

ABSTRACTS OF PAPERS

Note that in many instances the following abstracts summarise works that are in various stages of completion. Please do not quote without consulting the author(s).

Aleskerov, Fuad, Manfred J. Holler and Rita Kamalova.

Title **Power Distribution in the Weimar Reichstag in 1919-1933**

Abstract: We present an analysis of the distribution of voting power in the Reichstag of the Weimar Republic based on the outcomes of the nine general elections in the period 1919–1933. The paper contains a brief description of the political and electoral system of the Weimar Republic and a characterization of the main political actors and their political views. The power distributions are evaluated by means of the Banzhaf index and three new indices which take into account the parties' preferences to coalesce. A model is constructed to evaluate the parties' preferences with reference to the closeness of the ideological positions in a one-dimensional political space. Paper.

Claus Beisbart

Dortmund University

Title: Measurements of voting power and probability

Abstract: Measures of voting power (I-power) quantify the extent to which a voter can make a difference by using a probability. But probabilities and probabilistic statements are a matter of much philosophical controversy. There are various rival interpretations of probabilistic statements, some more subjectivist, others more objectivist. The aim of this paper is to see which interpretations are most fitting for understanding the measurement of voting power. The hope is that, in this way, we can also clarify how the values of the probabilities should be set. This issue is immediately relevant for deciding whether power should (in certain contexts, for certain purposes) be measured using empirical information (i.e. in an a posteriori way) or not. It is also relevant for the question how coalitions should be counted. Finally, the interpretation of the probabilities may have a bearing on the justification why voting power should be equalized (at least in certain contexts, for certain purposes, maybe). The paper goes through a number of well-known interpretations of probability and discusses their applicability to voting power. The results are as follows: The classical view according to which probabilities are normalized counts of possibilities is in a trivial way compatible with the way the Banzhaf measure and the Shapley-Shubik index are calculated. However, it does not give us any guidance as to how exactly we should individuate possibilities and count them (this is a well-known general criticism of the classical view). Subjective views of the probabilities make measures of voting power pure estimates. These estimates may either concern objective probabilities or non-probabilistic facts. In either case, subjective probabilities are not fundamental, and we can dispense with them. Frequentist interpretations of probabilities do not help either, because we may want to apply measures of voting power in cases in which no suitable actual frequencies are available. Hypothetical frequencies do not solve this problem, if Jeffrey is right in claiming that there is not fact of the matter what the hypothetical frequencies are. Propensity views raise a number of interesting questions for the measurement of voting power. I argue that they push us towards a posteriori voting power. In fact, any objectivist view that underwrites Lewis's Principal Principle does so. This is a problem for the Banzhaf measure and the Shapley-Shubik index. My conclusion comes in two parts: 1. If we want to underwrite the application of the Banzhaf measure and the Shapley-Shubik index, we should understand voting power as a count of possibilities. A fair case can be made that we should not call this a probability. The challenge is to find out how the possibilities should be individuated. I offer a suggestion, as far as normative assessments of voting rules are concerned. 2. Regarding descriptive purposes, some latter-day objectivist views of probabilities are fair candidates for a better understanding of voting power.

Sreejith Das

Birkbeck, University of London

Title: An Analysis of the Intrinsic Differences Between the Commonly Applied Voting Power Techniques

Abstract: Voting power science is a field of co-operative game theory concerned with calculating the influence a voter can exert on the outcome of a voting game. The techniques used to calculate voting power have names like the Shapley-Shubik index, and the Banzhaf measure. They are invaluable when used to design democratically fair voting games, however, there is currently no consensus over which technique is best. This paper will examine the relative merits of the different techniques. Ignoring the well known differences in probability models, this paper will focus upon the less well known differences in underlying measures. With the analysis showing that the Shapley-Shubik index is afflicted with a fundamental flaw, restricting its use in many real world voting games, it soon becomes apparent that the dissimilarities between the techniques extend far beyond their methods of counting voting coalitions.

Maria Ekes

Warsaw School of Economics

Title Banzhaf and Shapley indices in games with a coalition structure - a special case study

Abstract: The paper involves the comparison of the Shapley index and Banzhaf index in games with a coalition structure. We analyze two possible approaches in both cases - we calculate voters' power in a composite game or we apply the modification of original indices proposed by Owen for games with a priori unions. The behavior of both indices is compared basing on the voting game with 100 voters and different coalition structures. We analyze changes of the power (measured by means of Banzhaf index and Shapley index) implied by changes of the size and composition of coalition structures as well as by different methodology of measuring the voters' power (composite game versus game with a priori unions).

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Title Ordinal equivalence of power indices

Abstract: In this work the notion of power is analyzed from a qualitative point of view. We are interested in the hierarchies that power indices produce rather than in exact values for them. Hence, proportional notions of power as the ones introduced by Penrose, Banzhaf or Coleman (two measures in this latter case) become in this approach equivalent.

Different reasonable power indices provide different orderings of importance for voters, so that the evaluation of (a priori) power in (binary) voting systems is quite arbitrary since the rankings highly depend on the chosen particular power index. One is left with the hope that such discrepancies occur because the voting system at hand is rare enough. To better understand what happens, we analyze the ordinal equivalence of some well-known power indices.

Power indices based on symmetric probabilistic values (or semi-indices) with positive coefficients (regular semi-indices), as the Banzhaf index or the Shapley-Shubik index, share the same rankings of voters within the class of weakly complete games. Weakly complete games contain weighted and complete games and, therefore, the most common real binary voting systems. This partially solves the problem because power indices are mostly applied to non-complicated voting systems derived from real problems which almost always are weakly complete. However, the analytical problem of studying the ordinal equivalence, even for regular semi-indices, is still not solved outside weakly complete games. An extension is provided in this work, which implies the ordinal equivalence of the Banzhaf and Shapley-Shubik power indices.

The Johnston index also shows a good behavior since it is ordinally equivalent to Banzhaf and Shapley-Shubik indices in a sufficiently large class of games containing complete games. Necessary and sufficient conditions are given to determine such a class. Oppositely, families of power indices based only on minimal winning coalitions, as Holler or Deegan-Packel indices, show a quite undesirable odd ranking behavior. This study shows evidence in favor of either regular semi-indices or the Johnston index.

Holler, Manfred

University of Hamburg

Title Pathology or Revelation? – The Public Good Index

Abstract This paper sets out from a discussion of the well-known fact that the PGI violates the axiom of local monotonicity (LM). It argues that cases of nonmonotonicity indicate properties of the underlying decision situations which cannot be brought to light by the more popular power measures, i.e., the Banzhaf index and the Shapley-Shubik index, that satisfy LM. The discussion proposes that we can constrain the set of games such that LM also holds for the PGI. A discussion of causality follows. It suggests that the nonmonotonicity can be the result of framing the decision problem in a particular way and perhaps even ask the “wrong question.” Correspondingly, the PGI can be interpreted as an indicator. The probabilistic relationship of Banzhaf index and PGI identifies the factor which is responsible for the formal difference between the two measures and therefore for the violation of LM that characterizes the PGI, but not the Banzhaf.

Serguei Kaniovski

Title: **Theorems for Exchangeable Binary Random Variables with Applications**

Abstract: The presentation discusses (a) parameterizations of the joint probability distribution of correlated binary random variables, (b) the probability of at least k successes in n exchangeable correlated binary trials and (c) its bounds when the correlations are unknown. This probability finds wide application in reliability and decision theory. It can be used to compute voting power as the probability of casting a decisive vote when the votes are correlated. Empirical evidence refutes the assumption of independent votes required in the classic versions of the Condorcet Jury Theorem and the Penrose - Banzhaf measure of voting power.

Werner Kirsch

Jessica Langner,

Fakultät für Mathematik und Informatik, FernUniversität Hagen, Germany

Title: **The fate of the square root law for correlated voting**

Abstract: The square root law by Penrose gives an answer to the question of fair representation in two-tier voting systems. Under the assumption of independence of the voters the square root law can be based on two different lines of thought (as given in the book by Felsenthal and Machover). The first approach uses the Penrose-Banzhaf power index to compute the square root distribution of power as the unique one giving all voters the same share of power. The independence of the voters' behavior is implicitly reflected by the choice of this particular power index (as opposed to the Shapley-Shubik index, for example). In the second approach the democracy deficit is minimized in the sense of smallest mean square error (Second square root law). It turns out that a voting system with a square root distribution of weights is the optimal (i.e. fairest) voting system among all weighted voting systems. Here the independence assumption is explicitly used in the definition of the average ("mean" square error). In this paper we will discuss to which extend the independence result can be relaxed and to which extend the result has to be modified to include correlation structures of the voters. The models we consider are inspired by spin models from statistical physics. We compute the asymptotic behavior of the democracy deficit for a large class of models and relate this situation to certain power indices. Our models include those giving the Penrose-Banzhaf power index as well as what we call the Shapley-Shubik model but also many more models which can be tuned and compared to real data. Within this approach we may also define new classes of power indices realizing complex correlation structures between voters.

Jean-Francois Laslier

Title: Optimal Apportionment

Abstract. See the paper.

Leech, Dennis

University of Warwick

Title: Asymptotic behaviour of power indices

Abstract: What happens to voting power indices in limiting cases? On the one hand, in some real-world cases that have been studied the power indices (both Penrose-Banzhaf and the Shapley-Shubik indices) have been found to be nearly proportional to the weights (eg the EUCM, US Electoral College) and this has been interpreted as due to large numbers of voters (27, 51 respectively). On the other hand, we have the older literature from cooperative game theory, due to Shapley and his collaborators, showing that, where there are a finite number of voters whose weights remain constant in relative terms, and where the quota remains constant in relative terms, while the total number of voters increases without limit the powers of the voters with finite weight tend to limiting values that are, in general, not proportional to the weights. These theorems are supported by empirical studies of large voting bodies (eg. the IMF/WB boards, corporate shareholder control). This paper proposes a redefinition of the limiting case in terms of the Laakso-Taagepera index rather than the number of voters. It shows that the power indices converge to weights as this index of fragmentation increases without limit. This reconciles the different theoretical and empirical results that have been found for large voting bodies.

Maaser, Nicola

Bayreuth University

and Stefan Napel

Designing Decision Rules in Two-Tier Voting Systems

Abstract The paper studies two-tiered voting systems in which differently sized countries, states, or districts each elect a representative who later votes at a union level on their behalf. Considering a unidimensional spatial voting model and simple majority rule we investigate weighting rules at the union level that realize the ideals of equalizing individual influence on collective decisions, equalizing expected utility across individuals and maximizing the total expected utility of all individuals in the union. The latter can be interpreted as minimizing the mean deviation of the outcome of the indirect decision-making process from the outcome of a ‘direct democracy’ simple majority rule. We analyze artificial constituency configurations as well as the Council of the European Union.

Merlin Vincent

University of Caen

Gabriele Esposito, Mathieu Martin and Fabrice Barthelemy

Title. Fair apportionment in the Italian Senate: Which Reform should be Implemented

Abstract. In Italy, a deep change of the Senate's role was planned many times. In this paper we analyze the fairness of the 2007 reform proposal concerning the apportionment of the seats between the regions for the Italian Senate. Theory of power indices is used to compare the actual case with the proposed one. Four scenarios are proposed, according to whether (1) the senators belonging to the same region vote in block or according to party line and (2) the voting rule in a region is the current one, that is a proportional rule with a minimal number of seats for the winner, or a winner takes all system. Our objective is to determine which apportionment is closer to the equal distribution of power among the citizens. In addition, we will seek for apportionments that are closer to the ideal representation than the ones proposed by politicians. We will also derive the probability that these different apportionments produce a referendum paradox, i.e. exhibit a majority in the Senate different from the national popular majority. In order to do that, we will use Monte Carlo simulations.

JEL classification: C7, D7 Keywords: Power index, Banzhaf, Italian Senate, apportionment, voting paradox, Monte Carlo simulation.

Nicholas R. Miller

University of Maryland

**Title: A PRIORI VOTING POWER WHEN ONE VOTE COUNTS IN TWO WAYS,
WITH APPLICATION TO VARIANTS OF THE U.S. ELECTORAL COLLEGE**

Abstract: When we try to measure the *a priori* (absolute Banzhaf) voting power of individuals under variants of the U.S. Electoral College, two such plans present special difficulties: the Modified District Plan, under which a candidate is awarded one electoral vote for each Congressional District he carries and two electoral votes for each state he carries, and the National Bonus Plan, under which a candidate is awarded all the electoral votes of each state he carries (as at present) plus a “national bonus” of some fixed number of electoral votes (say 100) if he wins the national popular vote. This is because, under these arrangements, each voter casts a single vote that counts two ways: in the voter’s district and state under the Modified District Plan, and in the voter’s state and the nation as a whole under the National Bonus Plan. In his original analysis, Banzhaf (1968) evaluated voting power under the Modified District Plan by calculating a voter’s two-stage voting power first through the district vote and then through the state vote and then adding the two values (probabilities of decisiveness) together. Unfortunately, this approach cannot be justified, because it ignores interdependencies in the way district and state electoral votes may be cast — in particular, even though individuals cast statistically independent votes, the fact that they are casting votes that count in the same way in two tiers (districts and states or states and nation) induces a correlation between popular votes at the district and state levels within the same state. That this problem is serious is indicated by the fact that mean individual voting power under the District system, when calculated in the Banzhaf manner, considerably exceeds individual voting power under direct national popular vote, which Felsenthal and Machover (1998, pp. 58-59) show is a logical impossibility for a simple voting game. The proposed paper will explore this issue and present a method for calculating voting power when one vote count in two ways. It will be applied first to a simple example of district plus at-large representation and then to the Modified District and National Bonus variants of the Electoral College. While an analytic solution to such calculations may be possible, the difficulties appear to be formidable, and I proceed computationally by generating a very large sample of random (Bernoulli) elections, with electoral votes awarded to the candidates on the basis of each plan. This generates a database that can be manipulated to determine expected distributions of electoral votes for a candidate under specified contingencies with respect to first-tier voting, from which relevant second-tier probabilities can be inferred.

Maria Montero,

Nottingham University

Alex Possajennikov and Martin Sefton

Title: An Experimental Market for Votes

Abstract: In this paper we take an alternative approach to the measurement of power. Suppose a legislature must vote on a bill. There are two lobbyists, one of which favours the proposed bill whereas the other wants the status quo to prevail. The two lobbyists have identical budgets, and distribute them simultaneously across the voters in the legislature. The voters do not care about how they vote and do the bidding of whoever pays them most. The expected share of the budget given to a voter can be used as a measure of the voter's market value or P-power. If voters have different voting weights, the question arises of how the market value relates to these weights. We investigate this setup for the case of apex games, which are the simplest games with asymmetric voters. In an apex game there are n players, of which one is large and $n-1$ small. A proposal can be passed with the votes of the large player and at least 1 small player, or with the votes of all small players together, thus the large player can replace $n-2$ small players. Even though the large player and $n-2$ small players are interchangeable, the equilibrium market value of the lobbying game described above gives the large player more than $n-2$ times the value of a small player; power indices such as the Shapley-Shubik index (but not the nucleolus) make the same qualitative prediction. The equilibrium also predicts that lobbyists may distribute their budget over a coalition that is larger than minimal winning; this is due to the uncertainty about the strategy of the other lobbyist. We investigate this setup experimentally and find qualitative support for both theoretical predictions. The empirical market value of the large player is above the combined value of $n-2$ small players but below the equilibrium prediction. Similarly, the lobbyists try to bribe coalitions larger than minimal winning relatively often, though not as often as the theory predicts.

Stefan Napel

Bayreuth University

Title: Strategic vs. Non-strategic Approaches to Power in Voting Bodies

Abstract: The presentation discusses the measurement of power in voting bodies like the European Union's Council of Ministers from the a priori perspective of constitutional design using two distinct approaches: (1) applying traditional voting power indices; (2) carrying out strategic equilibrium analysis of particular legislative procedures, like the consultation procedure and the codecision procedure. It clarifies why both approaches lead to different power indications, and investigates the determinants of the differences' magnitudes. Depending on one's assumptions about external institutions, such as the European Commission or European Parliament, traditional indices turn out to deliver a good approximation of the relative strategic power inside an institution like the Council. Analysis of intra-institutional relative power by traditional indices may, therefore, be justified as a short-cut to more tedious strategic analysis. However, traditional indices have serious problems in capturing absolute intra-institutional power and may yield misleading comparative statics.

Nurmi, Hannu

Turku University

Title The Keys May Not Be under the Street Lamp: Aspects of Power Overlooked by Power Indices

Abstract The a priori indices of voting power concentrate on actor resource distributions and decision rules to determine the theoretical influence over outcomes by various actors. That these indices sometimes seem to be at odds with the intuitive distribution of real power in voting bodies follows naturally from their a priori nature. Indices based on actor preferences address this by equating an actor's voting power with the proximity of voting outcomes to his/her ideal point. With a simple argument using aggregation paradoxes we show that the preference-based indices may, in some circumstances, be just as misleading measures of power as the classic ones. Our main aim is to delineate the proper scope for power indices. In the pursuit of this aim we try to show that the procedures resorted to in making collective decisions are as important – if not more so – as the actor resource distribution. We review some results on agenda-systems to drive home this point. The proper role of power indices then turns out to be in the study of actor influences over outcomes when the actors are on the same level of aggregation and comparable in the sense of having similar sets of voting strategies. We finally discuss those aspects of power relationships in collective bodies that are ignored by the indices.

Puckelsheim, Friedrich

and Olga Ruff

Title A probabilistic re-view on F&M's "Measurement of Voting Power"

Abstract I would plan to talk on the interplay of a game-theoretic approach and a probabilistic approach to weighted voting decision rules, i.e. a melange of our two recent papers:

(with [O. Ruff](#)) A probabilistic synopsis of binary decision rules. *Social Choice and Welfare* **35** 501-516. [[Request copy](#)]

(with A. Käufel, [O. Ruff](#)) Abstentions in the German Bundesrat and ternary decision rules. Universität Augsburg, Institut für Mathematik, Preprint 4/2010. opus.bibliothek.uni-augsburg.de/volltexte/2010/1558/

Dieter Schmidtchen

Saarbrücken University

Title: On the Possibility of a Preference-based Power Index – The Strategic Power Index Revisited

Abstract. The SPI rests on a notion of power as the expected or average distance between players' ideal points and the equilibrium outcomes in policy games in which players have different abilities to affect the final outcome of the decision making procedure (Steunenberg/Schmidtchen/Koboldt 1999; Steunenberg/Koboldt/ Schmidtchen 1995, 1996). The smaller the mean distance between a player's ideal points in the policy space and the equilibrium outcomes, the more power is attributed to a player. In contrast to traditional power indices such as, for example, the Shapley-Shubik or the Banzhaf index this new method of power measurement is not based on the notion that players need to form some kind of majority or winning coalition and it employs the analytical tools of non-cooperative game theory. Actor preferences, the policy space, as well as the rules of the decision-making process, are fully integrated into the analysis. Since it allows players to act strategically, this index is labeled the *Strategic Power Index*. The calculation of the SPI is based on a two-step procedure: In the first step, the differences between a player's ideal points and the equilibrium outcomes are determined. Then, the expected or average distance is calculated. Taking expectations serves to level out the effect of luck on the equilibrium outcome of a specific policy game. The intuition is that the power of a player resides only in the game form or the rules of a game and not in the way a specific game is played.

Frank Steffen

University of Liverpool Management School

and René van den Brink

Tinbergen Institute, VU University

Title On the Measurement of Success and Satisfaction

Abstract In their volume 'The Measurement of Voting Power' Felsenthal and Machover (1998) point out that the notion and measurement of success has been part of the theory of voting power since the first scientific contribution in the field by Penrose (1946). However, it received relatively little attention as it was regarded to be 'virtually identical' with the notion of power differing only in using a different scale of measurement. Only recently Laruelle and Valenciano (2006) emphasized and vindicated the relevance of success for the normative assessment of collective decision-making mechanisms. For the purpose of their analysis they treat success, i.e. the correspondence between an actor's choice and the collective decision, and satisfaction, i.e. the correspondence between an actor's inclination and the collective decision, as synonyms. Although this is permissible for simultaneous decision-making mechanisms as in this case satisfaction coincides with success, in this paper we show that if we allow for sequential decision-making mechanisms, this coincidence no longer holds. It turns out that satisfaction entails success as one component. Making use of an action-based approach based on an extensive game form we develop a satisfaction and a success score and measure for sequential decision-making mechanisms. We illustrate some properties of these scores and measures by comparing and contrasting satisfaction, success, and power for the sequential decision-making mechanisms in different hierarchical organizations. understand voting power as a count of possibilities. A fair case can be made that we should not call this a probability. The challenge is to find out how the possibilities should be individuated. I offer a suggestion, as far as normative assessments of voting rules are concerned. 2. Regarding descriptive purposes, some latter-day objectivist views of probabilities are fair candidates for a better understanding of voting power.

Justyna Winnicka

Warsaw Economic School

Title Shapley index for games with r alternatives with a priori unions

Abstract This paper is about games with many alternatives extended with structure of a priori unions. It presents four methods of extending Shapley index to class of games with r alternatives with a priori unions. Two of the methods are based on classical Owen construction applied to some cooperative game derived from game with r alternatives. The second pair consists in applying Owen construction directly to game with r alternatives. After analysis two methods were abandoned because they led to results contrary to a common sense. We show that two other methods give the same index.

Karol Życzkowski

Wojciech Slomczynski

Jagiellonian University

Title: Square root voting system, optimal threshold and π

Abstract: The problem of designing an optimal weighted voting system for the two-tier voting, applicable in the case of the Council of Ministers of the European Union, is investigated. Various arguments in favour of the square root voting system, where the voting weights of member states are proportional to the square root of their population are discussed and a link between this solution and the random walk in the one-dimensional lattice is established. It is known that the voting power of every member state is approximately equal to its voting weight, if the threshold R for the qualified majority in the voting body is optimally chosen. We analyze the square root voting system for a generic 'union' of M states and derive in this case an explicit approximate formula for the level of the optimal threshold: $R \approx 1/2 + 1/\sqrt{\pi M}$. The prefactor $1/\sqrt{\pi}$ appears here as a result of averaging over the ensemble of unions with random populations. **Keywords:** Square root voting system, optimal threshold