

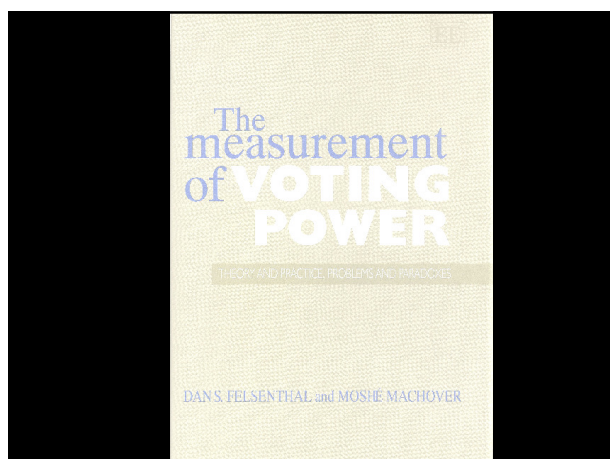
Voting Power in Practice Symposium
20-22 March 2011, London School of Economics
A prob. re-view on F&M's Meas. of Voting Power

Professor Dr. Friedrich Pukelsheim
Lehrstuhl für Stochastik und ihre Anwendungen
Universität Augsburg

A probabilistic 're-view' on
F&M's *Measurement of Voting Power*

1. The Book
2. Probability models

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Achievements of *The Book*:

- Conceptual meaning
- Mathematical theory
- Real applications

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Voting weight
↓
Voting power

Stimmgewicht
↓
Stimmkraft

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- Assembly
- Profile space
- Regions of growing acceptance

$$N = \{1, 2, \dots, n\}$$

$$\Omega_N = \{-1, 0, +1\}^N \ni a = (a_j)_{j \in N}$$

$$[a, 1_N] = \{b \in \Omega_N \mid a \leq b \leq 1_N\}$$

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$W_N \subseteq \Omega_N$ is a "decision rule" when

- (1) $1_N \in W_N$ and (2) $0 \notin W_N$ and
- (3) $\forall a \in W_N : [a, 1_N] \subseteq W_N$

Events of interest:

$$C_j = \{a \in \Omega_N : j \text{ is critical in } a\}$$

$$A_j = \{a \in \Omega_N : j \text{ agrees with } a\}$$

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Probabilities of interest:

- $P(W_N)$ efficiency of W_N
- $P(C_j)$ influence probability of j
- $\sum_{j \in N} P(C_j)$ influence sensitivity of W_N
- $P(C_j)/\sum$ power share of j
- $P(A_j)$ success probability of j

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Penrose/Banzhaf P_N (no abstentions):

Theorem (4 in Ruff/P. 2010)

If P is a partitioning of N and each bloc B in P has its decision rule W_B then, for all $j \in A \in P$:

$$P_N(C_j) = P_A(C_j | W_A) \cdot Q_P(C_A | W_P)$$

where $Q_P = \bigotimes_{B \in P} \text{Bernoulli}(P_B(W_B))$

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Penrose/Banzhaf P_N^t (with abstentions):

Theorem (5.2 in Birkmeier/Käufel/P. 2011)

With abstention probability $t \in [0, 1]$:

$$\sum_{j \in N} P_N^t(C_j(W_N)) = \frac{1}{1-t} E_{P_N^t}[\sigma_{W_N}]$$

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Shapley/Shubik S_N^t (with abstentions):

Theorem (in Diss. Birkmeier 2011)

With abstention probability $t \in [0, 1]$:

$$\sum_{j \in N} S_N^t(C_j(W_N)) = \frac{1-t^n}{1-t}$$

$$= 1 + t + t^2 + \dots + t^{n-1}$$

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