Iterated Belief Revision and Improvement Operators

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Preliminaries

- A Propositional Language $\mathcal{L}$ build from:
  - A set of propositional variables $\mathcal{P} = \{a, b, c, \ldots\}$
  - And a set of logical connectives $\wedge$, $\vee$, $\neg$, $\rightarrow, \ldots$
- An interpretation $\omega \in \mathcal{W}$ is a function from $\mathcal{P}$ to $\{0, 1\}$ (False/True)
- A model of a formula $\varphi \in \mathcal{L}$ is an interpretation that makes the formula true:

$$\text{mod}(\varphi) = \{\omega \in \mathcal{W} \mid \omega \models \varphi\}$$

- A belief base is a set of propositional formulas that represent the beliefs of the agent
- In the following we will consider that a belief base is a unique propositional formula (the conjunction of the set)

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  - Non-prioritized revision [Theoria 98]

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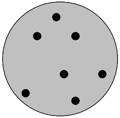

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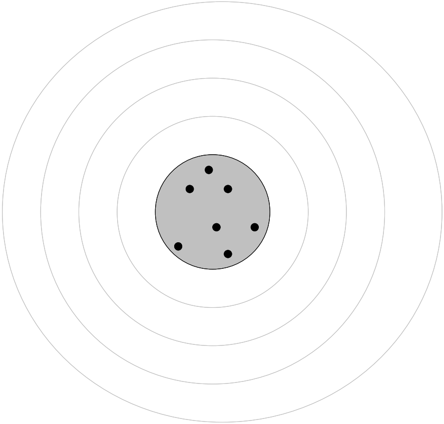

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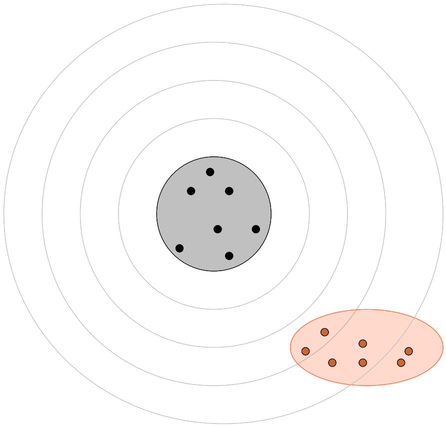

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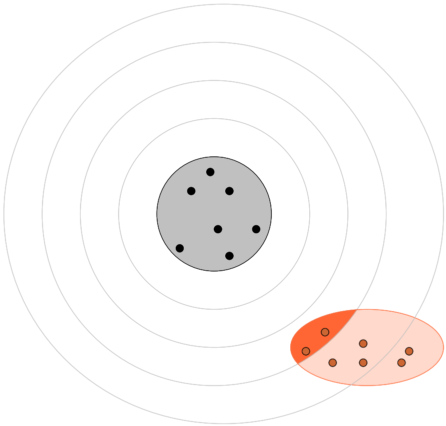

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Theorem (Katsuno-Mendelzon 91)

An operator $\star$ is a revision operator (ie. it satisfies (R1)-(R6)) if and only if there exists a faithful assignment that maps each base φ to a total pre-order $\leq_\varphi$ such that

$$\text{mod}(\varphi \star \alpha) = \min(\text{mod}(\alpha), \leq_\varphi).$$

A faithful assignment is a function mapping each base φ to a pre-order $\leq_\varphi$ over interpretations such that:

- If $\omega \models \varphi$ and $\omega' \models \varphi$, then $\omega \simeq_\varphi \omega'$
- If $\omega \models \varphi$ and $\omega' \not\models \varphi$, then $\omega <_\varphi \omega'$
- If $\varphi \equiv \varphi'$, then $\leq_\varphi = \leq_{\varphi'}$

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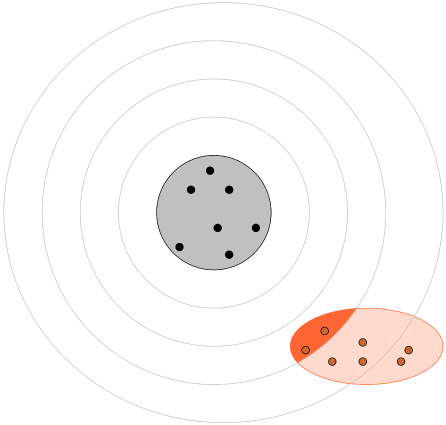

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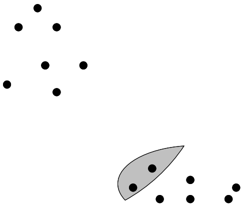

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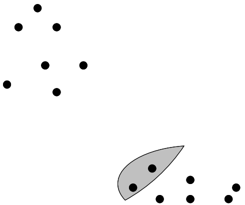

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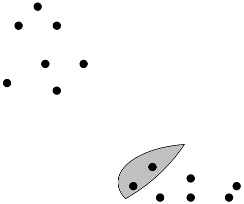

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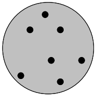

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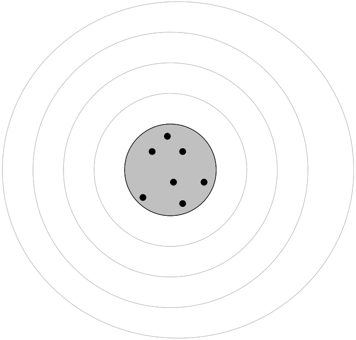

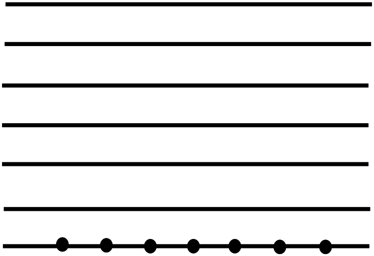

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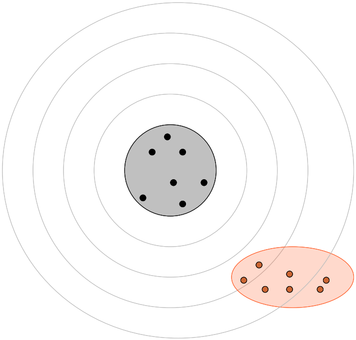

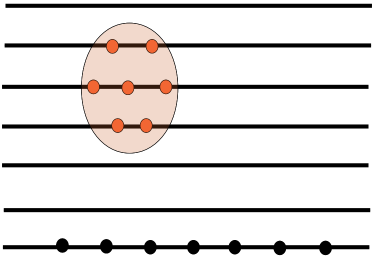

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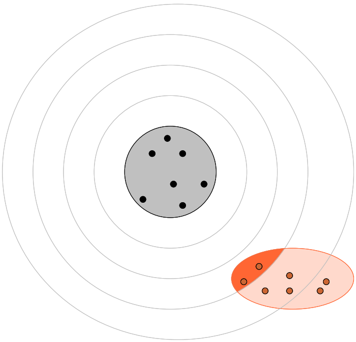

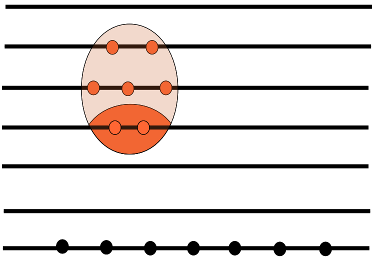

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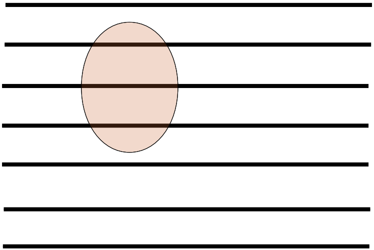

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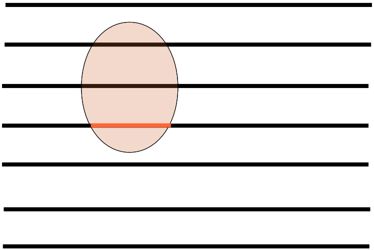

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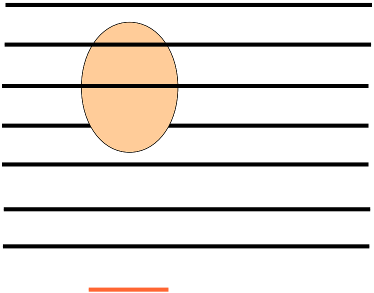

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AGM

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More on epistemic states

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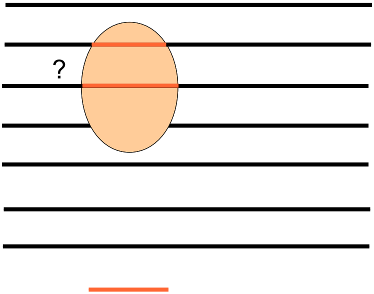

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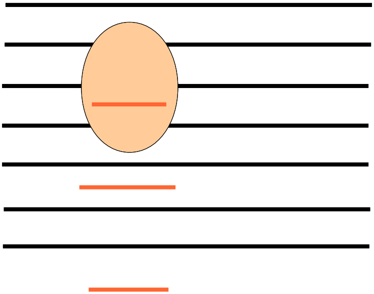

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Iterated Belief Revision

- [Jin & Thielscher 05,07, Booth & Meyer 06]
  - Improve plausibility of models of the new information α

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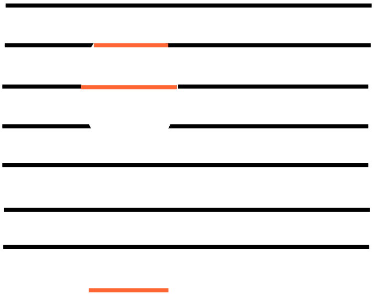

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- Boutilier's natural revision [Boutilier 96]
- Nayak's lexicographic operator
  [Nayak 94, Spohn 88]

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Weak Improvement Operators

Definition

An operator $\circ$ is said to be a weak improvement operator if it satisfies **(I1-I6)**:

(I1) There exists n such that $\Psi \circ^n \alpha \vdash \alpha$

(I2) If $\Psi \wedge \alpha \nvdash \bot$, then $\Psi \star \alpha \equiv \Psi \wedge \alpha$

(I3) If $\alpha \nvdash \bot$, then $\Psi \circ \alpha \nvdash \bot$

(I4) For any positive integer n if $\alpha_i \equiv \beta_i$ for all $i \leq n$ then

$$\Psi \circ \alpha_1 \circ \cdots \circ \alpha_n \equiv \Psi \circ \beta_1 \circ \cdots \circ \beta_n$$

(I5) $(\Psi \star \alpha) \wedge \beta \vdash \Psi \star (\alpha \wedge \beta)$

(I6) If $(\Psi \star \alpha) \wedge \beta \nvdash \bot$, then $\Psi \star (\alpha \wedge \beta) \vdash (\Psi \star \alpha) \wedge \beta$

More on (I4)

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Theorem

A change operator $\circ$ is a weak improvement operator if and only if there exists a strong faithful assignment that maps each epistemic state Ψ to a total pre-order on interpretations $\leq_\Psi$ such that

$$\mathrm{mod}(\Psi \star \alpha) = \min(\mathrm{mod}(\alpha), \leq_\Psi)$$

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Corollary

*If $\circ$ is a weak improvement operator, then $\star$ is an AGM/DP revision operator, i.e. it satisfies (R*1)-(R*6).*

Towards Improvement Operators

$$\Psi \star (\alpha \vee \beta) = \begin{cases} \Psi \star \alpha \textit{ or} \\ \Psi \star \beta \textit{ or} \\ \Psi \star \alpha \vee \Psi \star \beta \end{cases}$$

Definition

- We say that α is below β with respect to Ψ, given $\circ$, denoted $\alpha \prec^{\circ}_{\Psi} \beta$ if and only if $\Psi \star \alpha \vdash \Psi \star (\alpha \vee \beta)$ and $\Psi \star \beta \nvdash \Psi \star (\alpha \vee \beta)$
- The pair (α, β) is Ψ-consecutive, denoted $\alpha \lessdot^{\circ}_{\Psi} \beta$ if and only if $\alpha \prec_{\Psi} \beta$ and there is no formula γ such that $\alpha \prec_{\Psi} \gamma \prec_{\Psi} \beta$.

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Improvement Operators

Definition

A weak improvement operator is said to be an improvement operator if it satisfies **(I7 - I11)**:
(I7) If $\alpha \vdash \beta$ then $(\Psi \circ \beta) \star \alpha \equiv \Psi \star \alpha$
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Definition

Let $\circ$ be a weak improvement operator and $\Psi \mapsto \leq_\Psi$ its corresponding strong faithful assignment. A gradual assignment satisfies:

(S1) If $\omega, \omega' \in$ mod(α) then $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$
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(I11) If $\Psi \star \alpha \vdash \neg\beta$, $\alpha \wedge \beta \nvdash \perp$ and $\alpha \not\ll_{\Psi} \alpha \wedge \beta$ then $(\Psi \circ \beta) \star \alpha \nvdash \neg\beta$

Definition

Let $\circ$ be a weak improvement operator and $\Psi \mapsto \leq_{\Psi}$ its corresponding strong faithful assignment. A gradual assignment satisfies:

(S1) If $\omega, \omega' \in \text{mod}(\alpha)$ then $\omega \leq_{\Psi} \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$ (CR1)

(S2) If $\omega, \omega' \in \text{mod}(\neg\alpha)$ then $\omega \leq_{\Psi} \omega' \Leftrightarrow \omega \leq_{\Psi\circ\alpha} \omega'$ (CR2)

(S3) If $\omega \in \text{mod}(\alpha), \omega' \in \text{mod}(\neg\alpha)$ then $\omega \leq_{\Psi} \omega' \Rightarrow \omega <_{\Psi\circ\alpha} \omega'$ ([JT07,BM06])

(S4) If $\omega \in \text{mod}(\alpha), \omega' \in \text{mod}(\neg\alpha)$ then $\omega' <_{\Psi} \omega \Rightarrow \omega' \leq_{\Psi\circ\alpha} \omega$

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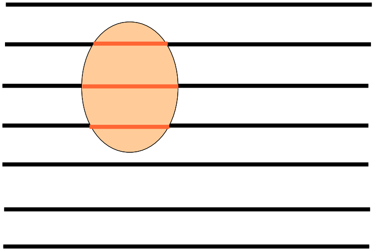
Proposition

Let $\circ$ be an improvement operator and $\Psi \mapsto \leq_\Psi$ its gradual assignment. Then for every formula α, the pre-order $\leq_{\Psi \circ \alpha}$ is completely determined by $\leq_\Psi$ and $\text{mod}(\alpha)$.

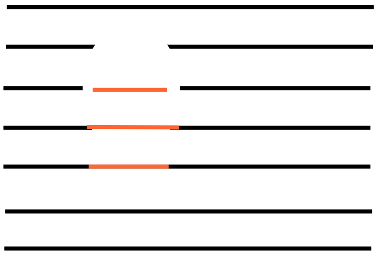
Example

Ψ



Example

$\Psi \quad \circ\, \alpha$



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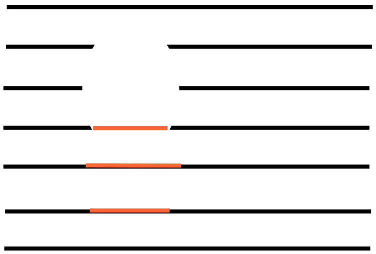
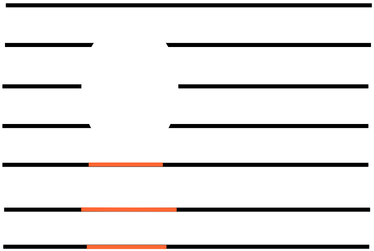
$\Psi \circ \alpha \circ \alpha$



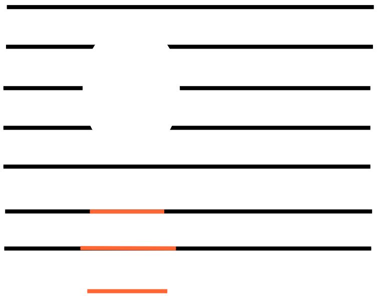
Example

$$\Psi \circ \alpha \circ \alpha \circ \alpha \equiv \Psi \dot{-} \neg\alpha$$



Example

$\Psi \circ \alpha \circ \alpha \circ \alpha \circ \alpha \equiv \Psi \star \alpha$



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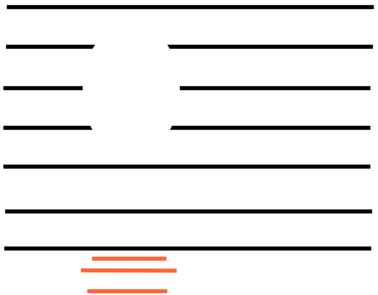

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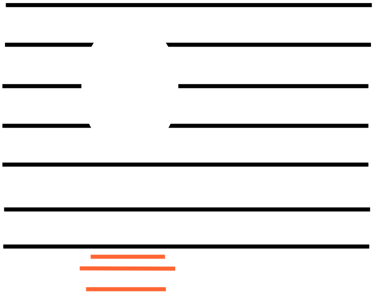

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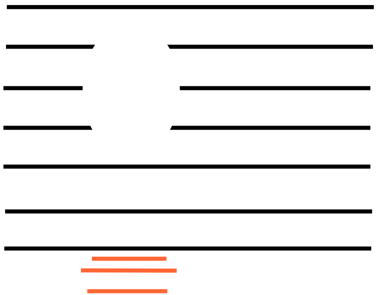

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Long Term Behaviour

- None of the iterated revision operators has a good long term behaviour
  - Leads to full meet revision
    - [Lehmann 95]
  - Leads to maxichoice revision
    - [Darwiche & Pearl 97, Nayak 94, Boutilier 96, Konieczny & Pino Pérez 00, Jin & Thielscher 07, Booth & Meyer 06]

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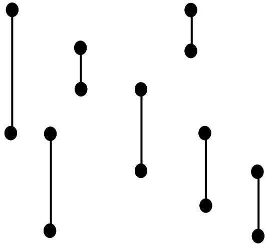
  - In some iterations new “levels” are created,
    in some iterations some “levels” are removed.

Conclusion

- New familly of change operators
  - Increase of the plausibility of the new information
  - No Primacy of Update
- Representation Theorem
- Good long term behaviour

- Others weak improvement operators
- Links with the good day/bad day approach [Booth, Meyer, Wong 06, Booth & Meyer 07]

Good Day / Bad Day [Booth, Meyer, Wong 06]



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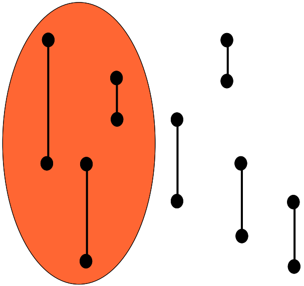

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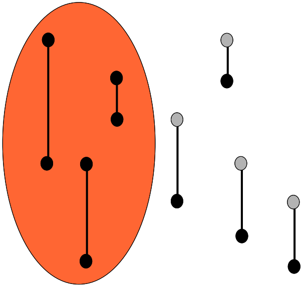

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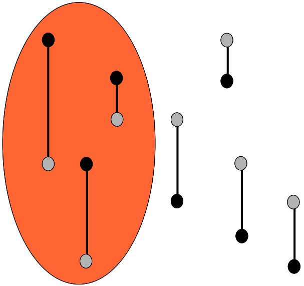

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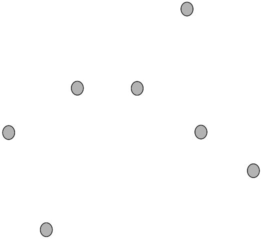

- Iteration ?
  - [Booth & Meyer 2007]
  - Requires to maintain $(<,<<,<<<)$

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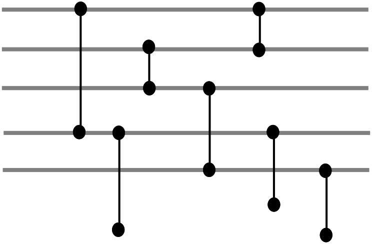

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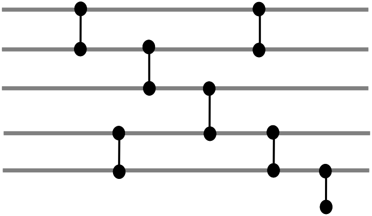

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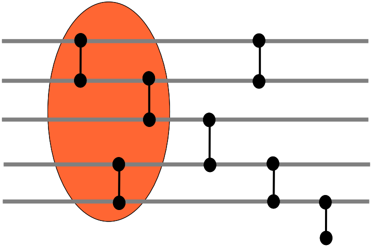

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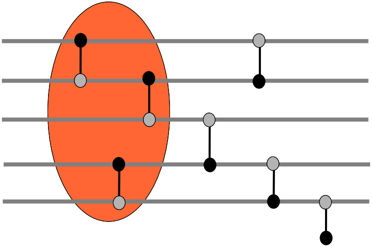

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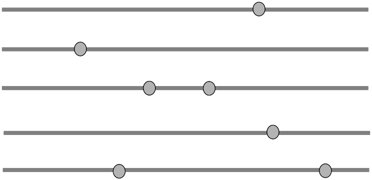

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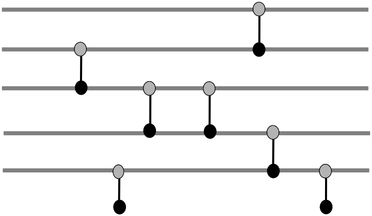

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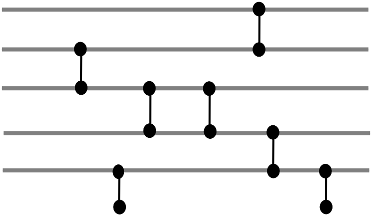


- Improvement operator is a particular case of good day / bad day Revision
  - More general
  - Requires more information

From $\leq_\psi$ to $\leq_{\psi\circ\alpha}$

$w \in \text{mod}(\alpha)$	$w' \in \text{mod}(\alpha)$	$w \leq_\psi w' \Leftrightarrow w \leq_{\psi\circ\alpha} w'$	**(S1)**
$w \in \text{mod}(\neg\alpha)$	$w' \in \text{mod}(\neg\alpha)$	$w \leq_\psi w' \Leftrightarrow w \leq_{\psi\circ\alpha} w'$	**(S2)**
$w \in \text{mod}(\alpha)$	$w' \in \text{mod}(\neg\alpha)$	$w <_\psi w' \Leftrightarrow w <_{\psi\circ\alpha} w'$ $w \simeq_\psi w' \Rightarrow w <_{\psi\circ\alpha} w'$ $w' \ll_\psi w \Rightarrow w \simeq_{\psi\circ\alpha} w'$ $w' <_\psi w \wedge w' \not\ll_\psi w \Rightarrow w <_{\psi\circ\alpha} w'$	**(S3)** **(S3)** **(S4) & (S5)** **(Lemma)**

Irrelevance of Syntax

Darwiche and Pearl (R*4):

Back

$$\text{If } \alpha \equiv \beta, \text{then } B(\Psi \circ \alpha) \equiv B(\Psi \circ \beta) \tag{1}$$

Booth and Meyer [06]:

$$\text{If } (\alpha \equiv \beta \text{ \& } \gamma \equiv \theta), \text{then } B(\Psi \circ \alpha \circ \gamma) \equiv B(\Psi \circ \beta \circ \theta) \tag{2}$$

(I4):

For any positive integer n if $\alpha_i \equiv \beta_i$ for all $i \leq n$ then (3)

$$B(\Psi \circ \alpha_1 \circ \cdots \circ \alpha_n) \equiv B(\Psi \circ \beta_1 \circ \cdots \circ \beta_n)$$

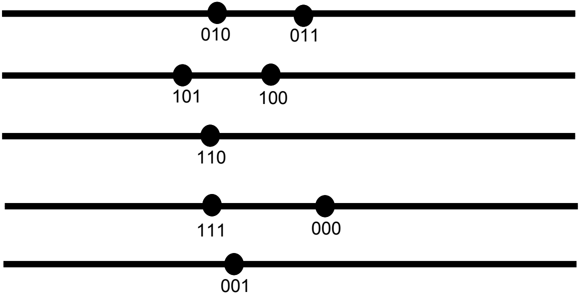
Example (Modification of [Booth & Meyer 06])

Let $\varphi_1 = p \vee \neg p$ and $\varphi_2 = q \vee \neg q$. Let Ψ such that $\Psi \circ \varphi_1 \neq \Psi \circ \varphi_2$. Note that this is compatible with RAGM plus (2), because the only constraint imposed by (2) is that $\leq_{\Psi\circ\varphi_1} = \leq_{\Psi\circ\varphi_2}$ but not that $\Psi \circ \varphi_1 = \Psi \circ \varphi_2$. Moreover we can take $B(\Psi \circ \varphi_1) = p \wedge q = B(\Psi \circ \varphi_2)$. Let Ψ_1, Ψ_2 be two epistemic states such that $B(\Psi_1) = p = B(\Psi_2)$, $00 <_{\Psi_1} 01$ and $01 <_{\Psi_2} 00$. Now, it is compatible with RAGM plus (2) to put $\leq_{\Psi\circ\varphi_1\circ p} = \leq_{\Psi_1}$ and $\leq_{\Psi\circ\varphi_2\circ\neg\neg p} = \leq_{\Psi_2}$. But then, by the representation, we have $B(\Psi \circ \varphi_1 \circ p \circ \neg p) = \neg p \wedge \neg q$ and $B(\Psi \circ \varphi_2 \circ \neg\neg p \circ \neg p) = \neg p \wedge q$, what clearly is a counter-example to (I4).

Plausibility pre-order

Consider $\mathcal{P} = \{b, f, r\}$
b : the animal is a bird
f : the animal flies
r : the animal is red

$B(\Psi) = \neg b \wedge \neg f \wedge r$



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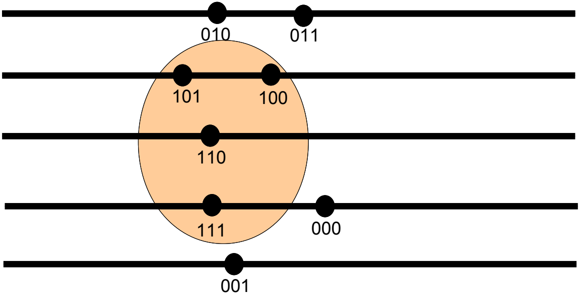

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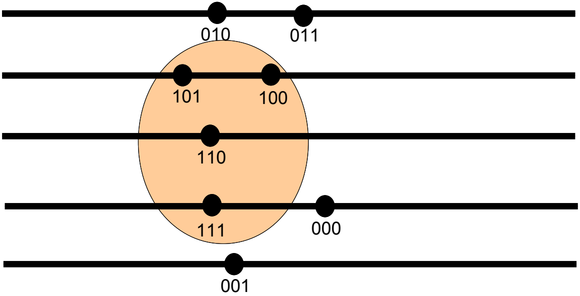

- $\Psi \star b \vdash f$

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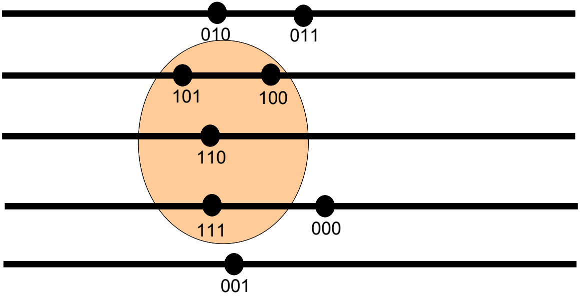

- $\Psi \star b \vdash f$
- Ramsey test:
  "First add the antecedent (hypothetically) to your stock of beliefs ; second make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is true."

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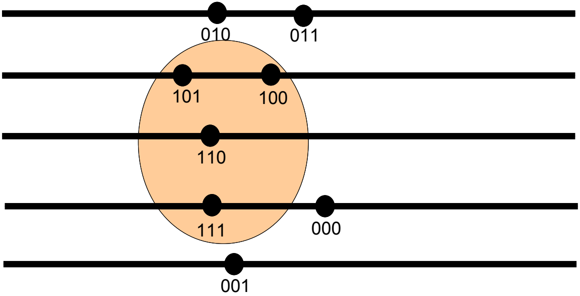

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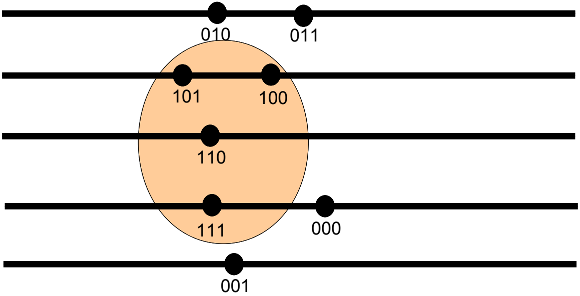

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- Conditional information
- Part of the agent's epistemic state