisfy. The violations of these properties are called 'paradoxes'. There exists a variety of paradoxes in the literature. See for instance Brams (1975), Brams and Affuso (1976, 1985a, 1985b), Deegan and Packel (1982), Kilgour (1974), Fisher and Schotter (1978), and Dreyer and Schotter (1980). More recently, Felsenthal and Machover (1995, 1998) and Felsenthal, Machover and Zwicker (1998) have critically discussed some of these paradoxes and proposed new ones, like 'the bloc paradox', 'the donation paradox' and 'the bicameral paradox'.

All variations of the traditional power index concept formally take the voting rule as the only explicit input for the assessment of power. That is to say, power indices map voting procedures, usually modelled as simple games, onto vectors whose coordinates are interpreted as the 'power' of the corresponding voter. These power measures leave aside the voters' voting behaviour, and whatever personal characteristics might condition it, as preferences over the issues or their interpersonal relations, etc. Consequently, the lack of basis for a positive or descriptive interpretation of these indices is obvious, as pointed out by some authors, as Garrett and Tsebelis (1999), because no information about the voters' behaviour enters the model.

In Laruelle and Valenciano (2001) a general positive or descriptive concept of voting power measurement based on *two* independent inputs in any real-world voting situation, the *voting rule* and the *voting behaviour* of the voters, is provided. The voters' voting behaviour, dependent on their preferences over the issues at stake, etc., is summarized by a probability distribution over the vote configurations. Then a priori voting power is defined as the probability of the different voters to 'exert power', that is, being decisive when a decision is to be made according to a given voting rule, in three precise different senses. These general measures include as particular cases the Shapley-Shubik index and the (non normalized) Banzhaf index, as well as some game theoretic solution concepts.

In this paper we test 'against each other' some of the best-established voting power paradoxes and the three general measures of voting power introduced in Laruelle and Valenciano (2001). This reciprocal test sheds some new light on the meaning of these so-called paradoxes and helps to understand better the concept of power in voting situations.

In particular it shows how the voters' behaviour influences their voting power. The coherence of the three alluded notions of voting power comes out reinforced by this test.

Manfred J Holler

How to sell power indices

There is no obvious answer to the problem about the right power measure. This paper suggests that we can learn "sales arguments" for power indices by looking at the history of the concepts of social welfare and, alternatively, the equilibrium notion in game theory. For instance, there isn't any unambiguous definition of social welfare. In general, we content ourselves with a tautology: social welfare is what social welfare functions measure. Irrespective of the multitude of social welfare functions and thus by the multitude of welfare concepts, a standard paper in microeconomics has a section dedicated to "welfare analysis"—which follows the sections headlined "basic model" and "equilibrium analysis".

This paper will also discuss the dominance of the Nash equilibrium concept in game theory. Although the Nash equilibrium is a questionable behavioral description for many game situations and often leads to inconclusive results, the large majority of game theorists agree that a game outcome has to concur with a Nash equilibrium—otherwise it does not make much sense. Why does such a wide consensus not exist for a specific power index?

Josep Freixas and William S. Zwicker*

Further thoughts on weighted voting, abstention, and multiple levels of approval

In modeling the type of voting intended to register collective approval or disapproval of a proposal, the literature on voting systems has confined itself almost exclusively to the mathematical structures known as simple voting games. By their nature, simple games allow for exactly two possible levels of approval in the "input" (voters must choose between "yes" and "no") and two levels in the "output" (the proposal either passes or does not pass). Yet in many of the real voting systems that have been modeled by these games, such as that of the United Nations Security Council, abstention plays a key role as a middle level of voter approval. Indeed, Felsenthal and Machover have remarked on the extent to which some authors "mis-report the rules as though abstention were not a distinct option" (p. 22 of Felsenthal and Machover, *The Measurement of Voting Power*, Edward Elgar, 1998).

One factor that may have retarded the study of voting games with abstention is the absence of a completely satisfactory definition of weighed voting in this context. In a weighted simple game with no abstentions allowed, each voter is assigned a numerical weight, and a proposal is approved if the total weight cast in favor meets or exceeds some preset threshold. There are two tempting ways to modify the definition in order to account for abstention: collective approval can require that the *ratio* of total weight cast in favor to total weight cast against meet or exceed a preset threshold, or that the *difference* between these totals meet or exceed such a threshold. These two approaches are distinct, though each reduces to the standard definition when no voters abstain. With abstentions allowed, the UN Security Council voting system fails to be weighted under either definition.

We argue in favor of a third definition of weighted voting with abstention, which is strictly more general than either of the two mentioned above. The UN Security Council system is weighted under this new definition. Moreover, we give a strong argument for mathematical naturality; our main result is a combinatorial characterization, in the form of the "grade trade robustness" property, of weighted voting with levels of approval. This result closely parallels the known characterization of standard weighted voting via "trade robustness."

The new definition, as well as its combinatorial characterization, generalizes to the structures we call "(j, k) games," in which there are j ordered levels of input approval and k ordered levels of output approval. While it is the (3,2) games and (3,3) games that are probably most relevant to legislative voting, these are not empty generalizations. Our examples of grading systems suggest that multi-level structures with larger j and k values arise naturally as models of certain types of decision making.

NOTE: There will be some added comments on the problem of measuring voting power in (j, k) games.

Ines Lindner

Computing power indices for voting games with several levels of approval

(Provisional)

In almost all voting games, the number of possible coalitions rises exponentially with increasing number of players which makes their computational determination only possible for very small games. The standard method for getting numerical values for power measures in (binary) weighted voting games is the recursion of Mann and Shapley (1962) which avoids this problem of exponential growth in computation time. The key computational idea (due to David Cantor) was to reformulate the problem, allowing to solve the problem by means of a generating function. This reduces the problem of computing the coefficients to multiplying out a polynomial which implies linear (instead of exponential) growth of computational effort with increasing number of voters. This paper extends the idea to voting games including abstentions. The method provided is

further extendable to voting games including several levels of approval. In addition, this paper gives a probabilistic characterization of power indices in voting games with several levels of approval. This provides approximation methods for power indices, primarily based on the central limit theorem.

Agnieszka Rusinowska and Harrie de Swart

On some properties of the Hoede–Bakker index

The paper concerns an analysis of the, to the best of our knowledge not well-known, power index, called the Hoede–Bakker index. Hoede and Bakker (1982) introduced the concept of decisional power. The essential point of the Hoede–Bakker index is the distinction between the inclination to say “yes” or “no” and the final decision apparent in a vote. As Hoede and Bakker noticed “In fact, it is the difference between inclination and final decision in which the exertion of power on an actor manifests itself”. In our paper, we investigate the properties of the Hoede–Bakker index and the relations between this index and other (well-known) power indices. In particular, we investigate whether this decisional power index displays some voting power paradoxes.

Hoede, C. & R. Bakker (1982) “A Theory of Decisional Power”, Journal of Mathematical Sociology 8: 309–322

M.R. Feix, D. Lepelley, V. Merlin and J.L. Rouet

The Probability of Conflicts in Two-stage Elections

(Preliminary version)

In a two-candidate election, it might be that a candidate wins in a majority of districts or states while he gets less votes than his opponent in the whole country. In Social Choice Theory, this situation is known as the compound majority paradox, or the referendum paradox.

Although occurrences of such paradoxical results have been observed worldwide (e.g. the US presidential elections, parliamentary elections in the United Kingdom, French local elections), no study evaluates theoretically the likelihood of such situations. In this study, we propose two probability models in order to tackle this issue, for the case where each state has the same population. In both electoral models, each party has a priori equal chance to win. Our results prove that the likelihood of this paradox rapidly tends to 20.5% for the first model and to 16.5 % for the second one when the number of states increases.

Antti Pajala

Power Index Website: a voting power and power index calculation WWW-resource

The purpose of this presentation is to introduce a World Wide Web resource dedicated to the study of voting power. The aim of the project is to gather and present Internet accessible information on voting power for experts in the discipline as well as for other interested visitors.

The resource includes two services. Firstly, general and detailed information on voting power and power indices can be accessed. A section to start with discusses the concept and ideas behind voting power introducing also the standard formal notation. The next section provides definitions and formalizations for all known power indices and related measures with an easy to approach example. Attention is also paid to the known similarities and connections among the indices as well as the power index paradoxes. A third section is a database collecting references to voting power literature.

Finally, a section collecting hyperlinks to other voting power resources in the WWW can be found.

The second service is a power index calculation and voting body analyse service called POWERSLAVE. The current implementation is capable of analysing voting bodies with a maximum of 20 actors within reasonable calculation times. For now POWERSLAVE produces the following results: Regarding individual actors the number of swings, Shapley-Shubik, absolute and std. Banzhaf, Coleman preventive power, Deegan-Packel, Holler, Zipke and Zipke nominators, Colomer and Johnston index values.

Regarding the voting body as a whole: Theil, Laakso and Herfindal-Hirschmann concentration and fragmentation scores, numbers of winning coalitions, losing coalitions, minimal winning coalitions and coalitions with a swing.

The Power Index Website can be reached at www-address:
http://powerslave.val.utu.fi

Dennis Leech

Power Indices as an Aid to Institutional Design

A priori voting power can be useful in helping to design a weighted voting system that has certain intended properties. Power indices can help determine how many weighted votes each member should be allocated, and what the decision rule should be. These choices can be made in the light of a requirement that there be a given distribution of

power, and/or a desired division of powers between individual members and the collective institution. These issues were discussed by Nurmi (1982) and Lucas (1983), and recent applied work on the topic has been done by Laruelle and Widgren (1998) and myself in two forthcoming papers.

This paper will focus on the 'inverse problem', which is the specific problem of choosing the weights given that the power indices and decision rule are fixed. Thus, given the weight vector $\mathsf{w} = (w_1,\ldots, w_n)$, the quota q, and the power index vector $\mathsf{p} = (p_1,\ldots, p_n)$, I will discuss the relationship between the w, q and p. How reasonable is it to assume a set of functional relationships that are written $\mathsf{p}(\mathsf{w}, q)$? How should we choose w and q in order to achieve given p? Suppose the desired power indices are $\mathsf{d} = (d_1,\ldots,d_n)$, then how do we solve the problem $\mathsf{d} = \mathsf{p}(\mathsf{w}, q)$? What is the nature of the solution? What is the role of normalisation? This paper tries to answer these technical questions by seeking to exploit similarities between the literatures on power indices and economic general equilibrium and investigates the relationship between the inverse problem and computable general equilibrium. The most importance reference on computable general equilibrium is the classic book by Arrow and Hahn (1971).

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