# Higher-order Information and Belief Dynamics:

## A New Perspective on Agreement Results

## Ongoing work with Cédric Dégremont (ILLC)

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February 12, 2009

## Overview

1. Introduction to Agreement Theorems (AT)
2. Three variations on the result:
   - Static agreements;
   - Agreements via conditioning;
   - Agreements via public announcements.

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2. Three variations on the result:
   - Static agreements;
   - Agreements via conditioning;
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## Highlights:

- The received view:
  - ATs undermine the role of private information.
- Instead (the point of view of Dynamic Epistemic Logic):
  - ATs highlight the importance of higher-order information.
  - ATs highlight the fact that updating beliefs by conditioning is different from updating after a "public announcement".

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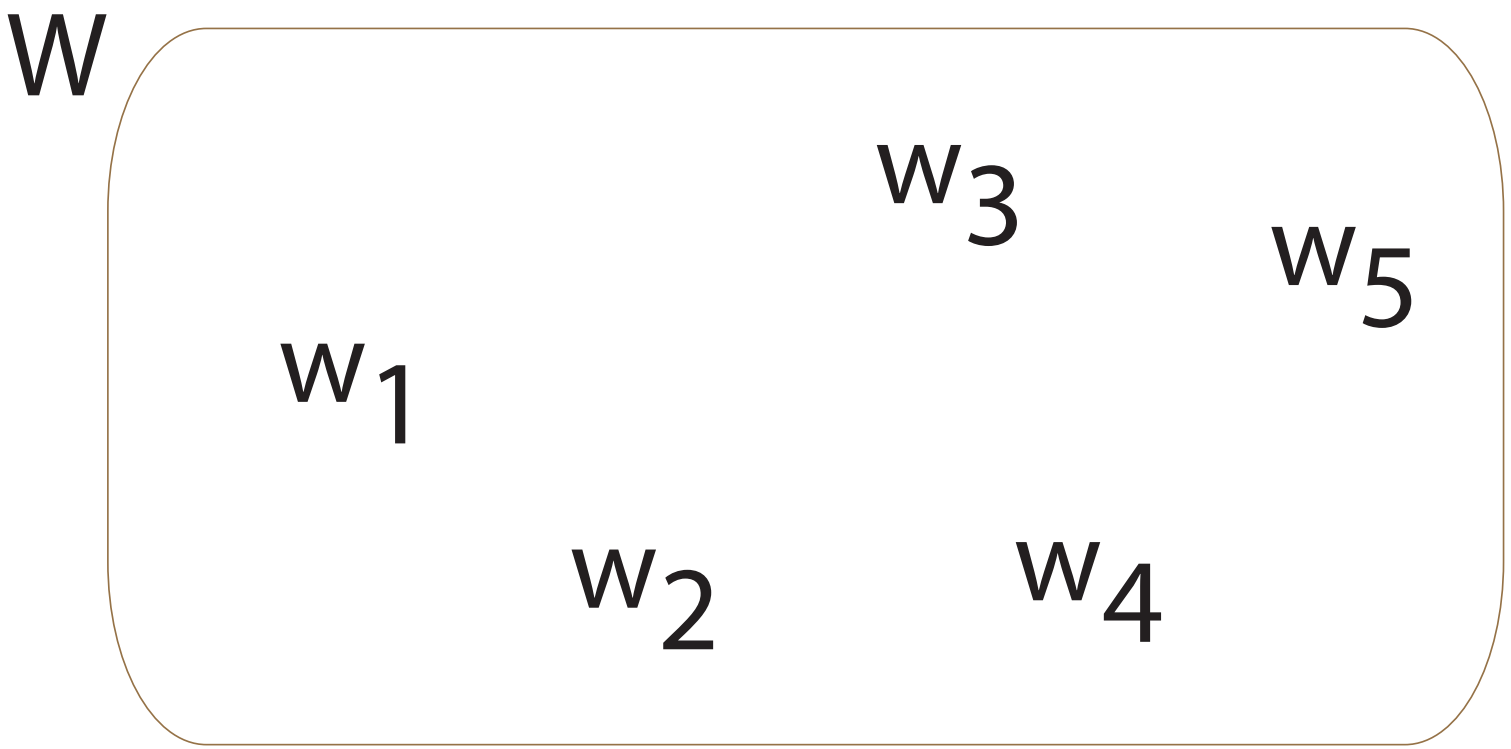
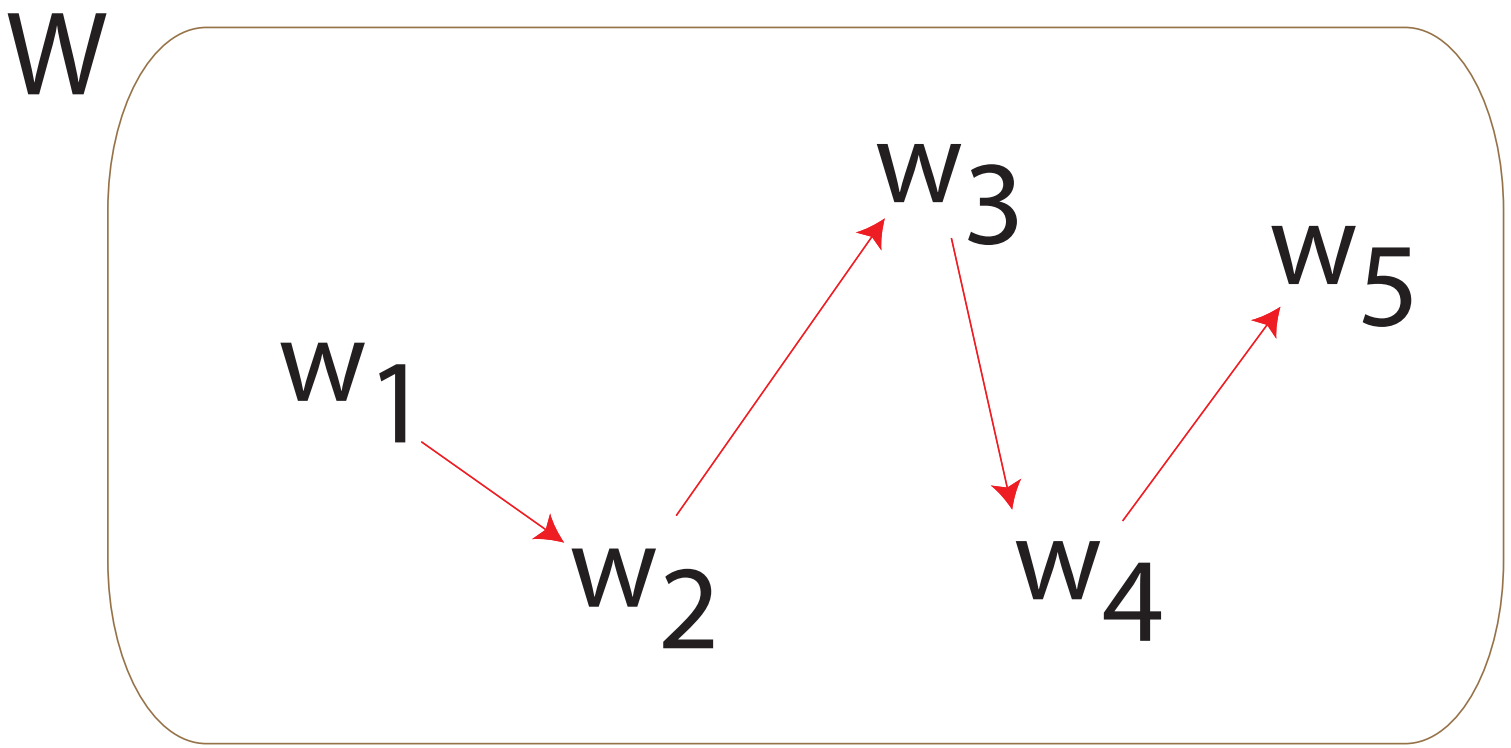
- Qualitative generalizations: [Cave, 1983], [Bacharach, 1985].
- Good survey: [Bonanno and Nehring, 1997].

# Models



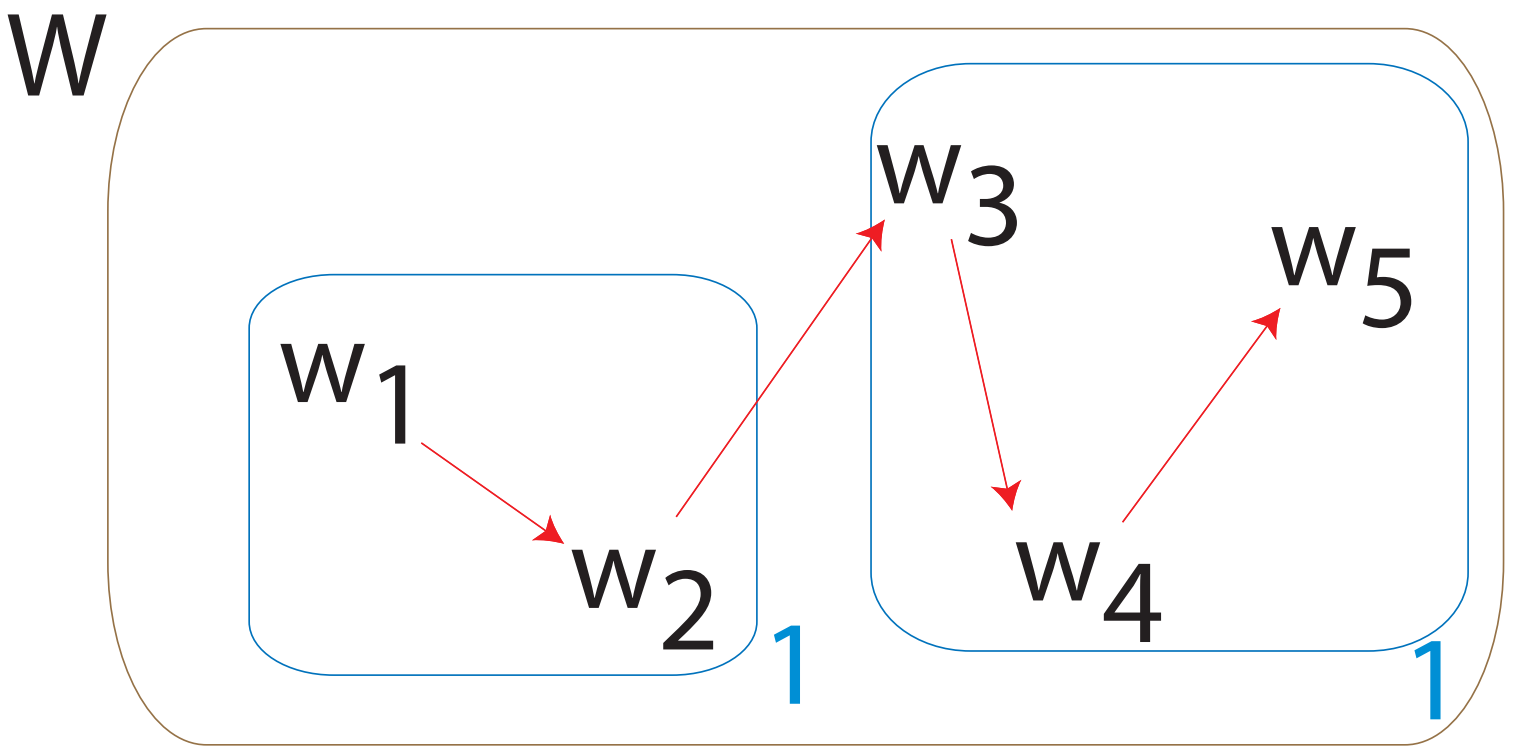

Set of States

See: [Board, 2004, Baltag and Smets, 2008]



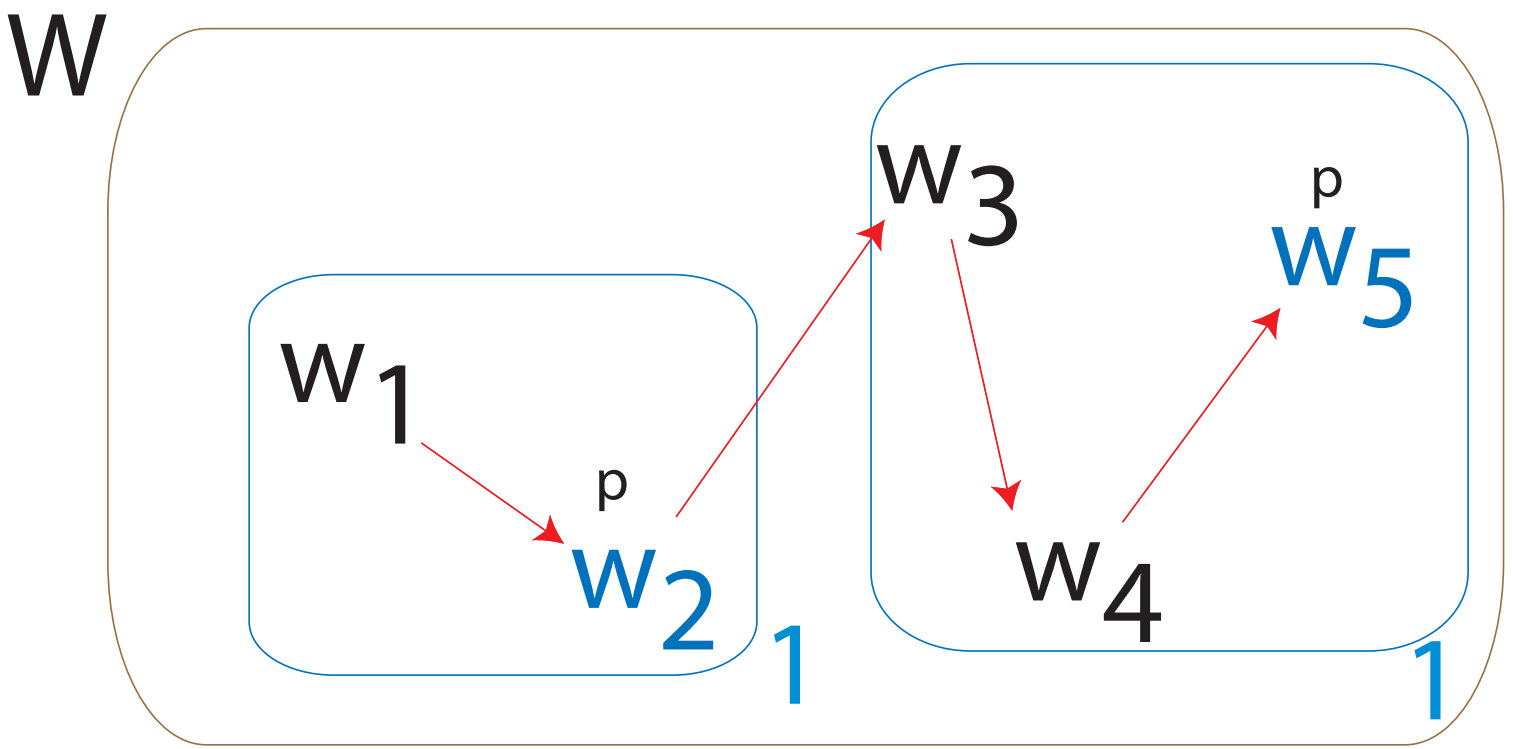

Plausibility Ordering. (Priors)

See: [Board, 2004, Baltag and Smets, 2008]



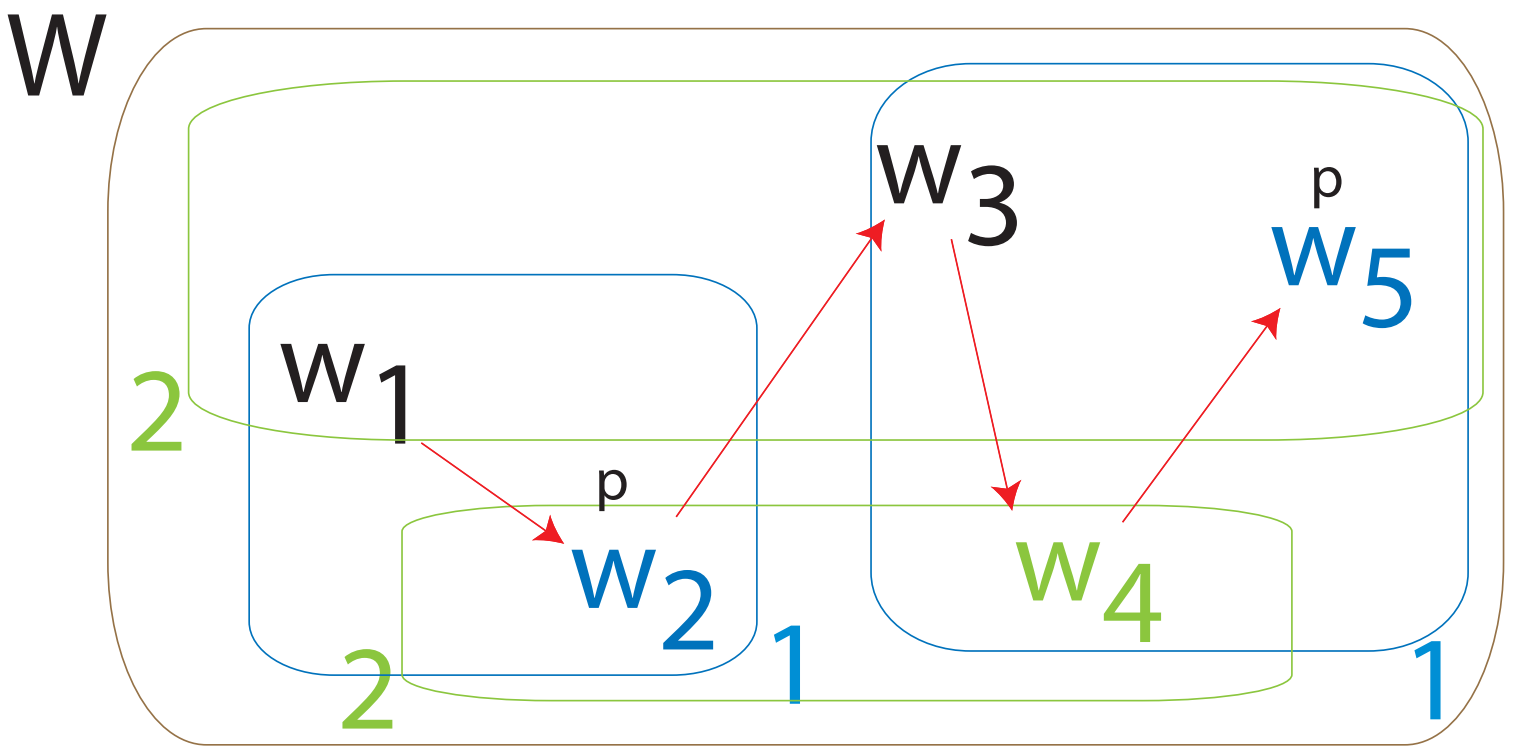

Information Partition.

See: [Board, 2004, Baltag and Smets, 2008]




Conditional Beliefs

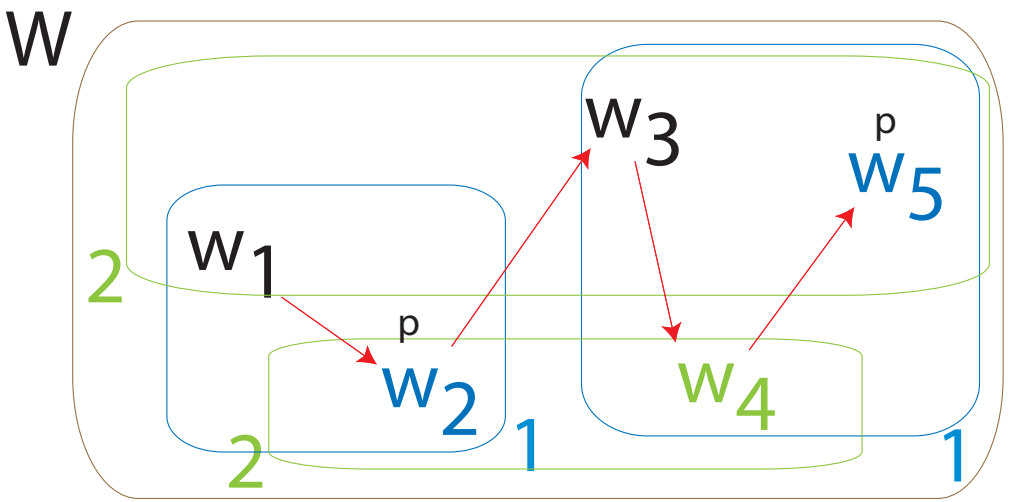
See: [Board, 2004, Baltag and Smets, 2008]

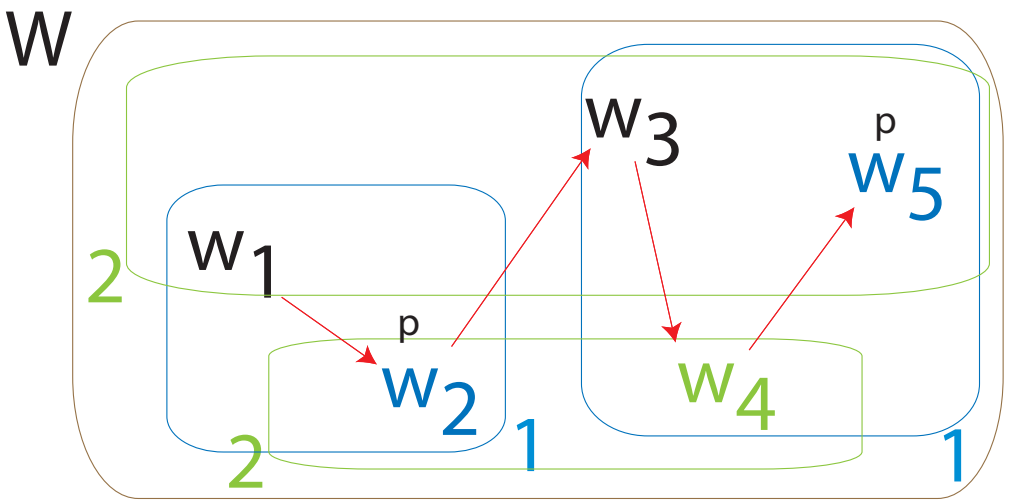



Multi-Agents and Higher-Order Information.

See: [Board, 2004, Baltag and Smets, 2008]

# The basic result



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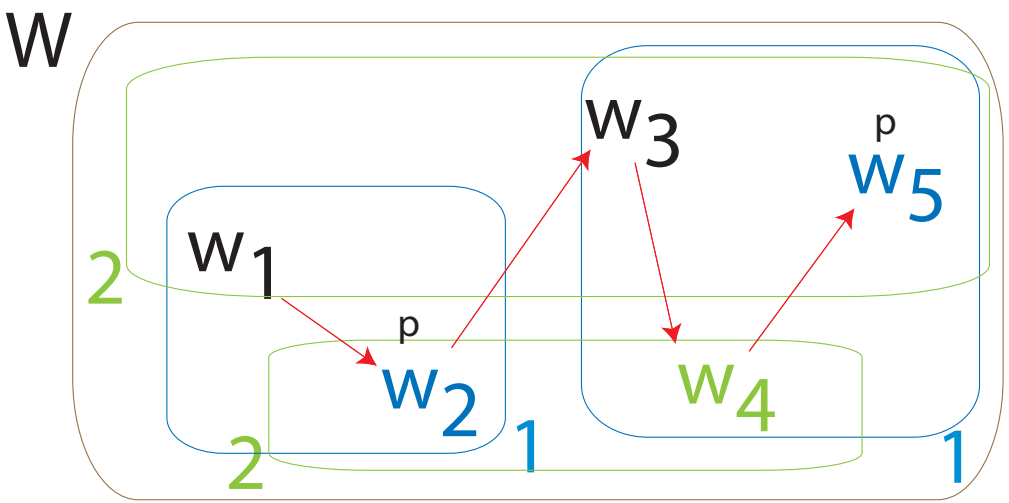
  Common priors and common knowledge of posteriors $\Rightarrow$ same posteriors.

- Why?

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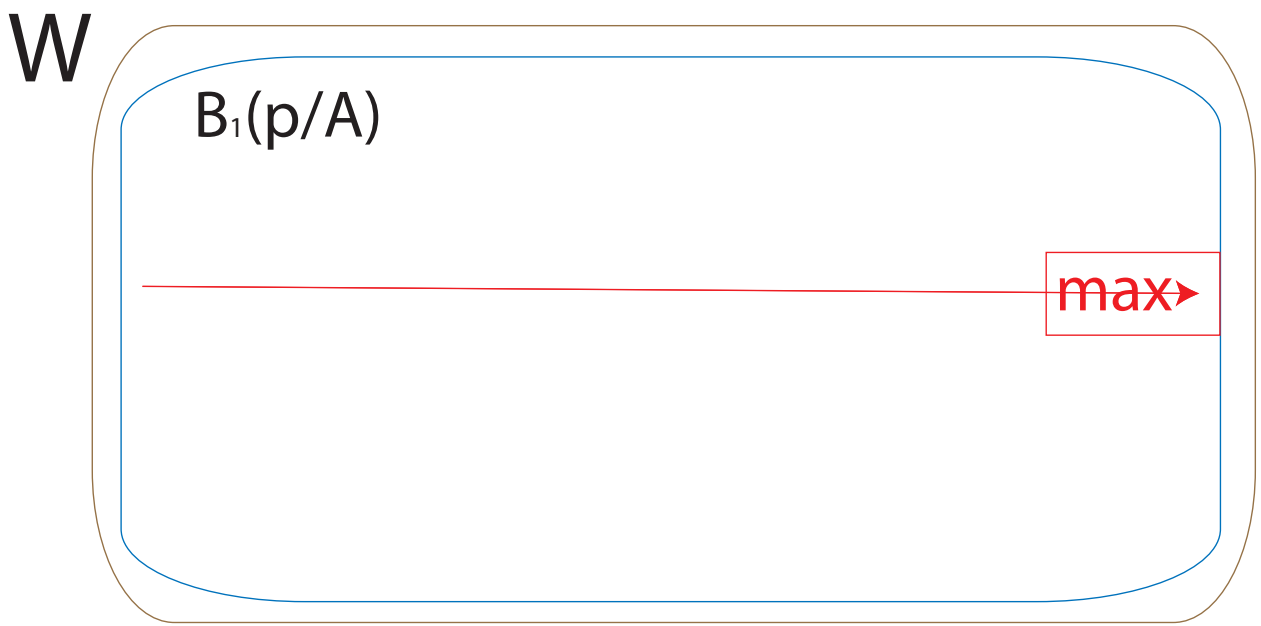
- Why?
  - Intuition: Independence between first- and (all levels!) of higher-order information is the exception rather than the norm.

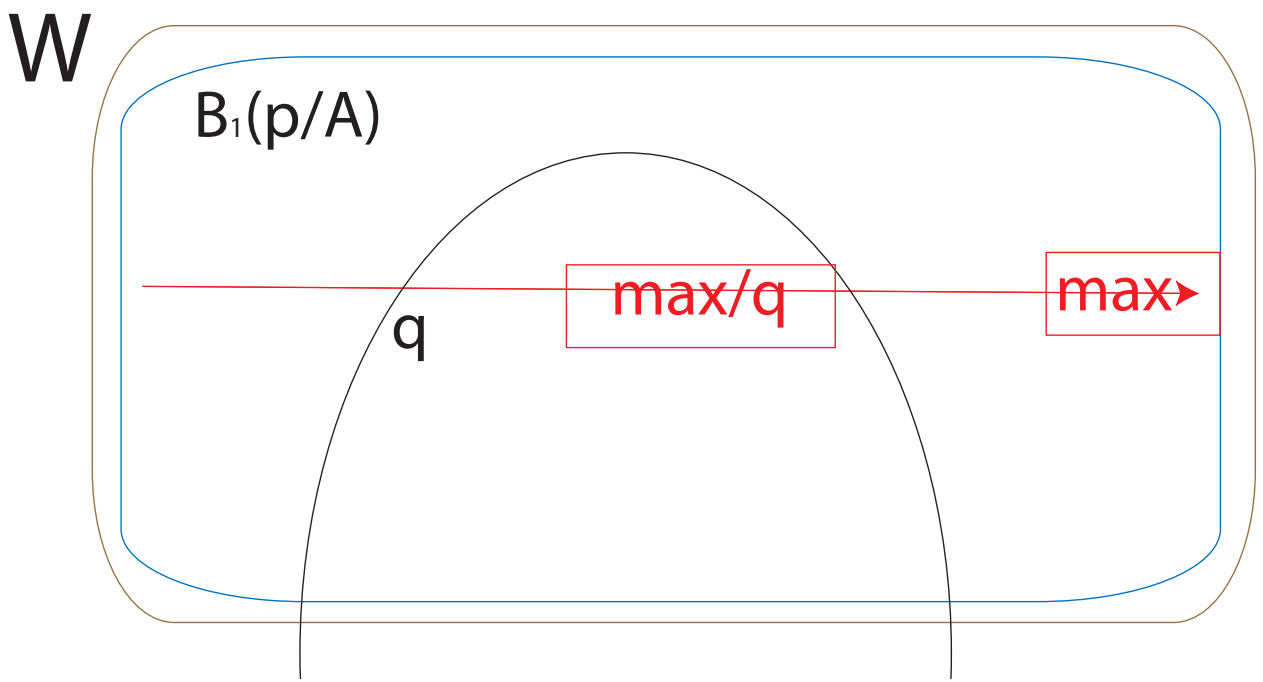
    Evidence for evidence matters.

# Agreements via dialogues

Two ways of analyzing the effect of incoming information




1. Conditioning.
   [Geanakoplos and Polemarchakis, 1982, Bacharach, 1985]

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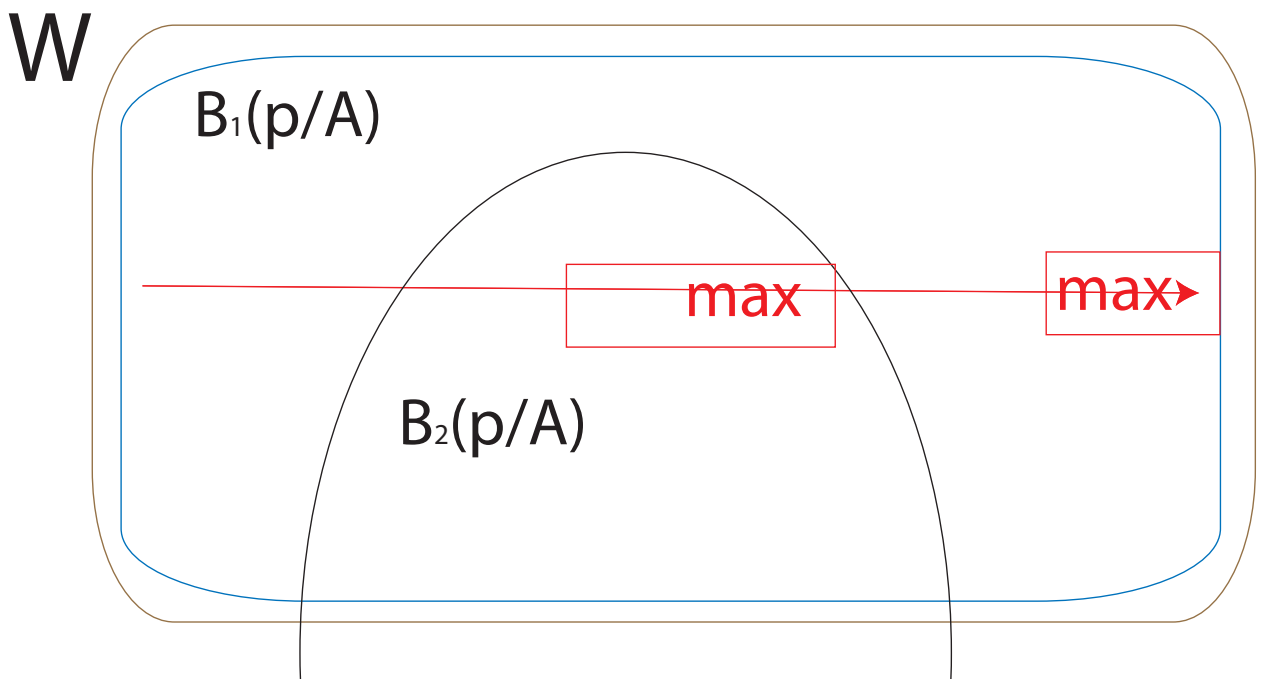
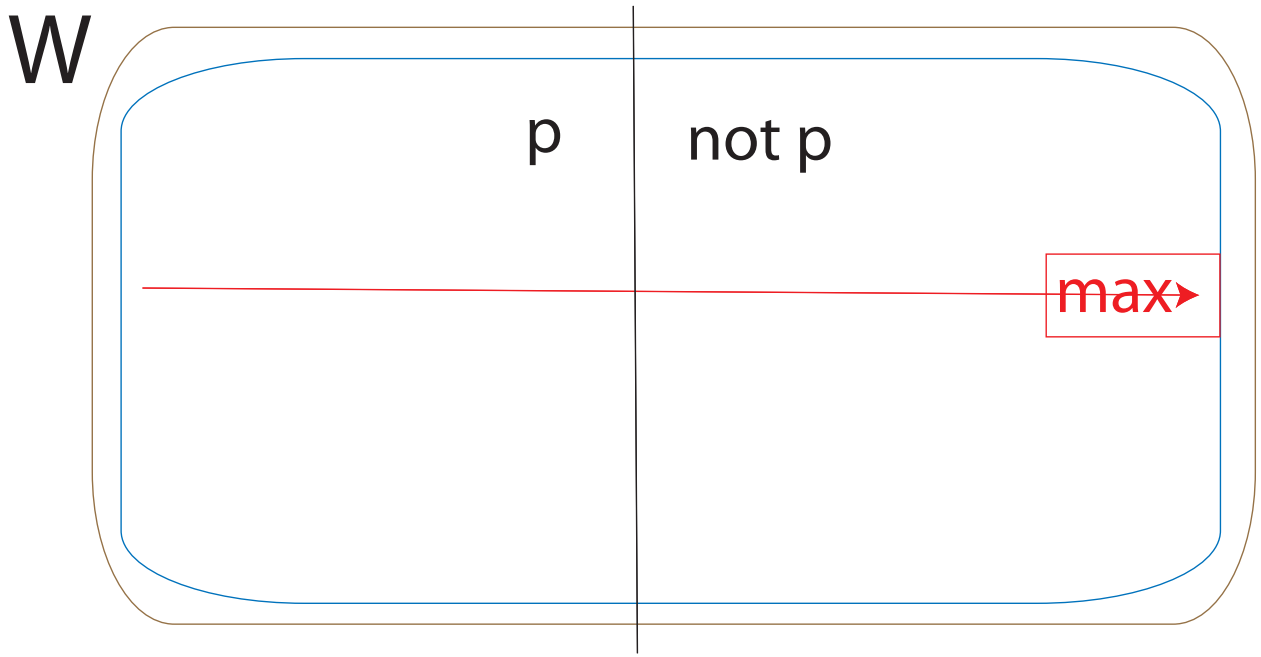
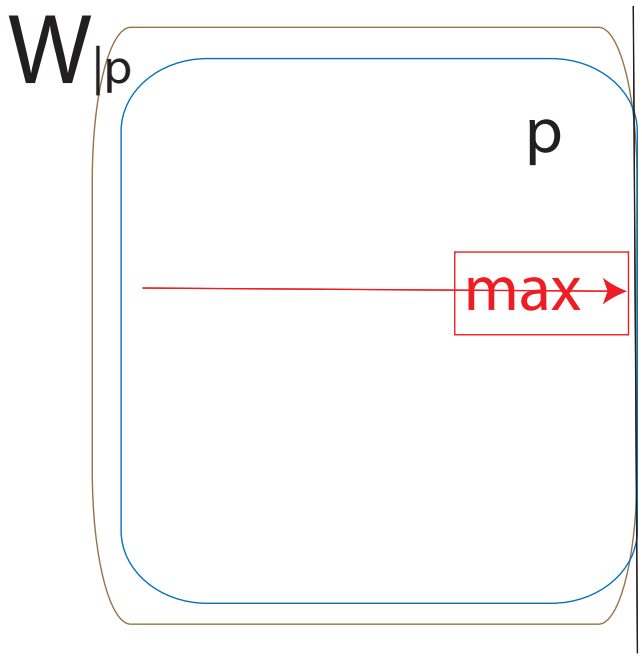



1. Conditioning. ✓

Repeated conditioning on the posteriors of the other agents eliminates all higher-order uncertainties about these.




1. Conditioning. ✓
2. Updating by "public announcements."
   [Gerbrandy, 1999, van Ditmarsch et al., 2007]

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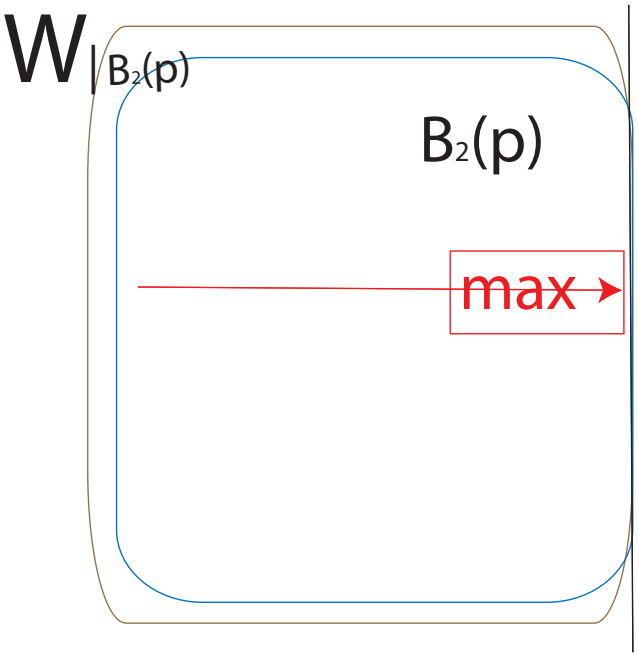


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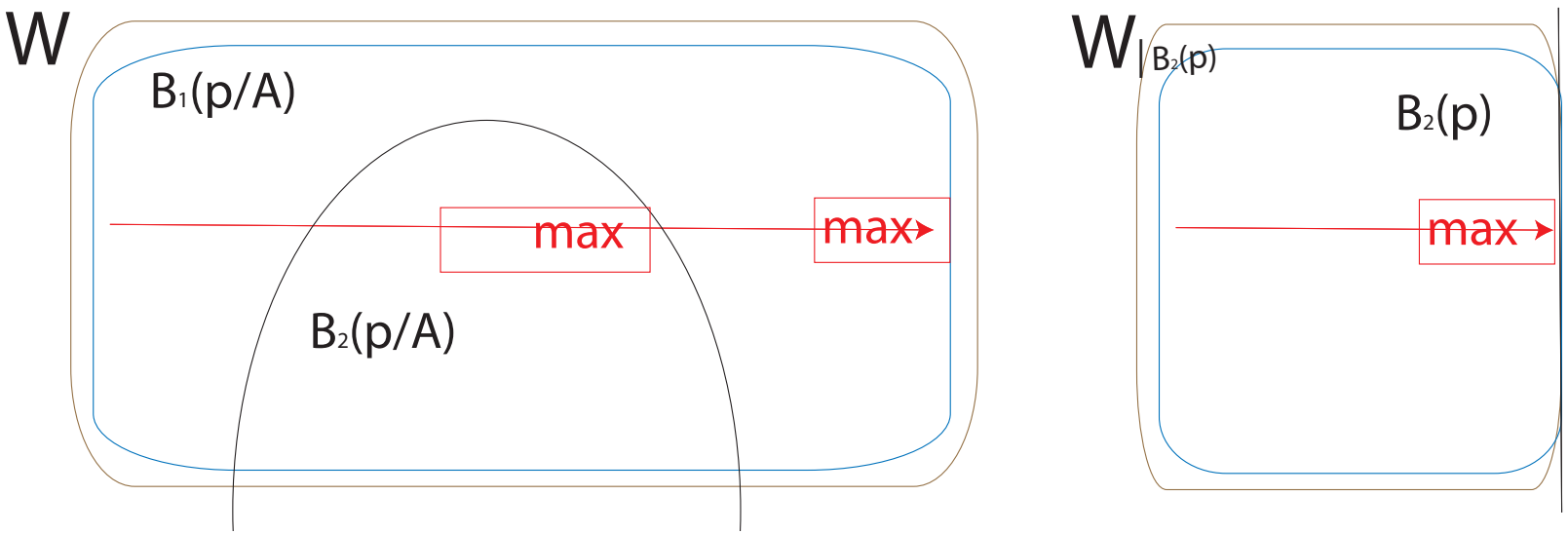
   Repeated public announcements of the posteriors of the others agents eliminates all higher-order uncertainties about these.

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$\Rightarrow$ (1) and (2) can lead to different agreements when the dialogue is about "epistemic facts".

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  - The inter-dependence of first- and higher-order information in interaction.

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  - Softer updates and changes of beliefs?
  - Connection with Lehrer-Wagner models?

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