A PRIORI VOTING POWER WHEN ONE VOTE COUNTS IN TWO WAYS, WITH APPLICATION TO TWO VARIANTS OF THE U.S. ELECTORAL COLLEGE

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Voting Power When One Vote Counts in Two Ways

- Problem: how to calculate individual *a priori* voting power in *two-tier* voting systems when voters cast a *single vote that counts in two different ways*, e.g., both within a district and at-large.

- Two U.S Electoral College variants raise this question:
  - *The (Modified) District Plan*: each candidate wins one electoral vote for each Congressional District he carries *and* two electoral votes for each state he carries;
  - *The National Bonus Plan*: 538 electoral votes are apportioned and cast as under the existing system at present but the candidate who wins the most popular votes nationwide is *also* awarded a *bonus* of some number additional electoral votes

Voting Power in the U.S. Electoral College

- Several years ago I had a commission to write an encyclopedia entry on “Voting Power in the U.S. Electoral College” and decided to include a chart displaying individual voting power by state.
  - I was confident I understood the properties and proper interpretations of voting power measures from Dan Felsenthal and Moshe Machover’s treatise on *The Measurement of Voting Power.*
  - I also believed that I could make all necessary calculations using the on-line *Computer Algorithms for Voting Power Analysis* created and maintained by Dennis Leech and Robert Leech.

Voting Power in the Electoral College (cont.)

- F&M show that we *s*hould use the (absolute) Banzhaf measure to measure voting power in this context.
- They also show that (unlike the relative Banzhaf index or the Shapley-Shubik index), the *absolute Banzhaf* value has a probabilistic interpretation that is directly meaningful and useful:
  - It is a voter's *a priori probability of casting a decisive vote*, i.e., one that determines the outcome of an election (for example, by breaking what otherwise would be a tie).
  - In this context, "*a priori* probability" means, in effect, *given that all other voters vote randomly*, i.e., vote for either candidate with a probability p = .5 (as if they independently flip fair coins).
  - I refer to such random two-candidate elections *Bernoulli elections*.

Voting Power in the Electoral College (cont.)

- Since absolute Banzhaf voting power can be interpreted as a probability, we can calculate individual voting power in two-tier systems, i.e., *double decisiveness*, by multiplying the voting powers in each tier together.

- For example, the *a priori* voting power of an individual voter under the existing Electoral College is:

the probability that the voter cast a decisive vote with his/her state

x

the probability that the state casts a decisive bloc of electoral votes in the Electoral College.

Voting Power in the Electoral College (cont.)

- The probability that a voter casts a decisive vote in the state is essentially
  - the probability that the state vote is tied (before the voter in question casts his/her decisive vote),
  - which, given the state has *n* voters, is approximately $\sqrt{(2/\pi n)}$.
- The probability that a state casts a decisive vote can be calculated using the *Voting Power Analysis* website to evaluate the Electoral College Game 51:538(270: 55, 34, …,1).
- I produced the following chart for the 2000 apportionment, which shows how individual voting power varies across states with different populations.
  - Note that mean individual voting power falls short of individual voting power under direct popular election (which of course is also equal across states).
  - Note also that voting power in the following a subsequent charts is expressed in adjusted/rescaled terms such that individual voting power in the least favored state is set at 1.0000 .

Figure 1 --- Voting Power under the Existing Electoral College

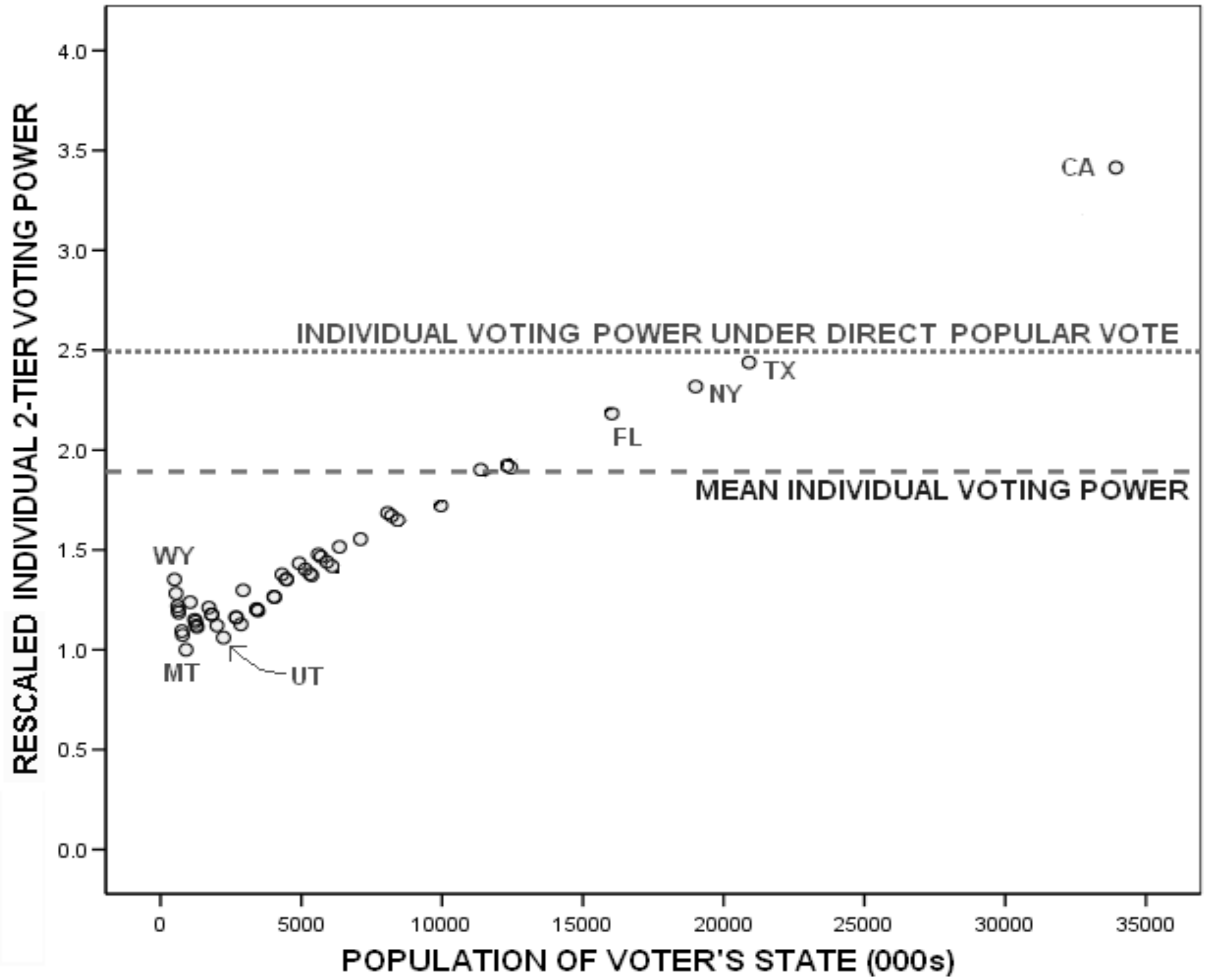


Figure 1 Individual Voting Power by State Population under the Existing Apportionment of Electoral Votes

Extension to EC Variants

- Having completed my encyclopedia entry, I thought it would be relatively easy and interesting to make similar charts for variants of the Electoral College, pertaining to
  - how electoral votes are apportioned among the states (though this issue has not been a matter of much public discussion), and
  - how electoral votes are cast within states,
    - concerning which there are various “Plans” that have been proposed as constitutional amendments,
      - though some can be adopted by states unilaterally.

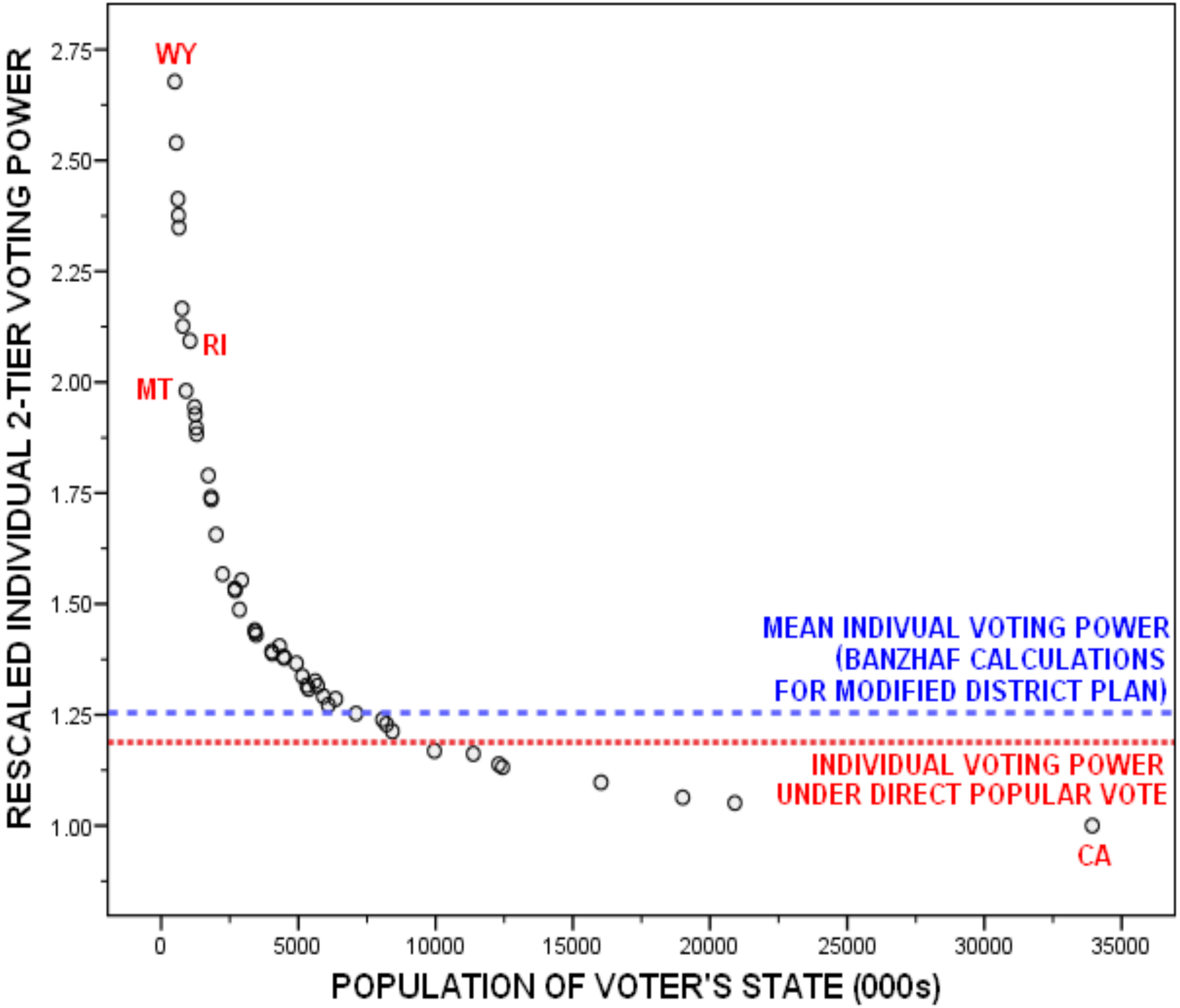
- However, as noted at the outset, two EC variants presented special problems.

Modified District Plan

- In his original work, Banzhaf (in effect)
  - determined each voter's probability of double decisiveness
    - through his/her district and the EC and
    - through his/her state and the EC, and then
    - summed these two probabilities.
- His table of results (for the 1960 apportionment) is comparable to the following chart (for the 2000 apportionment).

John F. Banzhaf, "One Man, 3.312 Votes: A Mathematical Analysis of the Electoral College," *Villanova Law Review*, Winter 1968.

Figure 2 --- Banzhaf Calculations for Modified District Plan



Problems with Banzhaf's Calculations

- There is a vexing problem: mean individual voting power so calculated exceeds voting power under direct popular vote.
- This is anomalous because Felsenthal and Machover (pp. 58-59) demonstrate that, within the class of ordinary voting games, mean individual voting power is maximized under direct popular vote.
- This anomaly was not evident in Banzhaf's original analysis, because
  - he reported only rescaled voting power values, and
  - he made no voting power comparison with direct popular vote (or with other Electoral College variants).
  - Recalculation of Banzhaf's results (using 1960 apportionment populations) shows that the same anomaly exists in that data.

- The same anomaly occurs with the National Bonus Plan, when voting power is calculated in the Banzhaf manner.

Figure 8A --- National Bonus (B = 101) Plan

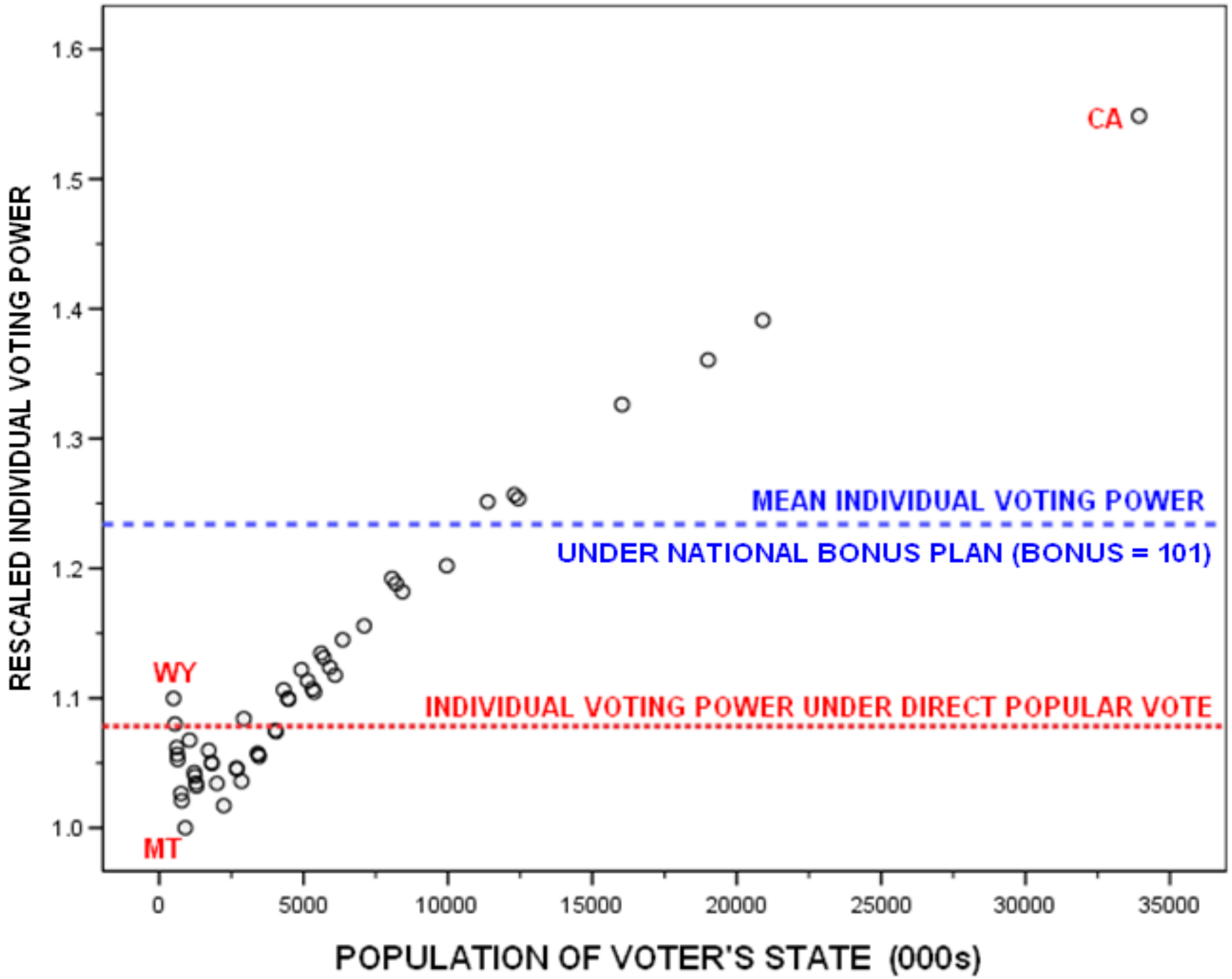


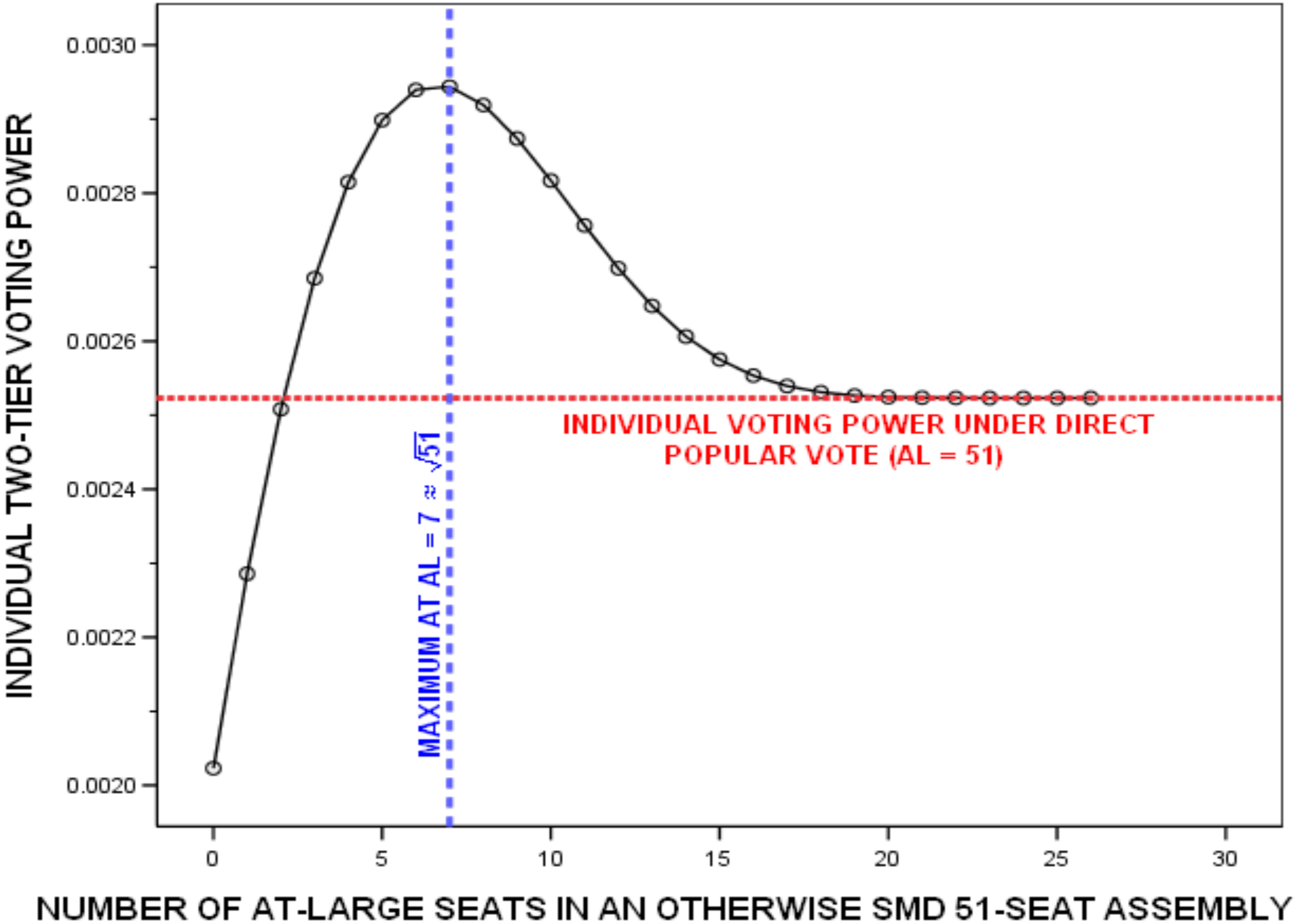
Figure 8A Individual Voting Power by State Population under the National Bonus Plan (Banzhaf/Edelman Calculations)

Voting Power and At-Large Representation

- I then discovered an article by Paul Edelman that seemed to show that individual voting power in two-tier voting systems can be enhanced beyond that entailed by direct popular vote rule by providing at-large representation as well as district representation.
- Specifically Edeleman shows that
  - if voters cast *two separate and independent votes* for district and at-large representatives, and
  - if the at-large representatives are elected on a winner-take-all basis and vote as a bloc in the top tier, then
  - individual voting power is maximized if the number of at-large representatives is equal to approximately the square root of the total number of representatives.

Paul Edelman, "Voting Power and At-Large Representation," ***Mathematical Social Science,*** **2004**

Figure 3 --- The Edelman At-Large Effect



Edelman (cont.)

- This is key assumption in Edelman's analysis: voters cast *separate and independent votes for district and at-large representation*.
- This assumption clearly does not apply (as Edelman notes) to the Electoral College variants, in which a voter casts a single vote that counts in two ways:
  - in the voter's district and state (Modified District Plan), or
  - in the voter's state and the nation as a whole (National Bonus Plan).
  - Moreover, even if these Plans were modified to allow separate independent votes, a voter would never have reason to "split" these votes.
- Edelman notes (as I just did) that Banzhaf computed voting power (in the Modified District Plan) as if voters were casting independent and separate votes.
  - "It is possible that, in a game as large as this modified presidential game, the power in the [Edelman setup] is the same as (or close to) the power in the standard [setup]."

A Warm-up Exercise

- As a warm-up exercise, let us consider the simplest case in which nine voters are partitioned into three uniform districts and elections are held under four distinct voting rules.
- Under all rules, voters cast a single vote that counts in two ways, i.e., first in the 'district' tier and second in the 'at-large' tier.
- With the U.S. Electoral College in mind, we may refer to first-tier votes as 'popular votes' and second-tier votes as 'electoral votes.'
  - *Pure District System*: there is l electoral vote for each district, and the candidate winning a majority of electoral votes (2 out of 3) is elected;
  - *Small At-Large Bonus System*: there is 1 electoral vote for each district plus 1 at-large electoral vote, and the candidate winning a majority electoral votes (3 out of 4) is elected (ties may occur in the second tier);
  - *Large At-Large Bonus System*: there is 1 electoral vote for each district plus a bloc of 2 at-large electoral votes, and the candidate winning a majority of electoral votes (3 out 5) is elected; and
  - *Pure At-Large System*: there no districts or, in any case, there is a bloc of 4 or more at-large electoral votes, so the districts are superfluous and the candidate winning a majority of the popular votes (e.g., 5 out of 9) is elected.

Table 1

All Possible Vote Profiles (Bipartitions) Confronting a Focal Voter *i* in District 1, Given a Total of Nine Voters Uniformly Partitioned into Three Districts

			Number of Times Voter i is Decisive (Total is i's Bz Score)			
Pop. Vote	***District Vote Profile***	***n****	DV	DV + 1 AL	DV + 2 AL	PV (all AL)
8-0	(2-0) (3-0) (3-0)	1	0	0	0	0
	Total	**1**	**0**	**0**	**0**	**0**
7-1	(1-1) (3-0) (3-0)	2	0	0	0	0
	(2-0) (2-1) (3-0)	3	0	0	0	0
	(2-0) (3-0) (2-1)	3	0	0	0	0
	Total	**8**	**0**	**0**	**0**	**0**
6-2	(0-2) (3-0) (3-0)	1	0	0	0	0
	(1-1) (2-1) (3-0)	6	0	0	0	0
	(1-1) (3-0) (2-1)	6	0	0	0	0
	(2-0) (3-0) (1-2)	3	0	0	0	0
	(2-0) (2-1) (2-1)	9	0	0	0	0
	(2-0) (1-2) (3-0)	3	0	0	0	0
	Total	**28**	**0**	**0**	**0**	**0**
5-3	(0-2) (3-0) (2-1)	3	0	0	0	0
	(0-2) (2-1) (3-0)	3	0	0	0	0
	(1-1) (3-0) (1-2)	6	6	3**	0	0
	(1-1) (2-1) (2-1)	18	0	0	0	0
	(1-1) (1-2) (3-0)	6	6	3**	0	0
	(2-0) (3-0) (0-3)	1	0	0	0	0
	(2-0) (2-1) (1-2)	9	0	0	0	0
	(2-0) (1-2) (2-1)	9	0	0	0	0
	(2-0) (0-3) (3-0)	1	0	0	0	0
	Total	**56**	**0**	**6****	**0**	**0**
4-4	(0-2) (3-0) (1-2)	3	0	1.5**	3	3
	(0-2) (2-1) (1-2)	9	0	4.5**	9	9
	(0-2) (1-2) (3-0)	3	0	1.5**	3	3
	(1-1) (3-0) (0-3)	2	2	2	2	2
	(1-1) (2-1) (1-2)	18	18	18	18	18
	(1-1) (1-2) (2-1)	18	18	18	18	18
	(1-1) (0-3) (3-0)	2	2	2	2	2
	(2-0) (2-1) (0-3)	3	0	1.5**	3	3
	(2-0) (1-2) (1-2)	9	0	4.5**	9	9
	(2-0) (0-3) (2-1)	3	0	1.5*	3	3
	Total	**70**	**40**	**55**	**70**	**70**
3-5	**Dual of 5-3**	**56**	**12**	**6**	**0**	**0**
2-6	**Dual of 6-2**	**28**	**0**	**0**	**0**	**0**
1-7	**Dual of 7-1**	**8**	**0**	**0**	**0**	**0**
0-8	**Dual of 8-0**	**1**	**0**	**0**	**0**	**0**
	Total [Bz Score]	**256**	**64**	**67**	**70**	**70**
	Bz Power		**.25**	**.26172**	**.27344**	**.27344**
	Edelman Bz Power***		**.25**	**.29004**	**.33008**	**.27244**

* *n* is the number of distinct voter combinations giving rise to the specified district vote profile.

** In these profiles, Banzhaf awards voter *i* "half credit" as *i*'s vote is decisive with respect to whether a particular candidate wins or there is a tie between the two candidates. (Under the other voting rules, ties cannot occur.)

*** $$\text{Edelman Bz Power} = \frac{\text{Prob. i decisive}}{\text{in district}} \times \frac{\text{Prob. district}}{\text{decisive in Tier 2}} + \frac{\text{Prob. i decisive}}{\text{at-large}} \times \frac{\text{Prob at-large}}{\text{decisive in Tier 2}}$$

AL = 1: $.5 \times .375 + .27344 \times .375 = .29004$

AL = 2: $.5 \times .25 + .27344 \times .7 = .33008$

TABLE 1

Crosstabulation of Warm-up Example

- Table 2 on the next slide summarizes Table 1 by crosstabulating the 256 voting combinations with respect to
  - whether the vote in *i*'s district (DV) is tied, thereby making *i*'s vote decisive within the district (column variable), and
  - whether the at-large vote (ALV) is tied, thereby making *i*'s vote decisive with respect to the at-large vote (row variable).
- We call each cell a *contingency*, and
  - the lower number in each cell indicates number of voter combinations giving rise to that contingency.
  - Note that the contingencies themselves pertain to characteristics of the first-tier vote only.
- The four top numbers in each cell pertain to the four distinct second-tier voting rules and indicate, for each voting rule, the number of combinations in which *i*'s vote is doubly decisive, and thereby contributes to *i*'s Banzhaf score.

Table 2

	DV Tied	DV Not Tied	Total
ALV Tied	40 / 40 / 40 / 40 **Contingency 1** 40	0 / 15 / 30 / 30 **Contingency 2** 30	40 / 55 / 70 / 70 70
ALV Not Tied	24 / 12 / 0 / 0 **Contingency 3** 88	0 / 0 / 0 / 0 **Contingency 4** 98	24 / 12 / 0 / 0 186
Total	64 / 52 / 40 / 40 128	0 / 15 / 30 / 30 128	64 / 67 / 70 / 70 256

All District / 1 A-L / 2 A-L / All A-L

Voting Power with a Single Vote Counting Two Ways

- In trying to solve the question of how to measure voting power when one vote counts in two ways, I focused initially on following example:
  - n =100,035 voters are uniformly partitioned into k = 45 districts (2223 voters per district),
  - each district has a single electoral vote, and
  - a bloc of 6 additional electoral votes elected at-large.

- Two relevant baselines:
  - At the lower extreme, given 51 districts and no at-large seats, individual two-tier voting power is .00202295.
  - At the upper extreme, with 25 or fewer districts (effectively direct popular vote), individual two-tier voting power is .00252269.

standard approximation for the probability of a tie vote $= \sqrt{2 / \pi n}$

Table 3A

- Here is the template for the this larger scale example.
- Since the number of voting combinations is impossibly large,
  - *proportions* rather than counts of combinations associated with each contingency are displayed and,
  - given random voting, these are also the *probabilities* of each type of tie (calculated as $\sqrt{(2/\pi n)}$).

	DV Tied	DV Not Tied	Total
PV Tied	**Contingency 1**	**Contingency 2**	.0025227
PV Not Tied	**Contingency 3**	**Contingency 4**	.9974773
Total	.0169227	.9830773	1.000000

Table 3B

- If district and at-large votes are *separate and independent* in the Edelman manner, we can calculate the probabilities of contingencies simply by multiplying the corresponding row and column probabilities, as shown 3B below.

	DV Tied	DV Not Tied	Total
ALV Tied	.0000427	.0024800	.0025227
ALV Not Tied	.0168800	.9805973	.9974773
Total	.0169227	.9830773	1.0000000

TABLE 3B

Table 3C

- But, given Edelman's assumptions, we need not be concerned with the interior cells at all. We need look only at the marginal proportions in the first row and first column and then (using *Voting Power Analysis*) take account of voting at the second tier.
  - Voting power of district: .080083
  - Voting power of at-large bloc: .628702
  - Individual voting power: .0029412

	DV Tied	DV Not Tied	Total
ALV Tied	.0000427	.0024800	× .628702 = .0015860 .0025227
ALV Not Tied	.0168800	.9805973	.9974773
Total	× .080083 = .0013552 .0169227	.9830773	$\boxed{.0029412}$ 1.0000000

TABLE 3C

Probabilities of Contingencies in the Single-Vote Setup

- The marginal probabilities are unchanged, as is shown in Table 4A.
  - The fact that voters cast the same vote for both district and at-large representation induces a degree of correlation between the vote in any district and the at-large vote, so we expect the probability that both votes are tied to be greater than the .0000427 in the Edelman setup.
- We can directly calculate the *conditional probability* that the at-large vote is tied, given that a district vote is tied.
  - Given that the vote in *i*'s district is tied, the overall at-large vote is tied if and only if there is also a tie in the residual at-large vote after the votes cast in voter *i*'s district are removed. By the standard approximation, this is .0025512 as shown in Table 4A.
  - We can now derive the unconditional probability that both types of ties occur simultaneously by multiplying this conditional probability by the probability that the district vote is tied in the first place, i.e., .0025512 × .0167366 = .0000432.
  - With the probability of Contingency 1 calculated, the probabilities of the other contingencies are determined by subtraction.

Tables 4A-B [Combined]

	DV Tied	DV Not Tied	Total
PV Tied	.0025512 $\Downarrow$ .0000432	.0024795	.0025227
PV Not Tied	.0168795	.9805978	.9974773
Total	.0169227	.9830773	1.0000000

- The probabilities of the contingencies differ only minutely from the Edelman setup (Table 3B), so
  - the substantially lower overall voting power arising from the single-vote setup relative to Edelman's results almost entirely from the workings of second-tier voting.

In Each Contingency, What is the Probability that a Vote is Decisive in the Second-Tier?

- Let's form some general expectations.

- *Contingency 1*, being the conjunction of two already unlikely circumstances, is extraordinarily unlikely occur;
  - but, if it does occur, a voter is very likely to be doubly decisive.
  - The voter is doubly decisive if and only if neither candidate has won a majority of 26 electoral votes from the 44 other districts.
  - Given the Bernoulli model, the electoral votes of the other 44 districts are likely to be quite evenly divided, so it is very likely that neither candidate has won 26 districts.

What is the Probability that a Vote is Decisive in the Second-Tier? (cont.)

- The second tier voting game is 45:51(26:7,1,...,1) and the Banzhaf voting power of the at-large plus one district bloc of 7 votes is .708785.
- It is tempting to conclude that a voter's probability of double decisiveness through Contingency 1 is therefore .0000432 × .708785 = .0000306.
- But this assumes that all second-tier voting combinations are equally likely and, in particular, that they are independent of votes at the district level.
  - This assumption is valid in the Edelman setup, but
  - it is not justified here, given that the district and at-large votes are the same vote counting the same way.

What is the Probability that a Vote is Decisive in the Second-Tier? (cont.)

- *Contingency 2* is much more likely to occur than Contingency 1, while a voter's probability of double decisiveness is only slightly less.
  - A voter is doubly decisive if and only if neither candidate has won a majority of 26 electoral from all 45 districts.
- So this contingency contributes much more than Contingency 1 to individual voting power.
- But again we must resist the temptation of multiplying the Banzhaf power at the at-large bloc in the voting game 46:51(26:6,1,…,1) by the probability of Contingency 2 to calculate individual voting power.

What is the Probability that a Vote is Decisive in the Second-Tier? (cont.)

- *Contingency 3* is still more likely to occur than Contingency 2, but a voter is far less likely to be doubly decisive in this contingency.
  - Voter *i* is doubly decisive if and only if there is a overall 25-25 electoral vote tie, and the probability of such an event is small for three reasons:
    - an *exact* tie in the second-tier electoral vote tie is required, because *i* is tipping only a single electoral vote;
    - the split in *district* electoral votes must be unequal in a degree that depends on the number of at-large seats (here 25-19) in order to create a tie in overall electoral votes, and such an unequal split is less likely than an equal split, since Bernoulli elections always produce 50-50 expectations; and
    - this rather unlikely 25-19 split in favor of one candidate in terms of district electoral votes must come about in the face of a popular vote majority in favor of the *other* candidate.

What is the Probability that a Vote is Decisive in the Second-Tier? (cont.)

- Thus in Contingency 3, voter *i* is doubly decisive only if *i*'s vote can bring about the kind of *election inversion* in which the candidate who wins with respect to district (but excluding at-large) electoral votes at the same time loses with respect to the overall at-large (popular) vote.
  - It is generally thought that election inversions are quite unlikely unless the (at-large/popular vote) election is very close.
  - But almost all large-scale Bernoulli elections are extremely close (in terms of the popular/at-large vote).
  - Indeed, if district and at-large votes are cast separately and independently in the Edelman manner, 50% of all Bernoulli elections entail election inversions.
  - But if the at-large vote is the district vote summed over all districts, a substantial correlation is induced between district and at-large votes, which considerably reduces the incidence of election inversions.
  - The following charts are based on the results of 30,000 simulated Bernoulli elections.

Edelman Setup Single-Vote Setup

Table 4A (based on a sample of 30,000 Bernoulli elections) Table 4B

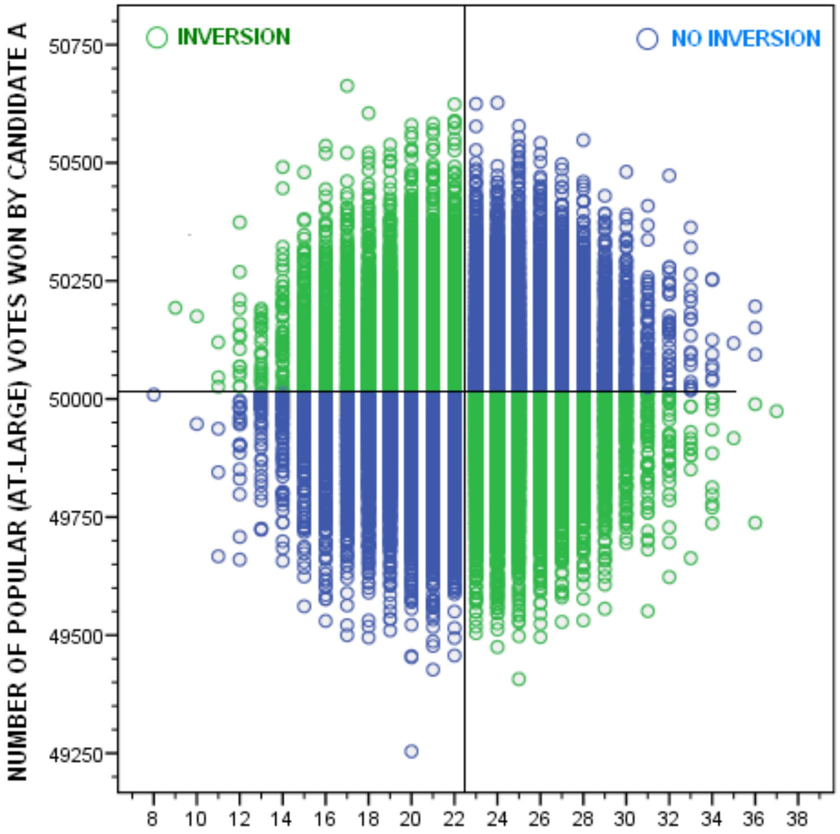


Figure 4A Two-Tier Bernoulli Election Outcomes with Separate and Independent Votes in Each Tier

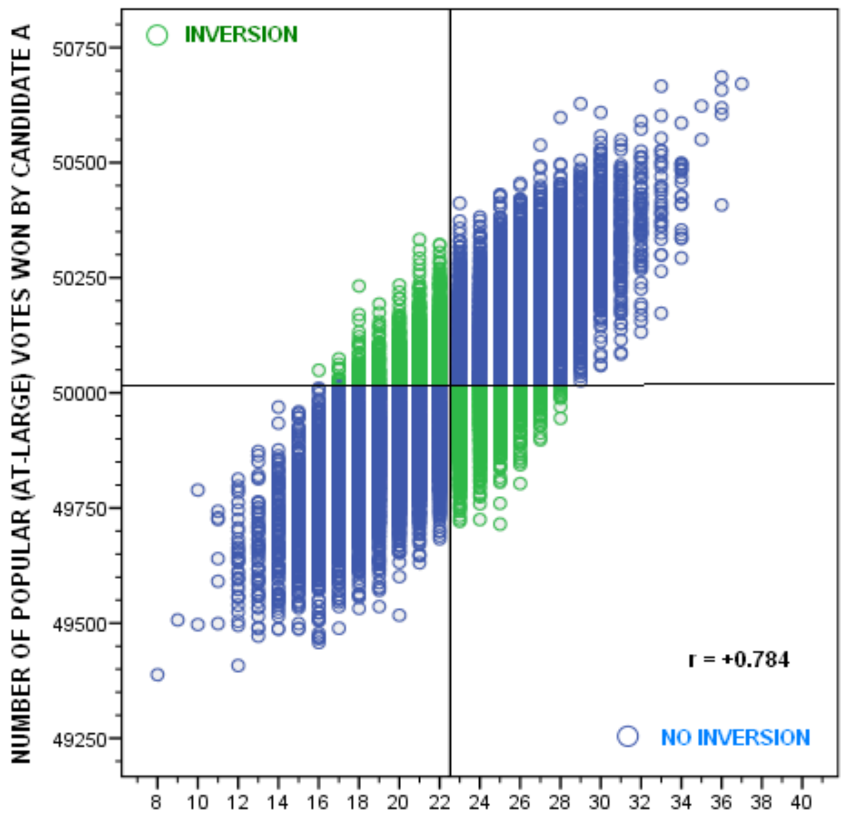


Figure 4B Two-Tier Bernoulli Election Outcome when One Vote Counts the Same Way in Both Tiers

Simulated Bernoulli Elections

- Having developed these expectations, how can we calculate or estimate actual probabilities?
- I proceeded on the basis of large-scale simulations.
- For the case of 45 districts and 6 at-large seats, I generated a sample of 1.2 million Bernoulli elections.
  - The simulations (generated by an SPSS syntax file) operate at the level of the district:
  - The vote for candidate A in each district is a number drawn randomly from a normal distribution with a mean of 2223/2 = 1111.5 and a standard deviation of $.5 \times \sqrt{2223} = 23.574$ and then rounded to the nearest integer.
- The next question is how to use the results of these simulations to estimate the relevant probabilities.

Table 4C

- The most direct approach is simply to duplicate the type of crosstabulation shown in Table 2 for the 9-voter example.

	DV Tied	DV Not Tied	Total
ALV Tied	49 Prob. of DD = .960784 51	2554 Prob. of DD = .862838 2960	2603 3011
ALV Not Tied	367 Prob. of DD = .018198 20,167	0 Prob. of DD = .000000 1,176,822	367 1,196,989
Total	416 20,218	2554 1,179,782	2970 $\boxed{\text{Prob. of DD = .002475}}$ 1,200,000

TABLE 4C

Table 4D

- Table 4C converted into the format of Tables 3A-C

	DV Tied	DV Not Tied	Total
PV Tied	.0000408 .0000425	.0021283 .0024667	.0021692 .0025092
PV Not Tied	.0003058 .0168058	.0000000 .9806850	.0003058 .9974908
Total	.0003467 .0168483	.0021283 .9831517	.0024750 1.0000000

TABLE 4D

Table 4DX [not in paper]

- A second approach makes use of what we know from exact calculations by replacing the sample relative frequencies of each contingency in the lower part of each cell in Table 4C with the exact probabilities displayed in Table 4A-B.
  - Individual voting power becomes .002492 (instead of .002475).
  - [This is correction of .002488 in the paper.]

	DV Tied	DV Not Tied	Total
ALV Tied	.0000408 Prob. Of DD = .944444 .0000432	.0021283 Prob. Of DD = .858359 .0024795	.0021692 .0025227
ALV Not Tied	.0003058 Prob. Of DD = .019117 .0168795	.0000000 Prob. Of DD = .000000 .9805978	.0003058 .9974773
Total	.0003467 .0169227	.0021283 .9830773	.0024750 Prob. Of DD = .002492 1.0000000

Using Simulated Data (cont.)

- A third approach uses looks in more detail at data from the simulations by examining the frequency distributions underlying the cells for Contingency 1 and 2
- In the total sample of 1.2 Bernoulli elections, the cells representing Contingencies 1 and 2 have only 51 and 2,960 cases, respectively.
- Given the small sample size for Contingency 1, the sample statistic of 49/59 is unreliable.
- A more reliable estimate of voter *i*'s probability of double decisiveness may be derived by supposing that the underlying distribution of districts won by Candidate A is normally distributed with a known mean of 22 (i.e., one half of the 44 districts other than voter *i*'s), rather that the sample statistic of 22.294118, and with the standard deviation of 2.032674 found in this sample.

Figure 5A --- Contingency 1

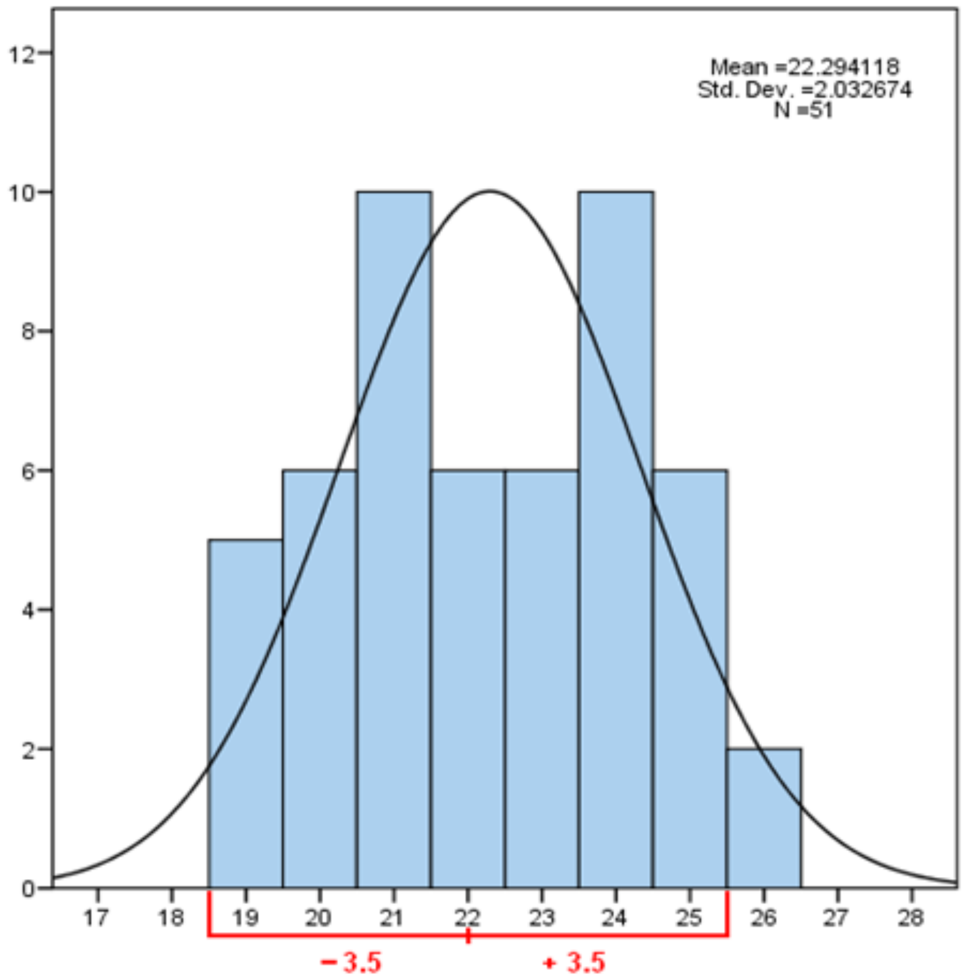


Figure 5A Distribution in Contingency 1

49/51 = .960785

Normal curve with

known mean = 22

estimated SD = 2.032674

Area beyond 3.5/2.032674 =

1.722 SDs from the mean

= .9124907

In this approach, the purpose of the simulations is to provide an estimate of the SD

Figure 5B --- Contingency 2

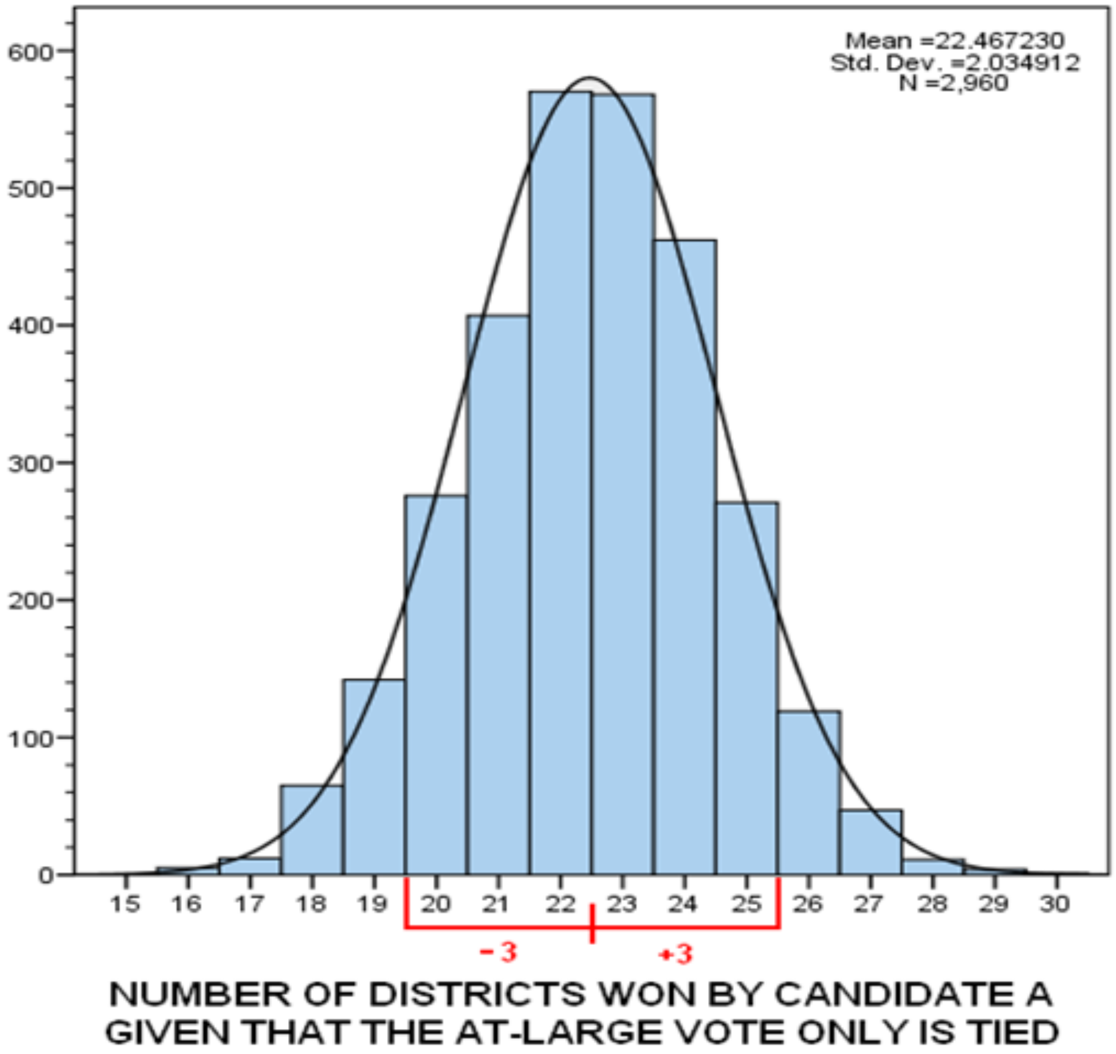


Figure 5B Distribution in Contingency 2

2554/2960

= .862834

Normal curve approach

= .859592

Figure 5C --- Contingency 3

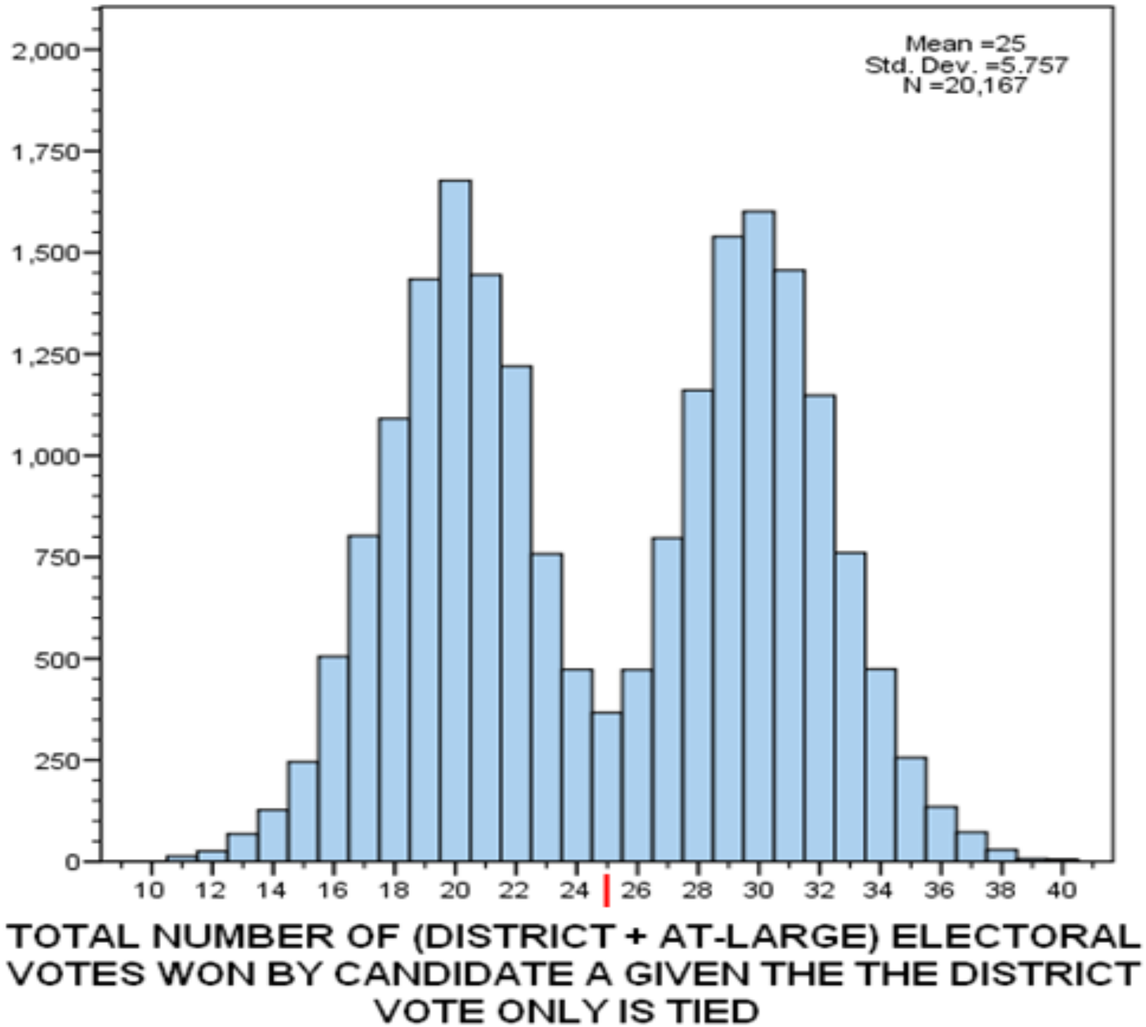


Figure 5C Distribution in Contingency 3

Here the normal curve approach obviously cannot be applied but the sample is so large that the sample statistic of

367/20,167

= .018198

is highly reliable.

Table 4E

- Using the normal curve approach to estimate probabilities of double decisiveness in Contingencies 1 and 2, the sample statistic in Contingency 3, and the known probabilities for the contingencies themselves, we get the (presumably best) estimate of individual voting power of .0024651 compared with
  - .0029412 in the Edelman setup, and
  - .0025227 under direct popular vote.

	DV Tied	DV Not Tied	Total
ALV Tied	× .914907 = .0000389 .0000425	× .859592 = .0021204 .0024667	.0021593 .0025092
ALV Not Tied	× .018198 = .0003058 .0168058	.0000000 .9806850	.0003058 .9974908
Total	.0003447 .0168483	.0021204 .9831517	$\boxed{.0024651}$ 1.0000000

TABLE 4E

Edelman vs. Single-Vote Probabilities of Second-Tier Decisiveness

- Remember that the probabilities of the contingencies (i.e., of first-tier decisiveness) are essentially the same under both setups.

- Comparing the previous charts with similar charts in which the popular/at-large vote is generated independent of the district votes (in the Edelman manner) shows how the differences probabilities of second-tier decisiveness arise.

Tables 5A vs. 6A --- Contingency 1

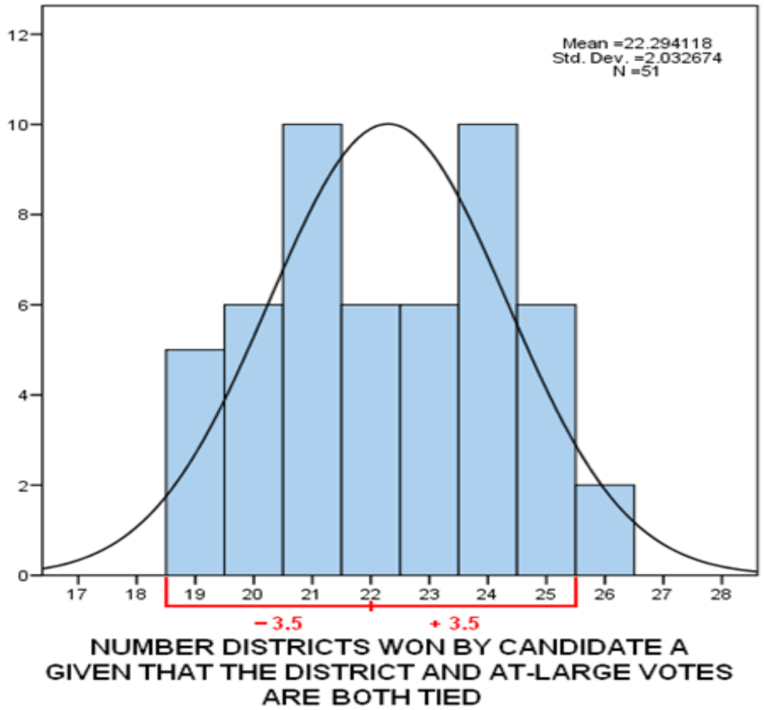


Figure 5A Distribution in Contingency 1

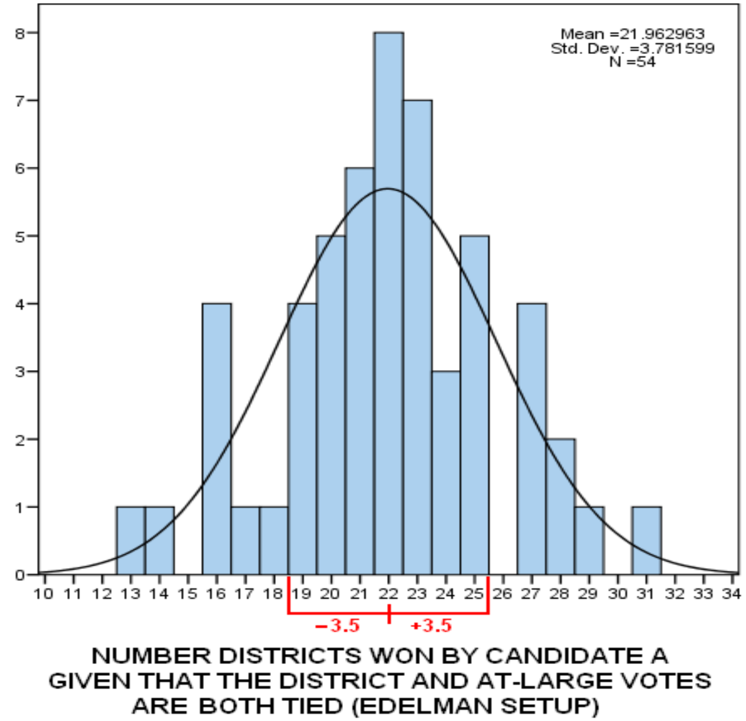
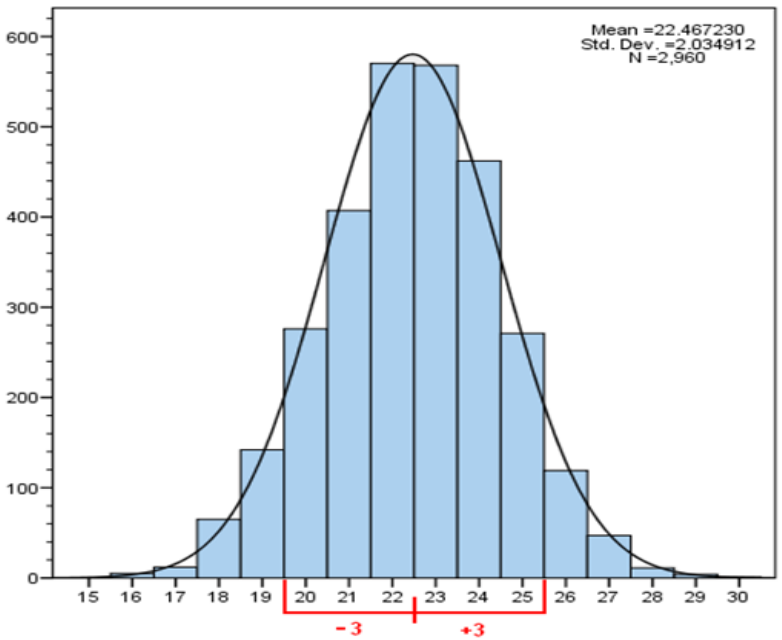


Figure 6A Distribution in Contingency 1

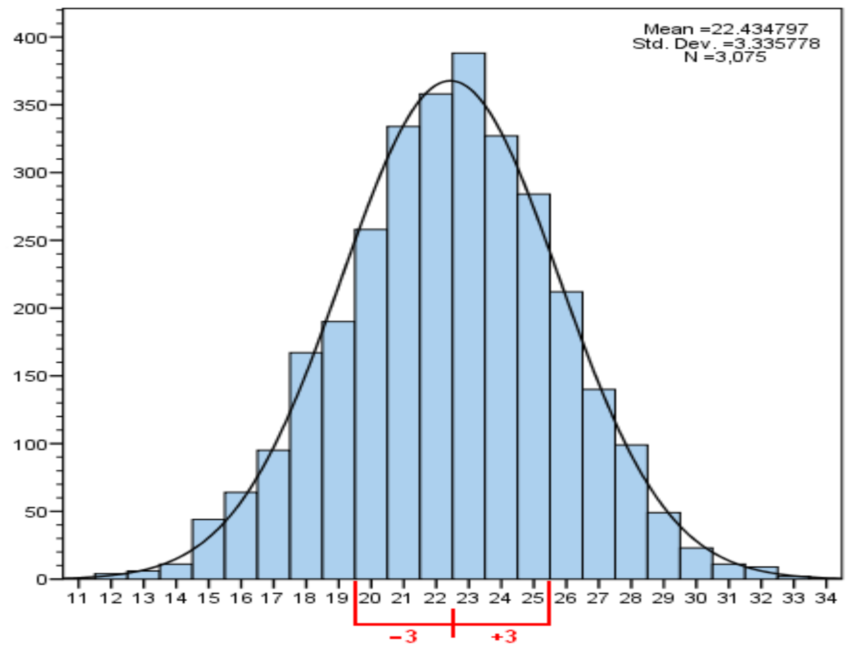
- In Contingency 1, a voter is actually less likely to be doubly decisive in the Edelman setup, as the spread in district electoral votes won by either candidate is substantially larger.
- This results from correlation between at-large votes won and number of districts won that results when each voter casts a single vote that counts twice rather that two separate and independent votes.

Tables 5B vs. 6A --- Contingency 2



NUMBER OF DISTRICTS WON BY CANDIDATE A GIVEN THAT THE AT-LARGE VOTE ONLY IS TIED

Figure 5B Distribution in Contingency 2

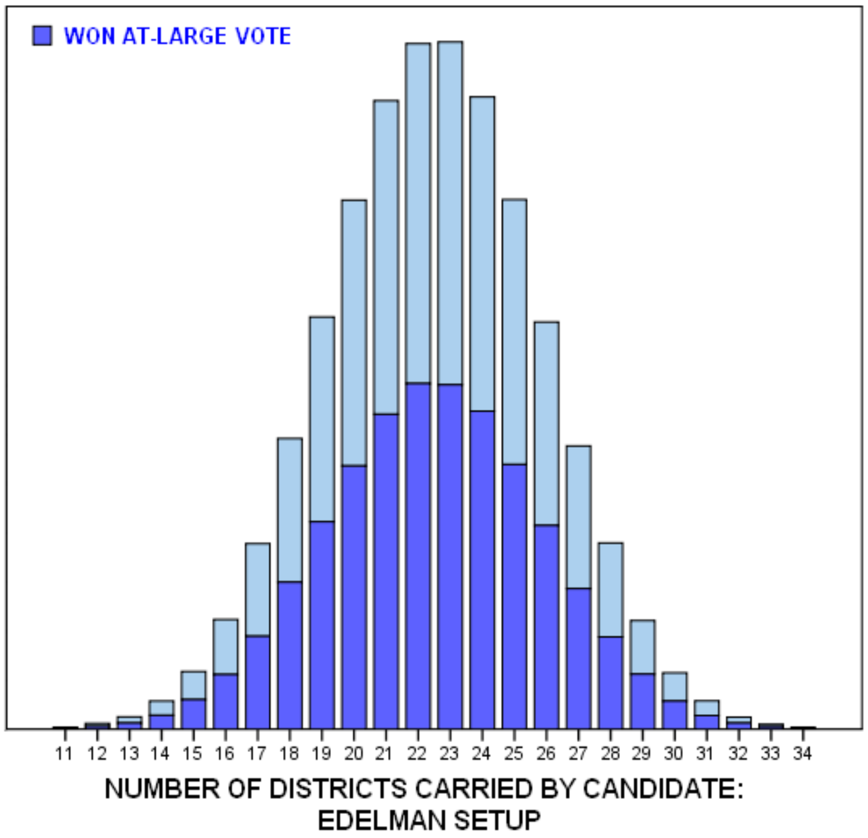


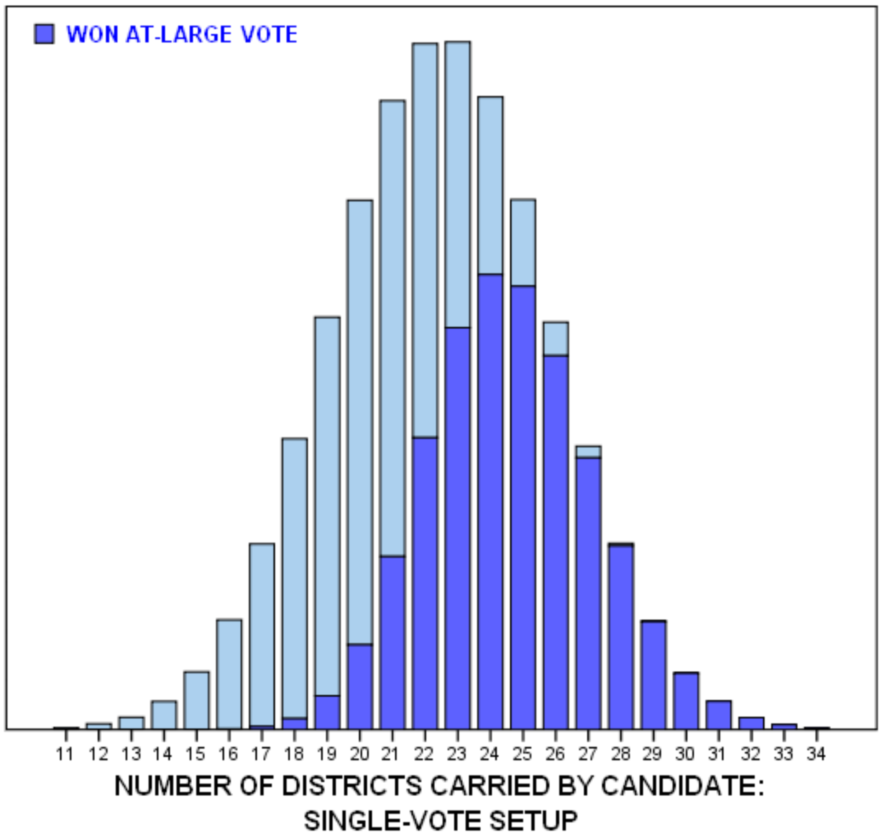
NUMBER OF DISTRICTS WON BY CANDIDATE A GIVEN THAT THE AT-LARGE VOTE ONLY IS TIED (EDELMAN SETUP)

Figure 6B Distribution for Contingency 2

- In Contingency 2, a voter is again less likely to be doubly decisive in the Edelman setup, as the spread in districts won by either candidate is again substantially larger.
- This again results from correlation between at-large votes won and number of districts won that results when each voter casts a single vote that counts twice rather that two separate and independent votes.

Districts Won and At-Large Winner: Edelman vs. Single-Vote





Figures 5C vs. 6C --- Contingency 3

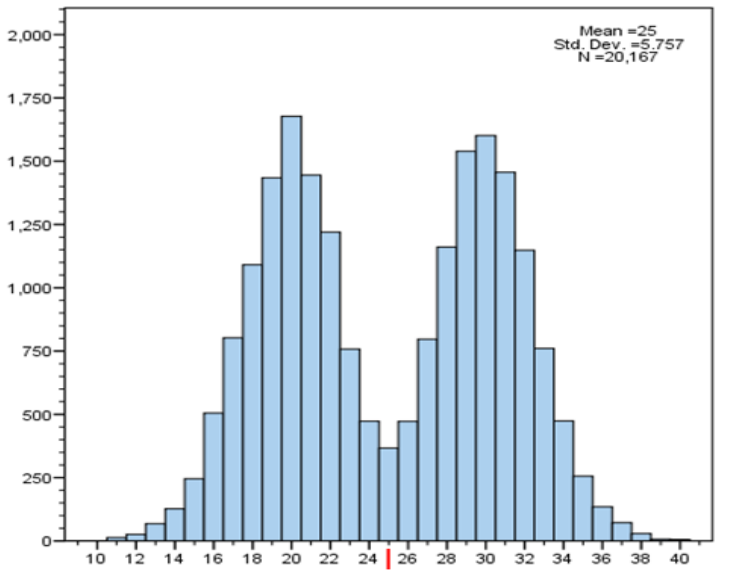


Figure 5C Distribution in Contingency 3

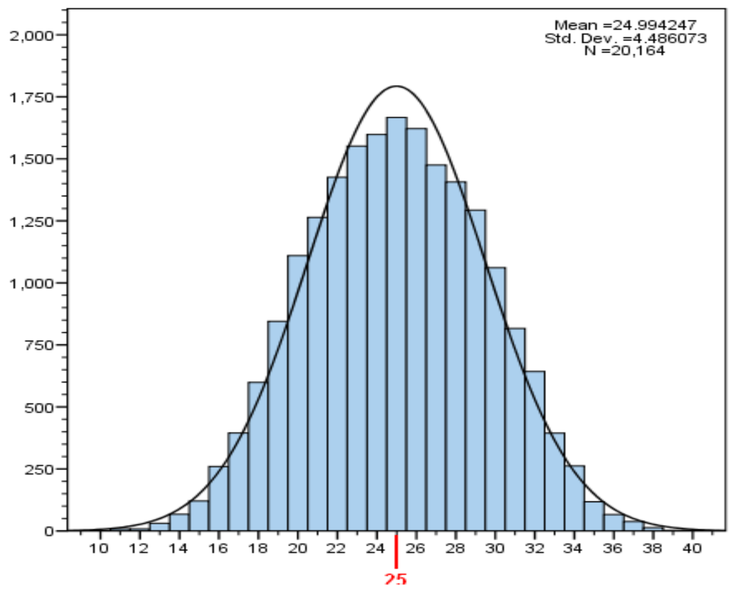


Figure 6C Distribution in Contingency 3

- In Contingency 3, the two setups result in quite different distributions of electoral votes won:
  - In the single-vote setup, the distribution is strikingly bimodal because, as a candidate wins more districts, he is more likely to win the at-large vote as well, whereas
  - in the Edelman setup no such correlation exists.
- With 6 at-large electoral votes out of 51, the Edelman setup produces a distribution that is unimodal but, relative to a normal curve, slightly “squashed” in the center.

Edelman vs. Single-Vote (cont.)

- Thus, the Edelman setup makes a even split of total electoral votes far more likely than does the single-vote setup and thereby greatly enhances the probability of double decisiveness in Contingency 3, which in turn is by far the most probable contingency that (in either setup) allows double decisiveness.

- In general, under the Edelman setup half the voters are expected to “split their votes,”
  - e.g., by voting for Candidate A at the district level and for Candidate B at the at-large level.

Other Magnitudes of At-Large Component

- I have conducted simulations for other odd values of the at-large component within a fixed total of 51 electoral votes.

- The simulations were run in blocks of 300,000 elections, and the voting power estimates produced by each sample are displayed individually for each at-large magnitude (along with means at each magnitude).
  - It is evident that blocks of this size still produce considerable sampling error, but the general pattern of the relationship between the magnitude of the at-large component and individual voting power is clear and in sharp contrast with the pattern of the same relationship in the Edelman setup.

Figures 3 vs. 7 --- Edelman vs. Single Vote

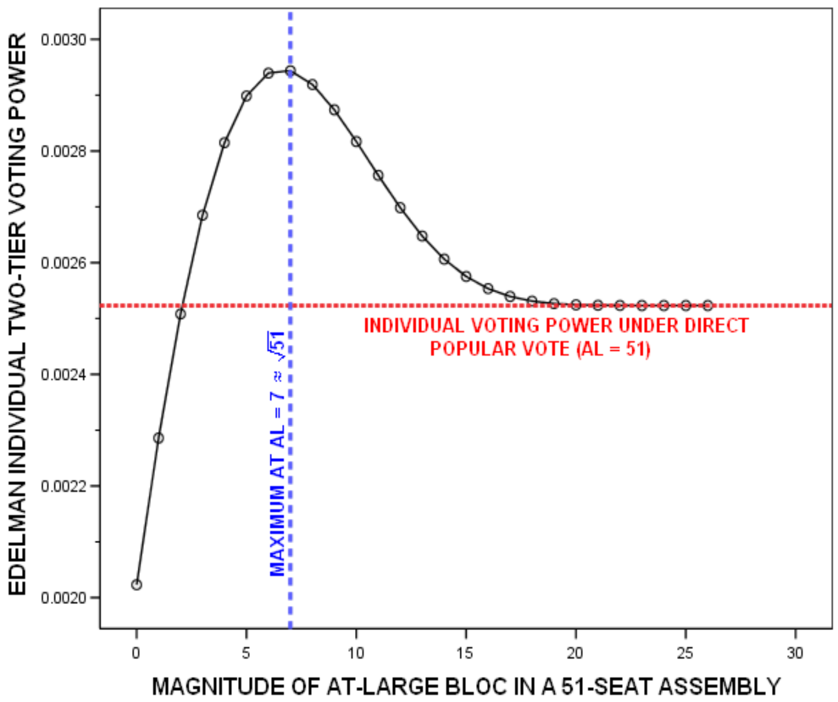


Figure 3 Individual Voting Power by Magnitude of of the At-Large Bloc (Edelman Calculations)

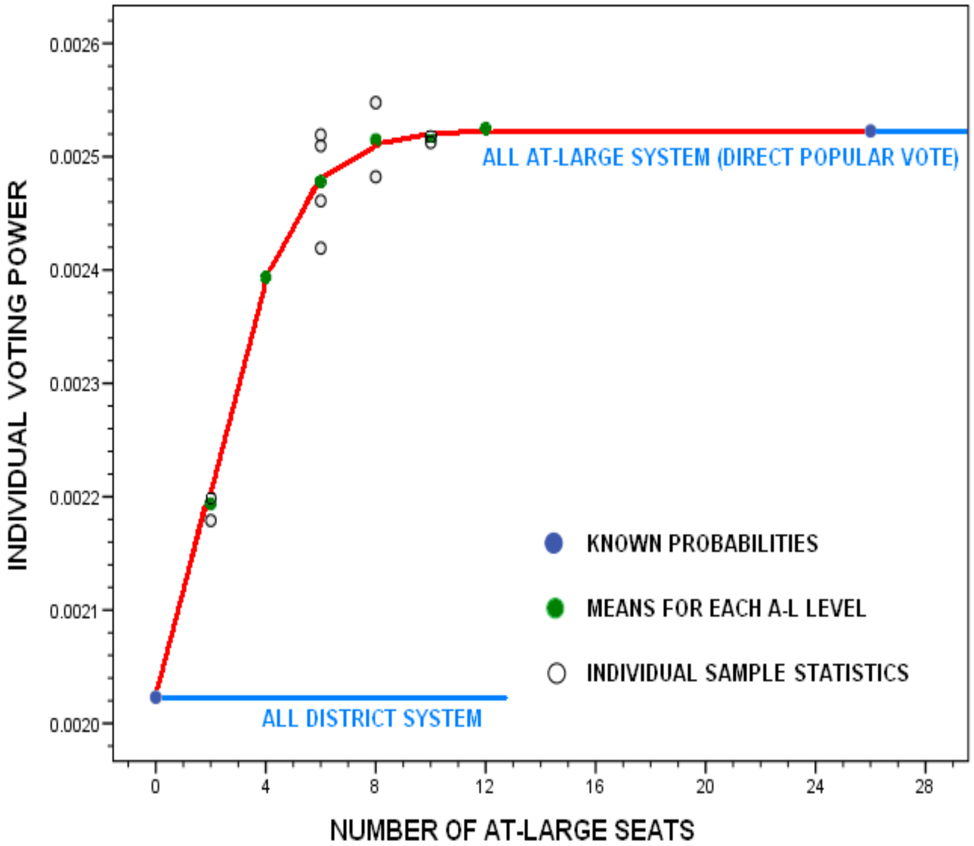


Figure 7 Indivial Voting Power By Magnitude of At-Large Component

National Bonus Plan

- The previous analysis can be applied directly to evaluating voting power under the National Bonus Plan.
- However, the relevant probabilities and simulation estimates must be separately determined for voters in each state, each with its own number of voters and electoral votes.
- While the calculations and simulations are in this respect more burdensome, the procedure is a straightforward extension of that set out in the previous section.
- This simulation data was based on a sample of 256,000 Bernoulli elections.
  - Sampling error presumably accounts for the relatively minor anomalies in the following charts, but again the overall pattern is clear enough.

Tables 8A vs. 8B --- National Bonus Plan (B = 101)

Banzhaf/Edelman vs. Single Vote

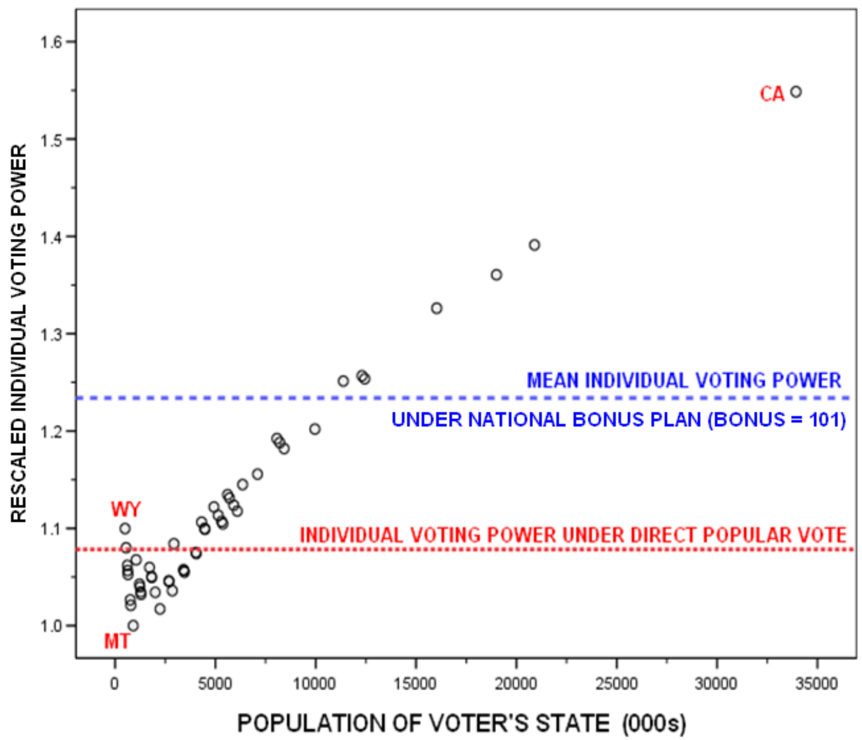


Figure 8A Individual Voting Power by State Population under the National Bonus Plan (Banzhaf/Edelman Calculations)

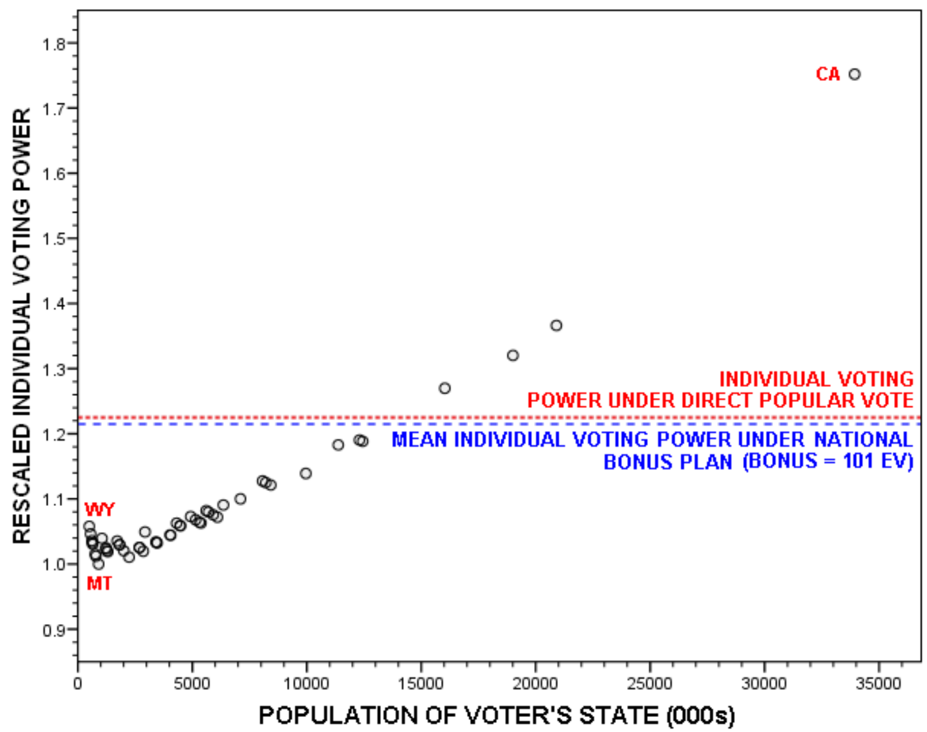


Figure 8B Individual Voting Power under the National Bonus Plan (Bonus = 101)

Tables 9A vs. 9B --- Varying the National Bonus

Banzhaf/Edelman vs. Single Vote

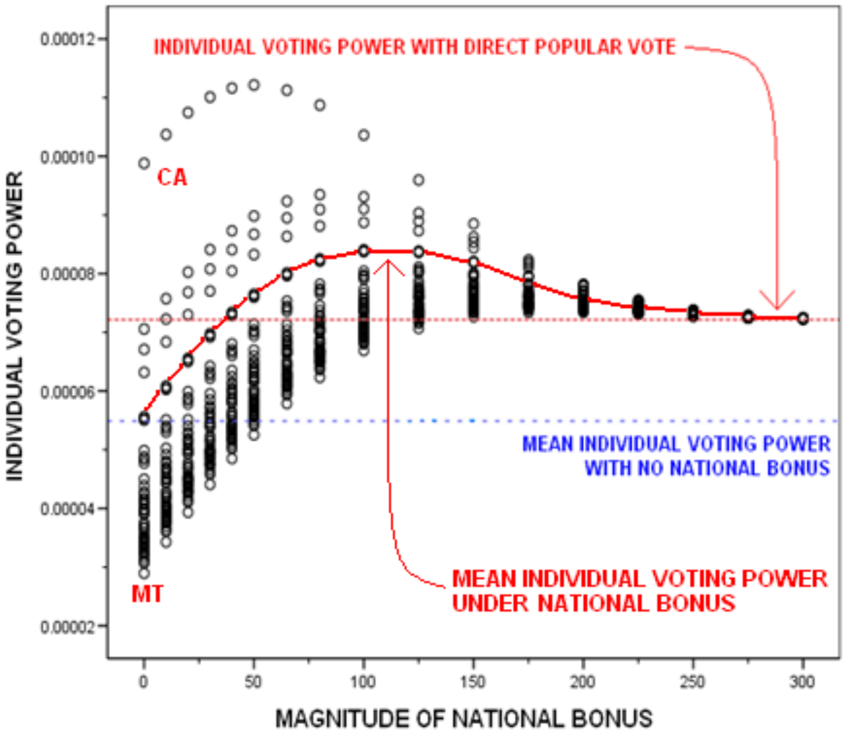


Figure 9A Individual Voting Power by Magnitude of National Bonus (Banzhaf/Edelman Calculations)

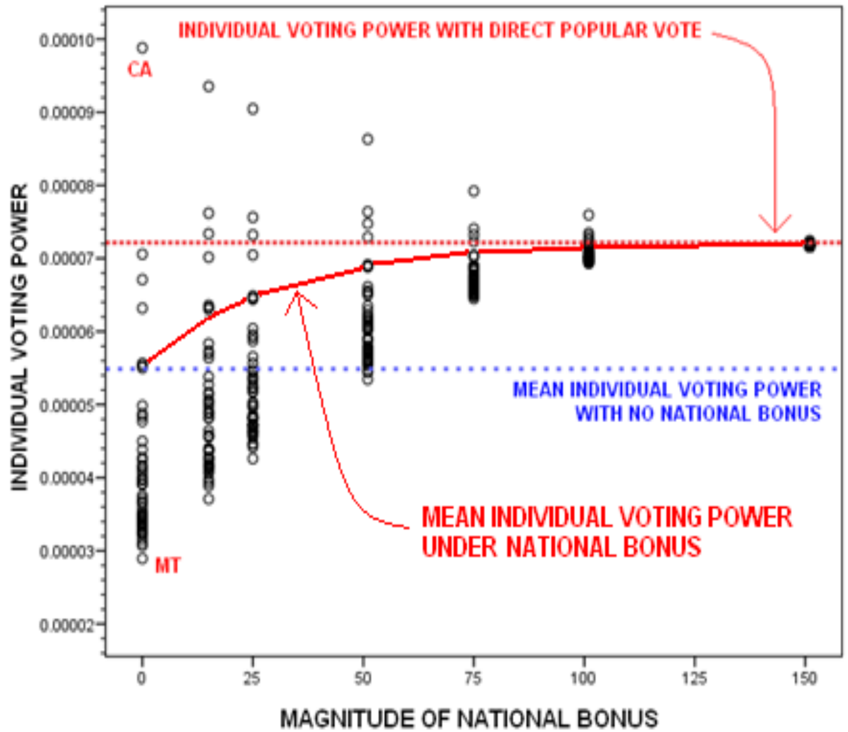


Figure 9B Individual Voting Power by Magnitude of National Bonus

The Modified District Plan

- Because each individual vote counts in two ways, there are logical interdependencies in the way in which district and state electoral votes may be cast.
  - Whichever candidate wins the two statewide electoral votes must also win at least one district electoral vote but, at the same time, need not win more than one. Thus
    - in a state with a single House seat, individual voting power under the Modified District Plan operates in just the same way as under the existing Electoral College, as its three electoral votes are always cast in a winner-take-all manner for the state popular vote winner.
    - in a state with two House seats, the state popular vote winner is guaranteed a majority of the state's electoral votes (i.e., either 3 or 4) and a 2-2 split is precluded.
    - in a state with three or more House seats, electoral votes may be split in any fashion, and
    - in a state with five or more House seats, the statewide popular vote winner may win only a minority of the state's electoral votes — that is, 'election inversions' may occur at the state, as well as the national, level.

The Modified District Plan (cont.)

- However, the preceding remarks pertain only to logical possibilities.
- Probabilistically, the casting of district and statewide electoral votes will to some degree be aligned in Bernoulli elections (and even more so in actual elections).
- Given that candidate A wins a given district, the probability that A also wins statewide is greater than 0.5 — that is to say, even though individual voters cast statistically independent votes, the fact that they are casting individual votes that count in the same way in two tiers (districts and states) induces a correlation between popular votes at the district and state levels within the same state.
- This correlation (or "quasi-bloc effect"), which as we have seen is perfect in the states with only one House seat, diminishes as a state's number of House seats increases, and therefore enhances individual voting power in small states relative to what it is under the Pure District Plan.

The Modified District Plan (cont.)

- I generated a sample of 120,000 Bernoulli elections, with electoral votes awarded to the candidates on the basis of the Modified District Plan.
- This generated a database that could be manipulated to determine frequency distributions of electoral votes for the focal candidate under specified contingencies with respect to first-tier voting, from which relevant second-tier probabilities could be inferred.
- Even with this large sample, few elections were tied at the district or state level, so the relevant electoral vote distributions were taken from a somewhat wider band of elections, namely by taking the average of district and state elections that fell within 0.2 standard deviations of an exact tie.
  - In a normal distribution, the ordinate at $\pm 0.2 \times$SDs from the mean is about .98 times that at the mean.

Figure 10A – Modified District Plan

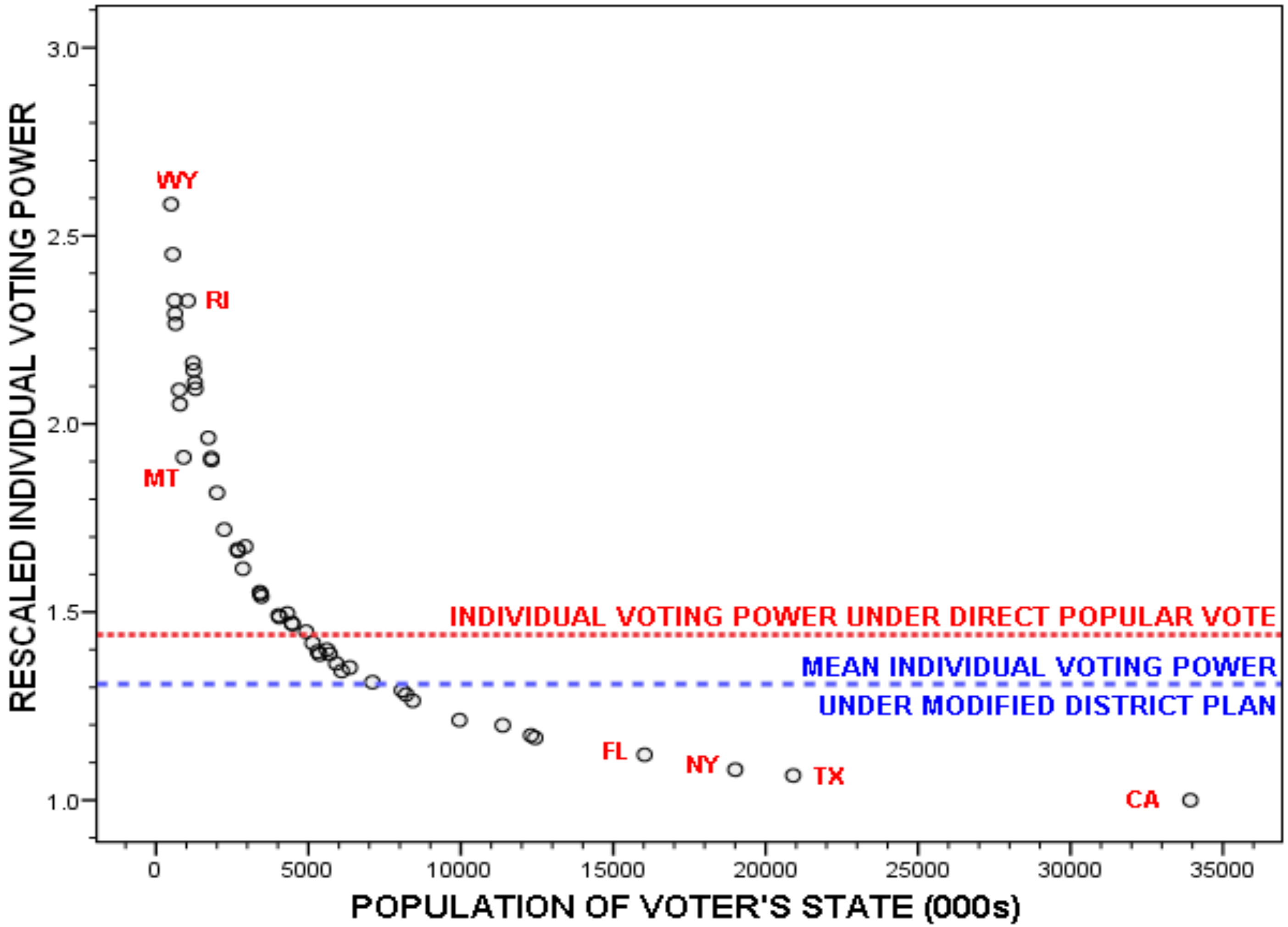


Figure 10A Individual Voting Power by State Population under the Modified District Plan

Figure 2 vs. Figure10A

Banzhaf/Edelman vs. Single Vote

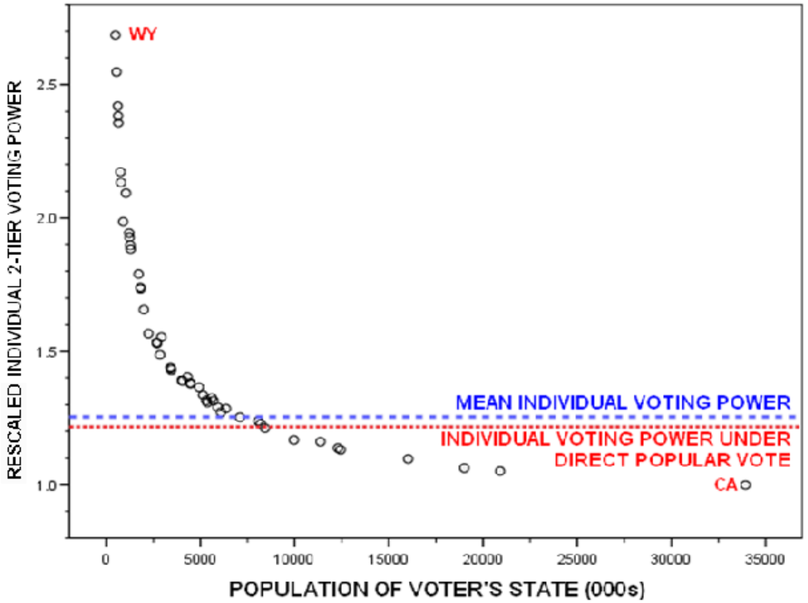


Figure 2 Individual Voting Power by State Population under the Modified District Plan (Banzhaf Calculations)

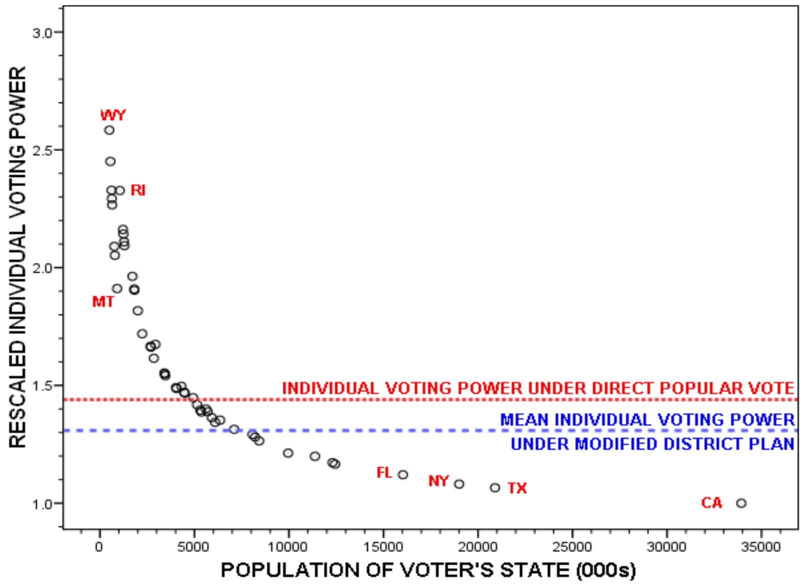


Figure 10A Individual Voting Power by State Population under the Modified District Plan

- Inequality in voting power as calculated here is slightly less than that when calculated in the Banzhaf/Edelman manner.
- However the main difference is that the (absolute) voting power of all voters in substantially less when calculated here, rather than in the Banzhaf manner,
  - as is indicated by the contrasting positions of the lines showing (rescaled) individual voting power under direct popular vote.

Figure 10A vs. Figure10B

Modified District Plan vs. Pure District Plan

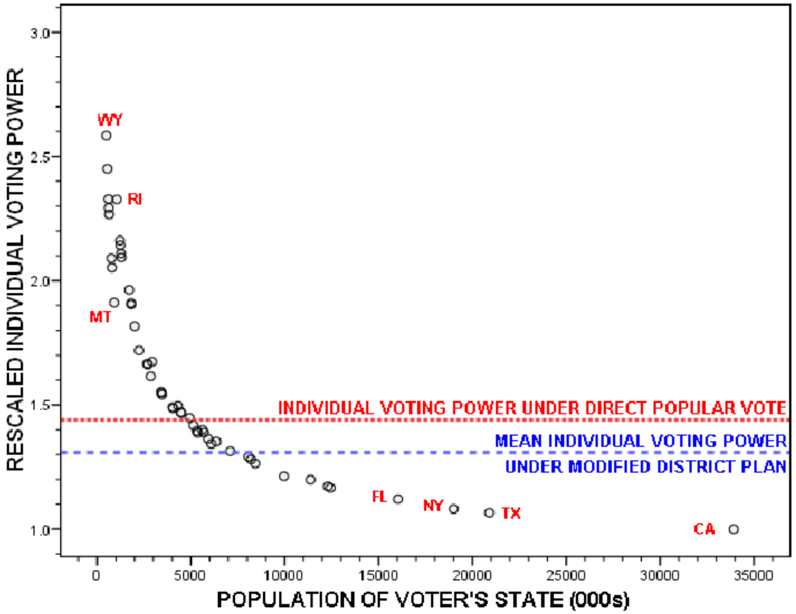


Figure 10A Individual Voting Power by State Population under the Modified District Plan

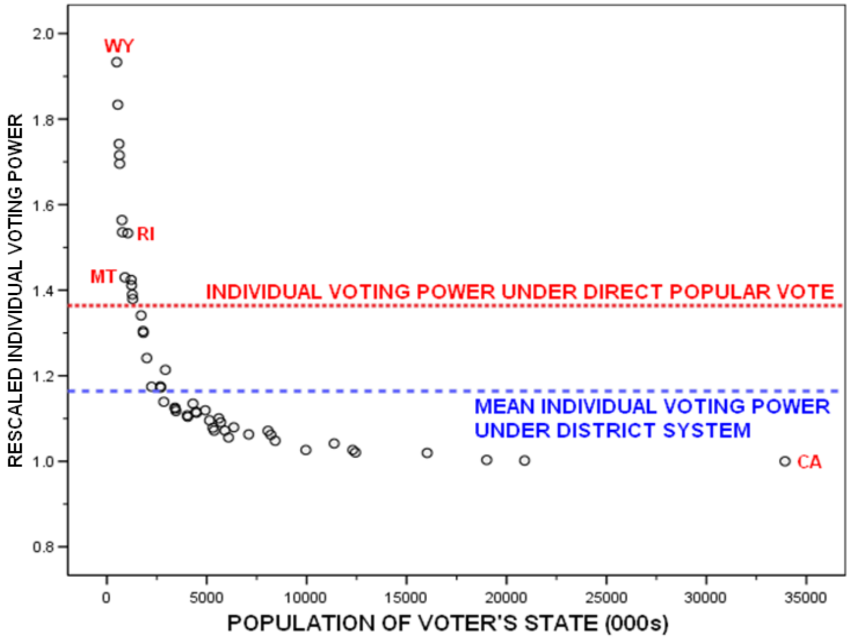


Figure 10B Individual Voting Power by State Population under the Pure District Plan

- It can be seen that, under the Modified District Plan,
  - the ‘bloc effect’ for the smallest states with three electoral votes,
  - the ‘semi-bloc effect’ for the next smallest states with four electoral,
  - and more generally the ‘quasi-bloc effect’ that diminishes as states get larger enhances the voting power of voters in these small states relative to that under the Pure District Plan.