The Direct Democracy Deficit in Two-tier Voting[‡]

– Preliminary Notes –

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Nicola Maaser[*]
and
Stefan Napel[*,†]

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Abstract

A large population of citizens have single-peaked preferences over a one-dimensional convex policy space with independently and identically distributed ideal points. They are partitioned into disjoint constituencies $\mathcal{C}_1, \ldots, \mathcal{C}_m$. Collective decisions are determined either in a directly democratic fashion and the implemented policy $X_{\mathcal{D}}$ corresponds to the ideal point of the (issue-specific) *population median*. Or decisions are taken in a two-tier voting system: one representative of each constituency $\mathcal{C}_i$, with his ideal point matching the *constituency median*, has a voting weight w_i; the collective decision $X_{\mathcal{R}}$ equals the ideal point of the *assembly pivot* defined by weight vector $(w_1, \ldots, w_m)$ and a 50%-quota. Monte-Carlo simulations indicate that proportionality of voting weights and the square root of population sizes minimizes the 'direct democracy deficit' of two-tier voting, i.e., the expected value of $(X_{\mathcal{D}} - X_{\mathcal{R}})^2$.

Keywords: weighted voting systems, majoritarianism, square root rules

1 Introduction

Democratic government of large political units such as modern nation states and supranational entities involves the use of political representatives who make decisions on behalf of the citizens. As democratic principles are being extended from city states to nation states and ever larger units – dubbed the "second democratic transformation" by Dahl (1994) – the question of whether representatives take the right decisions from the point of view of their citizens has been gaining importance.

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[*]Dept. of Economics, University of Bayreuth, Universitätsstr. 30, 95447 Bayreuth, Germany; phone: +49-921552880; e-mail: nicola.maaser@uni-bayreuth.de, stefan.napel@uni-bayreuth.de

[†]Corresponding author.

Discrepancies between the legislative outcome of a representative system and the policy preferences of citizens can be accounted for from at least two different perspectives: First, various frictions in political markets may leave citizens unable to effectively constrain the behavior of elected politicians. Arguably, these citizen-representative agency problems can be alleviated by more direct participation of citizens in the legislation process. In recent years, popular referenda and other direct democratic institutions are increasingly recommended as a complement or corrective to existing representative systems.[1]

Second, representatives are in many democracies elected in disjoint constituencies, and then participate in a governing body or *top-tier assembly* where each member casts a block vote for his or her constituency. We refer to such an arrangement as a *two-tier voting system*. In this case, the agreed policy will often deviate from citizen preferences even if no political market imperfections exist. In a frictionless median voter world, each representative will fully comply with the preferences of his district's citizens in the sense that he will adopt the policy position preferred by *district's median voter* when acting in the top-tier assembly. However, the compromise reached by these ideal representatives need not in general coincide with the outcome preferred by the *overall median voter* if the assembly uses a weighted voting rule.

In principle, the population median's preferences would prevail if policy decisions were made directly by an assembly of *all* citizens. Yet, at least at larger scales, representative democracy offers significant advantages. It relieves citizens from the burden of acquiring information on every issue and avoids the potentially high costs of involving the full population in all decisions.[2] Another important argument focuses on the negotiation possibilities in small bodies of representatives: political bargaining among representatives may bring about Pareto-superior solutions for all citizens which, due to transaction costs, could most probably not be reached and upheld when decisions are taken at the level of a large-scale citizenry (see Baurmann and Kliemt 1993).[3]

The aim of this paper is to study the links between the allocation of block voting rights, i.e., the *voting weights* of constituency representatives when top-tier decisions are taken according to simple majority rule by one representative each from every constituency, and the congruence of the outcomes produced by this two tier decision process and by direct democracy. We study the case in which all policy alternatives are elements of a one-dimensional policy space and individual voter preferences are single-peaked. The expected distance between the legislative outcomes of, first, indirect two-tier decision-making and, second, an ideal direct democracy under identical citizen preferences provides a measure of the *direct democracy deficit* which is implied by a particular allocation of voting weights.

[1] Another prominent argument is that direct democracy stimulates public deliberation processes (e.g., in the run-up to a referendum), which make citizens incorporate wider aspects of the issue at hand into their individual decision-making (e.g., Bohnet and Frey 1994).

[2] In fact, decision-making by the assembly of citizens persists only in two Swiss cantons and a number of Swiss and US municipalities.

[3] While trade between representatives reduces the transaction costs of political decisions, it also facilitates the exchange of votes via log-rolling arrangements resulting in pork-barrel politics (Weingast, Shepsle, and Johnsen 1981).

We seek to find the weight allocation rule which *minimizes* the direct democracy deficit when constituencies are – e.g., for geographical, ethnic, or historical reasons – not equally sized, and hence a weighted voting scheme is typically called for at the top tier.

In our setting, with single-peaked preferences over a one-dimensional policy space, the direct democratic outcome can easily be identified with the ideal point of the median individual. Finding the outcome of decision-making in the two-tier system is slightly more involved: first, the policy advocated by the representative of any given constituency is supposed to coincide with the ideal point of the respective constituency's median voter (technically, the unique element of the constituency's *core*). Second, the decision which is taken at the top tier is identified with the position of the *pivotal representative* (corresponding to the assembly's core), where pivotality is determined by the voting weights of all constituencies and a 50% decision quota. Consideration of the respective core is meant to capture the result of possible strategic interaction. As long as this is a reasonable approximation, the actual systems determining collective choices can stay unspecified. They could differ across constituencies.

Because the population size of a constituency affects the distribution of its median, the location of the top-tier decision in the policy space becomes a rather complex function of (the order statistics of) differently distributed random variables. A purely analytical investigation of the model is therefore unlikely to produce much insight even under far-reaching statistical independence assumptions. For this reason, we resort to Monte-Carlo approximations of the expected distance between the outcomes of representative and direct decision-making, considering randomly generated population configurations as well as recent EU population data. The main finding of our analysis is that the direct democracy deficit is minimized by the use of a simple *square root rule*, i.e., top-tier weights should be proportional to the square root of a constituency's population size.

The design of weighted voting rules has received considerable attention in the literature on indirect democracy already. Several – potentially conflicting – normative criteria have been applied to the problem of defining the weights of representatives from differently sized constituencies. For instance, the design of two-tier voting rules which minimizes the deviation of two-tier decision-making from direct democracy under simple majority rule and votes on *binary alternatives* has been studied by Felsenthal and Machover (1999). Their objective has been the minimization of the so-called 'mean majority deficit'.[4] The latter arises whenever the alternative chosen by the body of representatives is supported only by a minority of all citizens, and can be measured as the difference between the size of the popular majority camp and the number of citizens in favor of the assembly's decision. Felsenthal and Machover have shown that the mean majority deficit is minimal under a square root allocation of weights.[5] Feix et al. (2008) also consider majority votes

[4]As demonstrated by Felsenthal and Machover (1999), this is equivalent to maximizing the sum of citizens' indirect voting power measured by the Penrose-Banzhaf measure.

[5]Felsenthal and Machover (1999) refer to this decision rule as the *second square root rule* in order to distinguish it clearly from Penrose's (1946) (first) square root rule which requires representatives' voting power – rather than their weight – to be proportional to the square roots of their constituencies' population sizes.

on two alternatives. But while the investigation by Felsenthal and Machover focuses on the *number* of frustrated voters, Feix et al. seek to minimize the *probability* of situations where the decision taken by the representatives is at odds with the decision that citizens would have adopted in a referendum.[6]

In contrast to the mentioned studies, this paper investigates the difference between direct and representative outcomes for *non-binary* choices from a one-dimensional policy space. Our findings can be viewed as a confirmation of Felsenthal and Machover's result for many finely graded policy alternatives and with strategic interaction captured by the median voter theorem.

While we emphasize the context of direct vs. representative democracy, our work also makes a contribution to the optimal design of voting rules from an efficiency perspective. In the spatial voting model which we consider, minimization of the direct democracy deficit is equivalent to the maximization of the sum of individual citizens' expected utility provided that preferences over policy outcomes (i) have the same intensity across all citizens, and (ii) are representable by a utility function that is linearly decreasing in the Euclidean distance to the respective ideal policy.[7] This utilitarian ideal of maximizing total societal welfare has already been studied by Barberà and Jackson (2006) as well as Beisbart and Bovens (2007) and Beisbart et al. (2005). Beisbart et al. (2005) evaluate different decision rules for the Council of Ministers of the European Union according to an expected utility criterion under the presumption that decisions affect all individuals from a given country identically. Beisbart and Bovens (2007) also consider the welfarist objective of *equalizing* expected utility throughout society.

An alternative approach to assessing voting rules from a normative constitutional perspective focuses on the indirect *influence* of citizens. Analytical investigations of the objective to implement the "one person, one vote" principle in two-tier voting systems with a focus on influence (or *power*) date back to the seminal work of Penrose (1946). He identified a square root rule (see fn. 5) as the solution to the problem in binary settings (see Felsenthal and Machover 1998, Sect. 3.4). Maaser and Napel (2007) have studied the same objective for unidimensional spatial voting, and find that weights proportional to the square root of population sizes come close to ensuring equal representation also in that model. The following section draws on the presentation therein.

2 Model

Consider a large population $\{1, 2, \ldots, n\}$ of n *voters*. Assume a partition $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$ of the population into m *constituencies* with $n_j = |\mathcal{C}_j| > 0$ members each. Assume for

[6]This situation is known in the social choice literature as a *referendum paradox* (see e.g., Nurmi 1998).

[7]The policy corresponding to the median citizen's ideal point would maximize overall welfare under these assumptions (see e.g. Schwertman et al. 1990). If utility decreases quadratically in distance, the ideal point of the *mean* voter would maximize overall welfare (see e.g. Cramer 1946). Thus, a minimal direct democracy deficit would also guarantee maximal total expected utility under the additional assumption that the ideal points of all individuals come from the same symmetric distribution.

simplicity that n and all n_j are odd numbers. For $j = 1, \ldots, m$, let $\mathcal{C}_j$ be the set of labels of individuals from the jth constituency. Any citizen's preferences are single-peaked with *ideal point* λ^i (for $i = 1, \ldots, n$) in a bounded convex one-dimensional *policy space* $X \subset \mathbb{R}$.

For any random policy issue, let $\cdot : n$ denote the permutation of all voter numbers such that

$$\lambda^{1:n} \leq \ldots \leq \lambda^{n:n}$$

holds. In other words, $k : n$ denotes the k-th leftmost voter in the population and $\lambda^{k:n}$ denotes the k-th leftmost ideal point (i. e., $\lambda^{k:n}$ is the *k-th order statistic* of $\lambda^1, \ldots, \lambda^n$). Similarly, consider the restriction of the above ordering with respect to $k \in \mathcal{C}_j$, and let $\lambda^{k:n_j}$ denote the k-th leftmost ideal point of voters in constituency $\mathcal{C}_j$.

A policy $x \in X$ is decided on by a *council of representatives* $\mathcal{R}$ consisting of one representative from each constituency. Without going into details, we assume that the representative of $\mathcal{C}_j$, denoted by j, adopts the ideal point of his constituency's *median voter*, denoted by

$$\lambda_j \equiv \lambda^{(n_j+1)/2:n_j}.$$

In theory, elected representatives are fully responsive to their constituency's median voter. Practically, as a result of being in power, representatives tend to develop preferences (e.g., concerning their privileges) that differ from those of regular citizens. Electoral competition can prove insufficient to keep these divergences in check. Empirical evidence suggests that a representative may take positions that differ significantly from his district's median when voter preferences within that district are sufficiently heterogeneous (Gerber and Lewis 2004). A related problem arises when systematic abstention of certain social groups drives a non-negligible wedge between the median voter's and the median citizen's preferences, and non-voters go unrepresented.

In the top-tier assembly $\mathcal{R}$, each constituency $\mathcal{C}_j$ has *voting weight* $w_j \geq 0$. Any subset $S \subseteq \{1, \ldots, m\}$ of representatives which achieves a combined weight $\sum_{j \in S} w_j$ above $q \equiv 0.5 \sum_{j=1}^{m} w_j$, i.e., comprises a *simple majority* of total weight, can implement a policy $x \in X$.

Let $\lambda_{k:m}$ denote the k-th leftmost ideal point amongst all the representatives (i. e., the k-th order statistic of $\lambda_1, \ldots, \lambda_m$). Consider the random variable P defined by

$$P \equiv \min \Big\{ l \in \{1, \ldots, m\} : \sum_{k=1}^{l} w_{k:m} > q \Big\}.$$

Player $P : m$'s ideal point, $\lambda_{P:m}$, is the unique policy that beats any alternative $x \in X$ in a pairwise majority vote, i. e., it constitutes the *core* of the voting game defined by weights $(w_1, \ldots, w_m)$ and quota q. Without detailed equilibrium analysis of any decision procedure that may be applied in $\mathcal{R}$ (see Banks and Duggan 2000 for sophisticated non-cooperative support of policy outcomes inside or close to the core), we assume that the policy agreed by $\mathcal{R}$ is in the core. In other words, the policy outcome $X_{\mathcal{R}}$ from the two-tiered voting system is expected to equal the ideal point of the *pivotal representative* $P : m$, i.e.,

$$X_{\mathcal{R}} \equiv \lambda_{P:m}.$$

By contrast, consider a direct-democratic decision-making process under simple majority rule with assembly $\{1, \ldots, n\}$. In this case, the median voter model predicts that the policy outcome $X_{\mathcal{D}}$ will correspond to the preferences of the median voter in the total population, i.e.,

$$X_{\mathcal{D}} \equiv \lambda^{(n+1)/2:n} .$$

For an exogenously given partition $\mathcal{C}$ of the population and for any weighted voting rule $[q; w_1, \ldots, w_m]$ employed in $\mathcal{R}$, we define the random variable

$$\Delta = (X_{\mathcal{D}} - X_{\mathcal{R}})^2$$

which captures the *squared Euclidean distance* of the two-tier outcome from direct democracy. We call the expected value $\mathbf{E}[\Delta]$ the *direct democracy deficit* (of the system $\{\mathcal{C}, [q; w_1, \ldots, w_m]\}$). The direct democracy deficit would vanish if the two-tier system constituted an *unbiased* proxy for direct democracy.

The question we wish to answer in this setting is the following: *Which voting weight allocation rule approximately minimizes the expected difference between the policy outcomes of an indirect, two-tier voting system and a directly democratic system?* Or, more formally, we search for a 'simple' mapping w which assigns weights $(w_1, \ldots, w_m) = w(\mathcal{C}_1, \ldots, \mathcal{C}_m)$ to any given partition $\mathcal{C}$ of a large population $\{1, \ldots, n\}$ such that $(w_1, \ldots, w_m)$ is a solution (or approximates a solution) to the problem

$$\min_{(w'_1, \ldots, w'_m)} \mathbf{E}[\Delta] . \tag{1}$$

Our criterion for acceptably 'simple' mappings w will be that they are *power laws*, namely that $w_j = n_j^{\alpha}$ for all $j = 1, \ldots, m$ for some constant $\alpha \in [0, 1]$.

3 Analysis

Under the assumption that individual voters' ideal points are pairwise independent and come from an arbitrary identical distribution F (the *i.i.d. assumption*) with positive density f on the one-dimensional convex policy space X, the median position of the whole population has an asymptotically normal distribution with mean

$$\mu = F^{-1}(0.5) \tag{2}$$

and standard deviation

$$\sigma_{X_{\mathcal{D}}} = \frac{1}{2\, f(F^{-1}(0.5))\sqrt{n}} \tag{3}$$

(see, e.g., Arnold et al. 1992, p. 223).

Under the i.i.d. assumption for individual voters, the ideal points of the representatives, $\lambda_1, \ldots, \lambda_m$, are independently but *not* identically distributed (except in the trivial case

$n_1 = \ldots = n_m$). In fact, the median position λ_j in constituency $\mathcal{C}_j$ is asymptotically normally distributed with its mean given by (2) and standard deviation

$$\sigma_j = \frac{1}{2\, f(F^{-1}(0.5))\sqrt{n_j}}. \tag{4}$$

Finding optimal weights under criterion (1) requires some 'golden middle' between assigning equal weights to all constituencies and rendering the largest constituency in $\mathcal{R}$ a dictator. In particular, the median of the total population is more 'central' than the ideal policy of any particular constituency. According to (4), the distribution of a representative's ideal point is the more concentrated on the median of the underlying ideal point distribution F, the larger the constituency $\mathcal{C}_j$. So consider a weighted voting rule that makes the representative of the *largest* (i.e., most populous) constituency a dictator in $\mathcal{R}$. While this would result in a smaller direct democracy deficit than giving dictatorial power to any other representative, the gap between the representative outcome and the direct democratic outcome will be large whenever the ideal point of the median voter in the largest constituency happens to be 'extreme'. If, by contrast, voting weights are assigned uniformly, the representative of the random constituency with *median* top-tier ideal point is always pivotal, i. e., $P \equiv (m+1)/2$ for odd m. If representatives' ideal points were *i. i. d.*, the direct democracy deficit would, therefore, be minimized by giving equal weight to all representatives. Yet, this cannot be optimal in case of independently but not identically distributed $\lambda^1, \ldots, \lambda^m$. In order to solve problem (1) the optimal voting weights have to strike a balance between accounting for a large constituency's on average greater centrality and guarding against the possibility that any representative can put through an extreme policy.

Under our model assumptions, the outcome $X_{\mathcal{R}}$ is a random variable that coincides for any given policy issue with the ideal point of some council member $j \in \{1, \ldots, m\}$. Let X_j denote the policy outcome *conditional on* j being pivotal, and let Ω be the set of vectors of individual ideal points. Given a voting rule, define $\Omega_j \subseteq \Omega$ by

$$\Omega_j = \{\omega \colon X_{\mathcal{R}}(\omega) = X_j\}, \qquad j = 1, \ldots, m.$$

That is, Ω_j is the set of ideal point realizations $\omega \in \Omega$ such that representative j is pivotal in $\mathcal{R}$. Note that the Ω_j are disjoint sets. Then, $X_{\mathcal{R}}$ can be written as

$$X_{\mathcal{R}} = \sum_{j=1}^{m} X_j \, 1_{\Omega_j} \tag{5}$$

where 1_{Ω_j} is the indicator function of Ω_j.

Using (5), the direct democracy deficit can be written as

$$\begin{aligned} \mathbf{E}\,[\Delta] &= \mathbf{E}\left[(X_{\mathcal{D}})^2\right] - \mathbf{E}\left[2X_{\mathcal{D}} \sum_{j=1}^{m} X_j \, 1_{\Omega_j}\right] + \mathbf{E}\left[(\sum_{j=1}^{m} X_j \, 1_{\Omega_j})^2\right] \\ &\approx (\sigma_{X_{\mathcal{D}}})^2 - 2\mathbf{E}\left[X_{\mathcal{D}} \sum_{j=1}^{m} X_j \, 1_{\Omega_j}\right] + \sum_{j=1}^{m} \mathbf{E}\left[X_j^2 1_{\Omega_j}\right] \end{aligned} \tag{6}$$

where the second line follows by observing that $X_{\mathcal{D}}$ is approximately normally distributed.

Note that $\mathbf{E}\left[1_{\Omega_j}\right]$ is the probability that representative j is pivotal. It seems feasible in principle to provide an approximation for this probability as a function of weights and population sizes $n_1, \ldots, n_m$. However, we doubt the existence of a reasonable approximations for $\mathbf{E}\left[X_{\mathcal{D}} \sum_{j=1}^{m} X_j\, 1_{\Omega_j}\right]$ and $\mathbf{E}\left[X_j^2 1_{\Omega_j}\right]$. A purely analytical investigation of the model is therefore unlikely to produce much insight. For this reason, the following section uses Monte-Carlo simulation in order to approximate $\mathbf{E}[\Delta]$ for a given partition or *configuration* $\{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$ of the electorate and a fixed weight vector $(w_1, \ldots, w_m)$. Based on this, we try to identify the weights $(w_1^*, \ldots, w_m^*)$ which yield the smallest direct democracy deficit for the given configuration.

4 Simulation results

The goal of the simulations is to identify a rule for assigning voting weights to constituencies which approximately solves problem (1) for many different constituency configurations $\{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$. The *Monte-Carlo method* makes use of the fact that the empirical average of s independent realizations of $(x_{\mathcal{D}} - x_{\mathcal{R}})^2$ converges in s to the theoretical expectation $\mathbf{E}[(X_{\mathcal{D}} - X_{\mathcal{R}})^2]$ (by the *law of large numbers*).

The 'verbatim approach' to simulating the model involves the drawing of n random numbers $\lambda^1, \ldots, \lambda^n$ from some distribution F, and then – for any $j = 1, \ldots, m$ – the inference of a realization of representative j's ideal point as the median of the λ^i, $i \in \mathcal{C}_j$. In a second step, the realized positions of the representatives are sorted and, for a fixed allocation of weights, the realized pivotal position p is determined. The ideal point $\lambda_{p:m}$ is thus identified as the legislative outcome selected by $\mathcal{R}$. This is compared with the median ideal point in the population, i.e., the median of $\lambda^1, \ldots, \lambda^n$.

Instead of generating a vector of n individual ideal points in each iteration, an (approximate) realization of $(x_{\mathcal{D}} - x_{\mathcal{R}})^2$ can be obtained much more efficiently by using the following 'short-cut approach': When population sizes are large, the mentioned asymptotic results for order statistics imply that both the position of the population median and the position of a constituency's median can be well approximated by drawing random numbers from normal distributions with standard deviations as given by (3) and (4), respectively. We use this short-cut rather than the verbatim approach in parts of the analysis.

Regarding the assignment of voting weights, power laws

$$w_j = n_j^{\alpha}$$

with $\alpha \in [0, 1]$ provide a natural focus due to their simplicity and flexibility. For any given m and population configuration $\{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$ under consideration, we fix α and then approximate $\mathbf{E}[\Delta]$ by the empirical average of (verbatim or short-cut) realizations $(x_{\mathcal{D}} - x_{\mathcal{R}})^2$ in a run of 1 million iterations. This is repeated for different values of α, ranging from 0 to 1 with a step size of 0.1 or 0.02, in order to find the exponent α^* which yields the smallest direct democracy deficit for the given configuration.

$\mathbf{U}(10^3, 3\cdot10^3)$	$\mathbf{U}(10^3, 3\cdot10^3)$	$\mathbf{U}(10^3, 7\cdot10^3)$	$\mathbf{U}(10^3, 9\cdot10^3)$	$\mathbf{U}(10^3, 11\cdot10^3)$
0.5	**0.5**	**0.5**	**0.5**	**0.5**
(4.83×10^{-6})	(2.84×10^{-6})	(2.41×10^{-6})	(1.58×10^{-6})	(1.70×10^{-6})

Table 1: Optimal value of α for constituency sizes for uniformly distributed constituency sizes

4.1 Randomly generated configurations

Table 1 reports the optimal values of α that were obtained by applying the verbatim approach to five configurations. Each configuration consists of 15 constituencies where $n_1, \ldots, n_{15}$ were independently drawn from uniform distributions with increasing variance. For the sake of completeness, it shall be mentioned that individual ideal points were drawn from a standard normal distribution. The values of α in these simulations run from 0 to 1 in 0.1-intervals, and the corresponding value of $\mathbf{E}[\Delta]$ is estimated by simulations with 1 mio. iterations each. The size of the direct democracy deficit for the respective optimal power low is shown in parentheses.

The results in Table 1 are strongly suggestive. It seems that a square root rule also holds in the context of median voter-based policy decisions.

Table 2 includes results under various distributional assumptions (uniform, normal, and Pareto) with respect to constituency sizes. The values of α in Table 2 run from 0 to 1 in 0.02-intervals, and $\mathbf{E}[\Delta]$ is estimated again by simulations with 1 mio. iterations. It turns out that $\alpha \approx 0.5$ is no longer the general clear winner from the considered set of parameters $\{0, 0.02, ..., 0.98, 1\}$. In particular, when the variance of constituency sizes is relatively small, as is the case for the configurations from $\mathbf{U}(3\cdot10^6, 10^7)$ and $\mathbf{N}(10^7, 2\times10^6)$, little scope for discrimination between constituencies exists.

Figure 1 illustrates that the objective function is indeed very flat. Finding its minimum via Monte-Carlo techniques is thus particularly sensitive to the remaining estimation errors.

4.2 EU Council of Ministers

5 Concluding remarks

References

Arnold, B. C., N. Balakrishnan, and H. N. Nagaraja (1992). *A First Course in Order Statistics*. New York: John Wiley & Sons.

Banks, J. S. and J. Duggan (2000). A Bargaining Model of Social Choice. *American Political Science Review 94*(1), 73–88.

Distribution of constituency sizes	(I) α^*		(II) α^*	
$\mathbf{U}(0, 10^8)$	**0.52**	2.87×10^{-9}	**0.5**	2.59×10^{-9}
$\mathbf{U}(3 \cdot 10^6, 10^7)$	**0.34**	2.17×10^{-8}	**0.12**	1.91×10^{-8}
$\mathbf{N}(10^7, 4 \times 10^6)$	**0.46**	1.33×10^{-8}	**0.50**	1.29×10^{-8}
$\mathbf{N}(10^7, 2 \times 10^6)$	**0.22**	1.32×10^{-8}	**0.4**	1.43×10^{-8}
$\mathbf{P}(1.0, 500000)$	**0.56**	6.27×10^{-8}	**0.52**	4.70×10^{-8}
$\mathbf{P}(1.8, 500000)$	**0.54**	7.34×10^{-9}	**0.54**	1.57×10^{-8}

Table 2: Optimal value of α for constituency sizes from two independent draws from different distributions (size of the direct democracy deficit in parentheses)

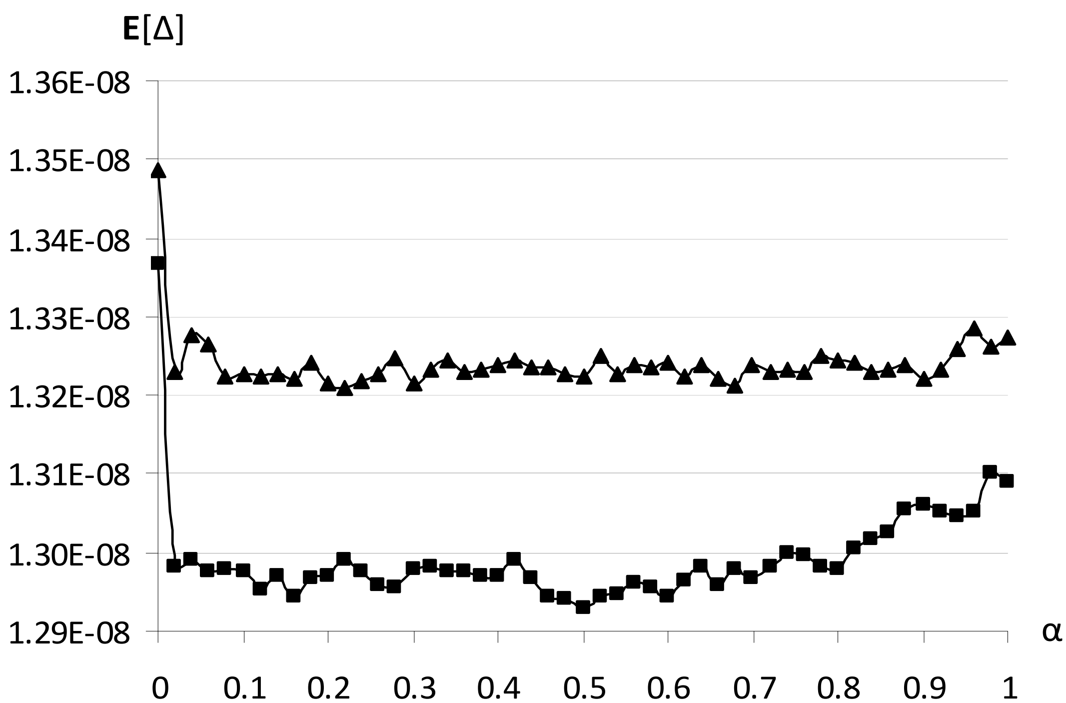

Figure 1: Direct democracy deficit in normal-distribution runs with $m = 30$ and two different levels of population size variance (the triangle- and square-line refer to constituency sizes from $\mathbf{N}(10^7, 2 \times 10^6)$ and $\mathbf{N}(10^7, 4 \times 10^6)$, respectively)

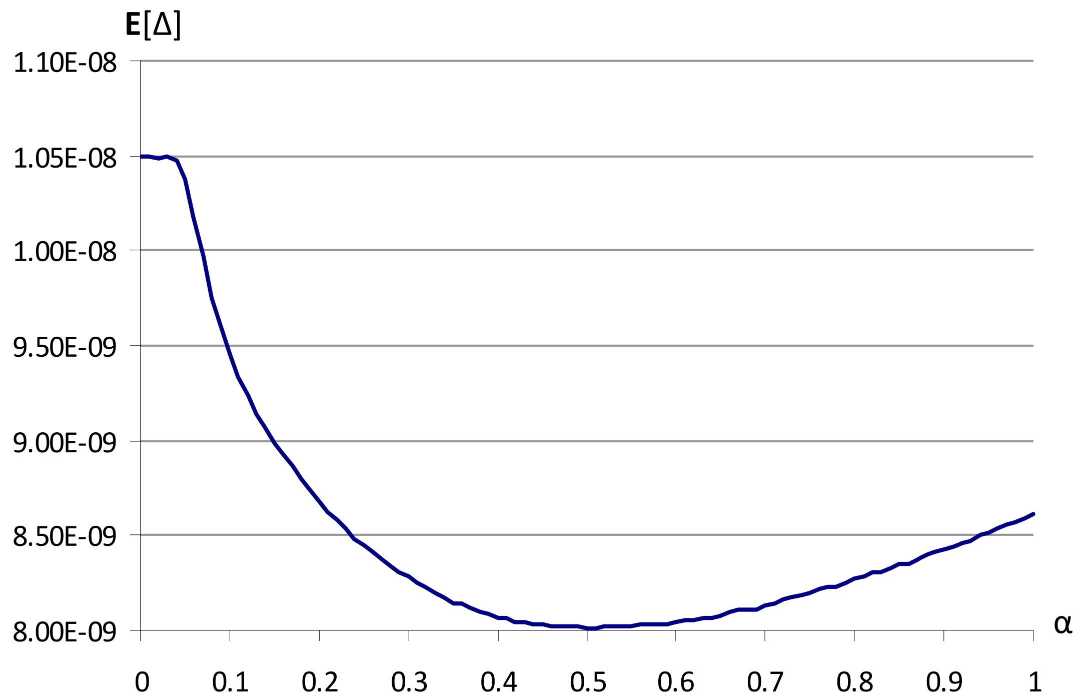

Figure 2: Average squared distance between direct democracy outcome and representative democracy outcome in EU27

Barberà, S. and M. O. Jackson (2006). On the weights of nations: Assigning voting weights in a heterogeneous union. *Journal of Political Economy 114*(2), 317–339.

Baurmann, M. and H. Kliemt (1993). Volksabstimmungen, Verhandlungen und der Schleier der Insignifikanz. *Analyse und Kritik 15*(.), 150–167.

Beisbart, C. and L. Bovens (2007). Welfarist evaluations of decision rules for boards of representatives. *Social Choice and Welfare 29*(4), 581–608.

Beisbart, C., L. Bovens, and S. Hartmann (2005). A utilitarian assessment of alternative decision rules in the Council of Ministers. *European Union Politics 6*(4), 395–419.

Bohnet, I. and B. S. Frey (1994). Direct-Democratic Rules: The Role of Discussion. *Kyklos 47*(3), 341–354.

Cramer, H. (1946). *Mathematical Methods of Statistics*. Princeton: Princeton University Press.

Dahl, R. A. (1994). A Democratic Dilemma: System Effectiveness versus Citizen Participation. *Political Science Quarterly 109*(1), 23–34.

Feix, M. R., D. Lepelley, V. Merlin, J. L. Rouet, and L. Vidu (2008). Majority Efficient Representation of the Citizens in a Federal Union. mimeo, Université de Caen.

Felsenthal, D. and M. Machover (1998). *The Measurement of Voting Power – Theory and Practice, Problems and Paradoxes*. Cheltenham: Edward Elgar.

Felsenthal, D. and M. Machover (1999). Minimizing the Mean Majority Deficit: The Second Square-root Rule. *Mathematical Social Sciences 37*(1), 25–37.

Gerber, E. R. and J. B. Lewis (2004). Beyond the Median: Voter Preferences, District Heterogeneity, and Political Representation. *Journal of Political Economy 112*(6), 1364–1383.

Laruelle, A. and F. Valenciano (2008). Bargaining in Committees of Representatives: The 'Neutral' Voting Rule. *Journal of Theoretical Politics 20*(1), 93–106.

Maaser, N. and S. Napel (2007). Equal Representation in Two-tier Voting Systems. *Social Choice and Welfare 28*(3), 401-420.

Nurmi, H. (1998). Voting Paradoxes and Referenda. *Social Choice and Welfare 15*(3), 333-350.

Penrose, L. S. (1946). The Elementary Statistics of Majority Voting. *Journal of the Royal Statistical Society 109*, 53–57.

Schwertman, N. C., A. J. Gilks, and J. Cameron (1990). A Simple Noncalculus Proof That the Median Minimizes the Sum of the Absolute Deviations. *The American Statistician 44*(1), 38–39.

Weingast, B. R., K. A. Shepsle, and C. Johnsen (1981). The Political Economy of Benefits and Costs: A Neoclassical Approach to Distributive Politics. *Journal of Political Economy 89*(4), 642–666.