Decisions with conflicting and imprecise information

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Introduction

Decision under uncertainty

- consult experts
- decide on the basis of the collected information

Question

How to decide on the basis of an information coming from several experts?

Problem

- Experts might be imprecise
- Experts might disagree

Conflict and imprecision

	Expert 1	Expert 2
Situation A	$\frac{1}{2}$	$\frac{1}{2}$
Situation B	$\frac{1}{4}$	$\frac{3}{4}$
Situation C	$\left[\frac{1}{4}, \frac{3}{4}\right]$	$\left[\frac{1}{4}, \frac{3}{4}\right]$

Linear aggregation rule

$F_A = F_B = \frac{1}{2}$: Fails to grasp conflict

Union rule

- $F_B = F_C = \left[\frac{1}{4}, \frac{3}{4}\right]$
- Fails to grasp imprecision
- Smithson (1999), Cabantous (2007): "conflict aversion hypothesis"

Aggregation and preferences

Traditional approach

- gather information
- aggregate information
- decide

Consider directly preferences with several sources of information

Plan

1. Representation theorem
2. Comparative imprecision and conflict aversion
3. Example

Setup

- Ω: finite state space
- $P \in \mathscr{P}$: compact convex subset of probas on Ω
- $x \in X$: pure outcomes
- $\Delta(X)$: simple lotteries
- $\mathscr{F} = \{f : \Omega \to \Delta(X)\}$: acts

Domain of preferences

$$\mathscr{F} \times \mathscr{P} \times \mathscr{P}$$

- $(f, P_1, Q_1) \succcurlyeq (g, P_2, Q_2)$
- $(f, P) \succcurlyeq^* (g, Q) \Leftrightarrow (f, P, P) \succcurlyeq (g, Q, Q)$

Preliminaries: one source of information

GHTV (2008)

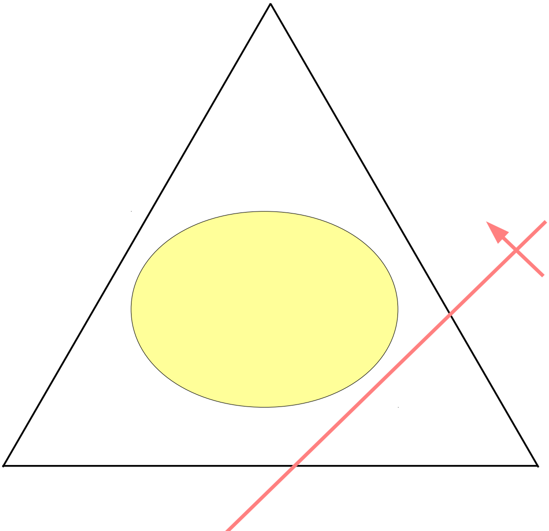
The preference $\succsim^*$ is represented by:

$$V(f,P) = \min_{p \in \varphi(P)} \sum_{\omega} u\left(f(\omega)\right) p(\omega).$$

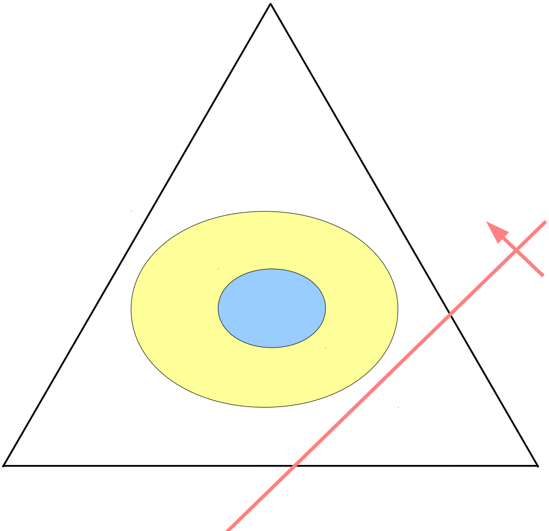
Risk and imprecision

- φ: attitude towards imprecision
- u: attitude towards risk

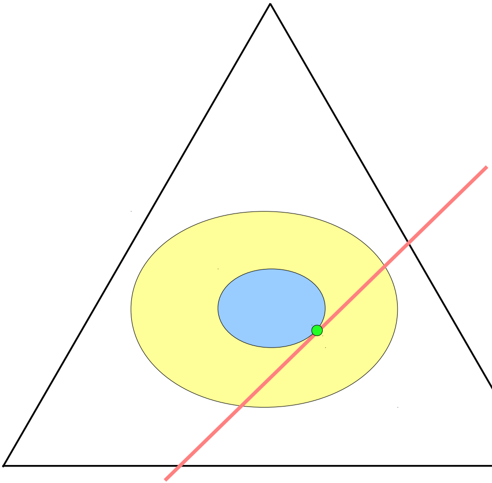
Preliminaries: one source of information



Preliminaries: one source of information



Preliminaries: one source of information



Axioms

Extension of GHTV

$$V(f, P, Q) = \min_{p \in \psi(P,Q)} \sum_{\omega} u\left(f(\omega)\right) p(\omega).$$

Dominance

$$\left.\begin{array}{l} (f, P_1) \succsim^* (g, P_2) \\ (f, Q_1) \succsim^* (g, Q_2) \end{array}\right\} \Rightarrow (f, P_1, Q_1) \succsim (g, P_2, Q_2)$$

Axioms

Consensus

- $\alpha P + (1-\alpha)Q = \{\alpha p + (1-\alpha)q : p \in P, q \in Q\}$
- Interpretation: the "correct" set is P with proba α and Q with proba $(1-\alpha)$
- Both experts agree that the other expert might be right with proba α
- $(P, Q) \to (\alpha P + (1-\alpha)Q, \alpha Q + (1-\alpha)P)$
- $\alpha \to \frac{1}{2}$: reach consensus

Conflict Aversion

$$(f, \alpha P + (1-\alpha)Q, \alpha Q + (1-\alpha)P) \succcurlyeq (f, P, Q).$$

Representation

Theorem

There exist u linear, φ continuous and linear, $\Pi \in \Delta(\{1,2\})$ symmetric such that:

$$V(f,P,Q) = \min_{\pi \in \Pi} \pi(1) \left(\min_{p \in \varphi(P)} \sum_{\omega} u(f(\omega))p(\omega) \right) + \pi(2) \left(\min_{p \in \varphi(Q)} \sum_{\omega} u(f(\omega))p(\omega) \right).$$

Equivalent representation

$$V(f,P,Q) = \min_{p \in \Pi \otimes (\varphi(P),\varphi(Q))} \sum_{\omega} u(f(\omega))p(\omega)$$

$$\Pi \otimes (\varphi(P), \varphi(Q)) = \{\pi(1)p + \pi(2)q | \pi \in \Pi,\ p \in \varphi(P),\ q \in \varphi(Q)\}$$

Comparative imprecision aversion

Definition

Suppose there exist two prizes $\bar{x}, \underline{x}$ in X st both a and b strictly prefer $\bar{x}$ to $\underline{x}$. We say that $\succcurlyeq_b^*$ is more averse to imprecision than $\succcurlyeq_a^*$ whenever for all $f \in \mathscr{F}^b_{\bar{x},\underline{x}}$, $p \in \Delta(\Omega)$, $P \in \mathscr{P}$

$$(f, \{p\}) \succcurlyeq_a^* (f, P) \Rightarrow (f, \{p\}) \succcurlyeq_b^* (f, P).$$

Proposition

The following assertions are equivalent:

1. $\succcurlyeq_b^*$ is more averse to imprecision than $\succcurlyeq_a^*$,
2. for all $P \in \mathscr{P}$, $\varphi_a(P) \subset \varphi_b(P)$.

Comparative conflict aversion

Definition

$\succsim_b$ is *more averse to conflict than* $\succsim_a$ if for all $f \in \mathscr{F}$, $P, Q \in \mathscr{P}$, $\alpha \in (0,1)$, st both a and b prefers (f, P) to (f, Q) if :

$$(f, \alpha P + (1-\alpha)Q, \alpha P + (1-\alpha)Q) \succcurlyeq_a (f, P, Q)$$

then,

$$(f, \alpha P + (1-\alpha)Q, \alpha P + (1-\alpha)Q) \succcurlyeq_b (f, P, Q).$$

Proposition

The following assertions are equivalent:

1. $\succcurlyeq_b$ is more averse to conflict than $\succcurlyeq_a$,
2. $\Pi_a \subseteq \Pi_b$.

Two states

Parametric form of φ and Π

If φ is symmetric:

$$\varphi(P) = \{(1-\theta)c(P) + \theta p | p \in P\},$$

$$\Pi = \left\{ (1-\alpha)\left(\frac{1}{2}, \frac{1}{2}\right) + \alpha(t, 1-t) \,\middle|\, t \in [0,1] \right\},$$

Two states

Proposition

Assume $|\Omega| = 2$ and $\varphi(P)$ is symmetric. Then the following are equivalent:

(*i*) For all $P, Q \in \mathscr{P}$, there exists $R \in \mathscr{P}$ such that $\Pi \otimes (\varphi(P), \varphi(Q)) = \varphi(R)$;

(*ii*) $\theta \geq \alpha$.

"conflict aversion hypothesis" $\Leftrightarrow \alpha > \theta$

Two states

Proposition

Assume $|\Omega| = 2$ and $\varphi(P)$ is symmetric. Then the following statements are equivalent:

(*i*) For all $P, Q \in \mathscr{P}$, $\Pi \otimes (\varphi(P), \varphi(Q)) = \varphi\,(\Pi \otimes (P, Q))$;

(*ii*) $\theta = 1$.

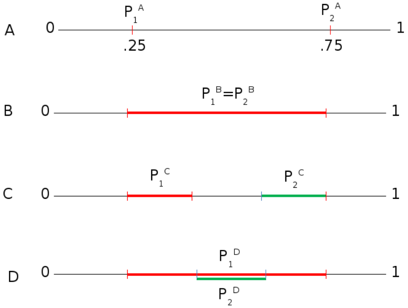
Proposition

Assume $|\Omega| = 2$ and $\varphi(P)$ is symmetric. Then the following statements are equivalent:

(*i*) For all $P, Q \in \mathscr{P}$, $\Pi \otimes (\varphi(P), \varphi(Q)) = \varphi(\mathrm{co}(P \cup Q))$;

(*ii*) $(\alpha, \theta) = (1, 1)$.

Illustration



$1 \geq \theta \geq \frac{3\alpha}{\alpha+2}$	$\frac{3\alpha}{\alpha+2} \geq \theta \geq \frac{4\alpha}{3+\alpha}$	$\frac{4\alpha}{3+\alpha} \geq \theta \geq \alpha$	$\alpha \geq \theta \geq 0$
A	*A*	*D*	*D*
C	*D*	*A*	*B*
D	*C*	*C*	*C*
B	*B*	*B*	*A*