Aberrant Events and Enlivened Decision Trees

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Outline

1. Introduction
2. A Classical Example
3. ZNK Paradigm
4. Enlivened Trees
5. Normative Theory and Descriptive Models
6. Conclusions

Abstract, I

The existing literature on decision theory, games, and statistics is based on a framework of complete extensive form models. These were pioneered by Zermelo for two-person games of perfect information (like Chess and Go), then von Neumann (later with Morgenstern, improved by Kuhn) for extensive form games with finitely many players, and by Kolmogorov for representing stochastic processes as probability measures over function space.

Abstract, II

This framework completely overlooks some serious issues concerning the need to revise any practical decision model in the light of "aberrant" events that could not be included in a necessarily simplified original model.
There is a consequent value of additional flexibility (or sometimes rigidity) because such revisions may be needed.
Estimating the value of this additional flexibility, however, is a major problem in induction.
Finally, this approach suggests that changing behaviour may be due to an enlivened decision model rather than changes in preferences *per se*.

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1. Introduction
   - Timescales
   - Dynamic Models
   - Aberrance
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Timeless Questions

Many of the most important questions that economics faces have remained essentially the same for a long time. For example:

1. How well, or badly, do markets allocate resources?
   Where do they succeed, or fail?
   What can be done about their failures?
2. How much better could markets allocate resources, if they could be better organized and regulated?
3. What do we mean by a good allocation of resources?

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Timeless Models

This first set of questions
have been extensively but incompletely answered
in what I shall call "normal form" models.
A notable example is the framework that Arrow and Debreu devised
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The economic system is treated as some kind of game
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Von Neumann's invariance hypothesis

Decision trees are equivalent to influence diagrams; they play a fundamental role in the theory and practice of decision making.

Multi-agent versions represent games in extensive form, the fundamental tool of all modern economic analysis.

The early article of von Neumann on game theory claimed that decision trees, or extensive form games, could be reduced to games in normal or strategic form where each player has a single choice of strategy, specifying a complete plan for the whole game.

This claim could be called the "invariance hypothesis", requiring that solutions to the extensive form of a game be invariant to changes that preserve the associated strategic form.

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Nash Equilibrium

The later work by Nash concerned
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applied to games in strategic form.

In fact, Nash's equilibrium theory added a model
of how a game would be played
to the von Neumann and Morgenstern description
of the strategic form model of how it could be played,
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Of course one can use standard normal form concepts
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Dynamic Form Models

A second set of questions cannot be answered without going beyond this normal form setting. For example:

1. What should we expect credit markets and the banking system to contribute toward a desirable allocation of resources?
2. Could anything be done to improve the adaptability of our market system, and overcome the apparent fragility of financial resources?

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Extensive form decision trees

The invariance hypothesis has significant implications
when applied to an unrestricted domain
of finite single person decision trees, including subtrees,
with consequences belonging to a fixed domain
attached to each terminal node.
Indeed, this is the subject of previous work
on "consequentialist" normative decision theory.
By now, however, game theorists generally recognize
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This does not rule out the possibility,
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The existing literature completely overlooks
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A Third Timescale

1. Is there any reason to revise our expectations of how well financial markets and the banks can perform in the light of the "credit crunch" and other recent events that were apparently unforeseen, if not also unforeseeable?
2. How well should we expect our economic system to adapt to such events? Or to increasingly pessimistic forecasts of the effect of economic activity on climate change, as well as of the effect of climate change on economic activity?
3. How should economic models treat technical progress, or the more general concept of innovation set out almost 100 years ago in Schumpeter's early work, *The Theory of Economic Development*?

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New Questions

Questions of this third class have rarely been asked,
at least by economists and decision or game theorists,
let alone answered.

Answering them more fully, I argue, requires
going beyond either normal or extensive form models
into the largely unexplored area of "enlivened form" models
involving games whose rules change over time in ways that,
even if they could in principle be modelled in advance,
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Circe, by John William Waterhouse (1891)

Kirke Tells Odysseus of the Sirens

καὶ τότε δή μ' ἐπέεσσι προσηύδα πότνια Κίρκη:

"'ταῦτα μὲν οὕτω πάντα πεπείρανται, σὺ δ' ἄκουσον,
ὥς τοι ἐγὼν ἐρέω, μνήσει δέ σε καὶ θεὸς αὐτός.
Σειρῆνας μὲν πρῶτον ἀφίξεαι, αἵ ῥά τε πάντας
ἀνθρώπους θέλγουσιν, ὅτις σφεας εἰσαφίκηται.

Then queenly Circe spoke to me and said: 'All these things have thus found an end; but do thou hearken as I shall tell thee, and a god shall himself bring it to thy mind. To the Sirens first shalt thou come, who beguile all men whosoever comes to them.'

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'So far so good,' said she, when I had ended my story, 'and now pay attention to what I am about to tell you — heaven itself, indeed, will recall it to your recollection. First you will come to the Sirens who enchant all who come near them.

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Whoso in ignorance draws near to them and hears the Sirens' voice, he nevermore returns, that his wife and little children may stand at his side rejoicing, but the Sirens beguile him with their clear-toned song, as they sit in a meadow, and about them is a great heap of bones of mouldering men, and round the bones the skin is shrivelling.

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ἀνδρῶν πυθομένων, περὶ δὲ ῥινοὶ μινύθουσι.

Whoso in ignorance draws near to them and hears the Sirens' voice, he nevermore returns, that his wife and little children may stand at his side rejoicing, but the Sirens beguile him with their clear-toned song, as they sit in a meadow, and about them is a great heap of bones of mouldering men, and round the bones the skin is shrivelling.

If any one unwarily draws in too close and hears the singing of the Sirens, his wife and children will never welcome him home again, for they sit in a green field and warble him to death with the sweetness of their song. There is a great heap of dead men's bones lying all around, with the flesh still rotting off them.

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Getting Past the Sirens

ἀλλὰ παρεξελάαν, ἐπὶ δ' οὔατ' ἀλεῖψαι ἑταίρων
κηρὸν δεψήσας μελιηδέα, μή τις ἀκούσῃ τῶν ἄλλων:

But do thou row past them, and anoint the ears of thy comrades with sweet wax, which thou hast kneaded, lest any of the rest may hear.

Therefore pass these Sirens by, and stop your men's ears with wax that none of them may hear;

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Hearing the Sirens in Safety

ἀτὰρ αὐτὸς ἀκουέμεν αἴ κ' ἐθέλῃσθα,
δησάντων σ' ἐν νηὶ θοῇ χεῖράς τε πόδας τε
ὀρθὸν ἐν ἱστοπέδῃ, ἐκ δ' αὐτοῦ πείρατ' ἀνήφθω,
ὄφρα κε τερπόμενος ὄπ' ἀκούσῃς Σειρήνοιιν.
εἰ δέ κε λίσσηαι ἑτάρους λῦσαί τε κελεύῃς,
οἱ δέ σ' ἔτι πλεόνεσσι τότ' ἐν δεσμοῖσι διδέντων.
αὐτὰρ ἐπὴν δὴ τάς γε παρὲξ ἐλάσωσιν ἑταῖροι,
ἔνθα τοι οὐκέτ' ἔπειτα διηνεκέως ἀγορεύσω,

But if thou thyself hast a will to listen, let them bind thee in the swift ship hand and foot upright in the step of the mast, and let the ropes be made fast at the ends to the mast itself, that with delight thou mayest listen to the voice of the two Sirens. And if thou shalt implore and bid thy comrades to loose thee, then let them bind thee with yet more bonds.

but if you like you can listen yourself, for you may get the men to bind you as you stand upright on a cross-piece half way up the mast, and they must lash the rope's ends to the mast itself, that you may have the pleasure of listening. If you beg and pray the men to unloose you, then they must bind you faster.

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Painting by John William Waterhouse (1891)

The Potential Addict Example

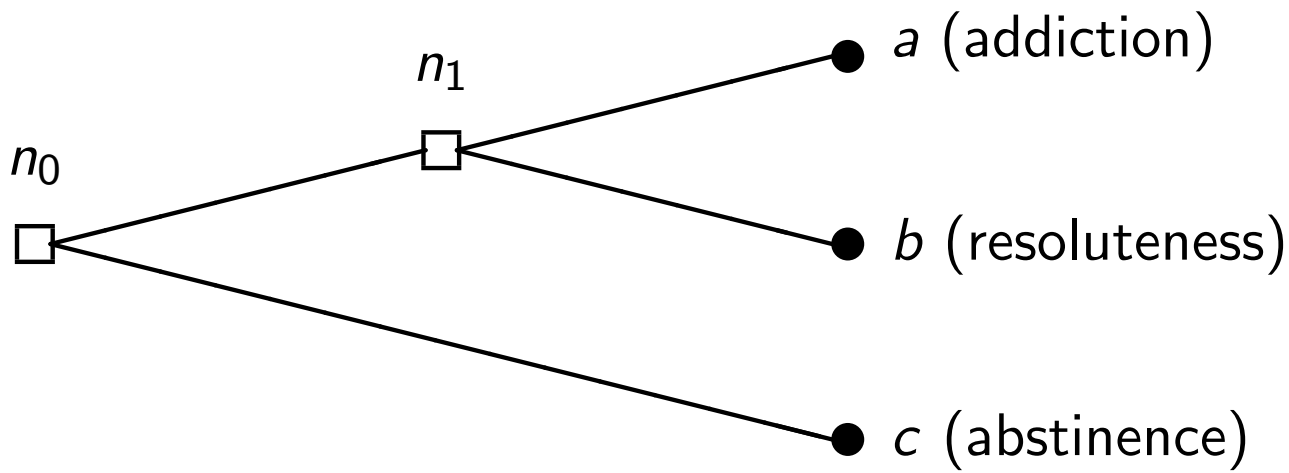

Figure: The potential addict's decision tree

Because the activity is addictive, typically one has $a \succ_1 b$ according to the (strict) preference relation $\succ_1$ that applies at node n_1.

But at node n_0, option b seems best and a seems worst, so $b \succ_0 c \succ_0 a$.

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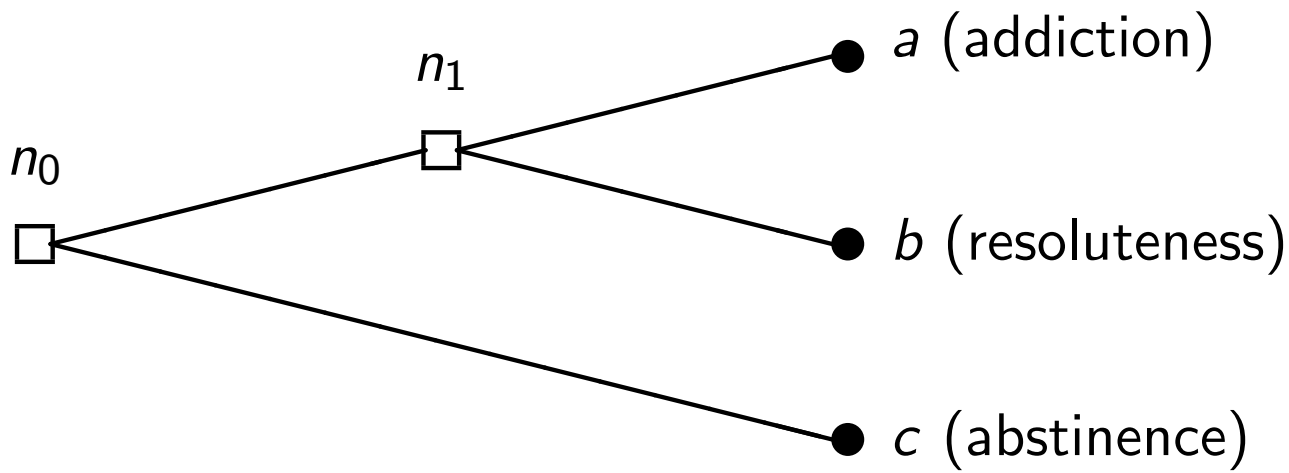

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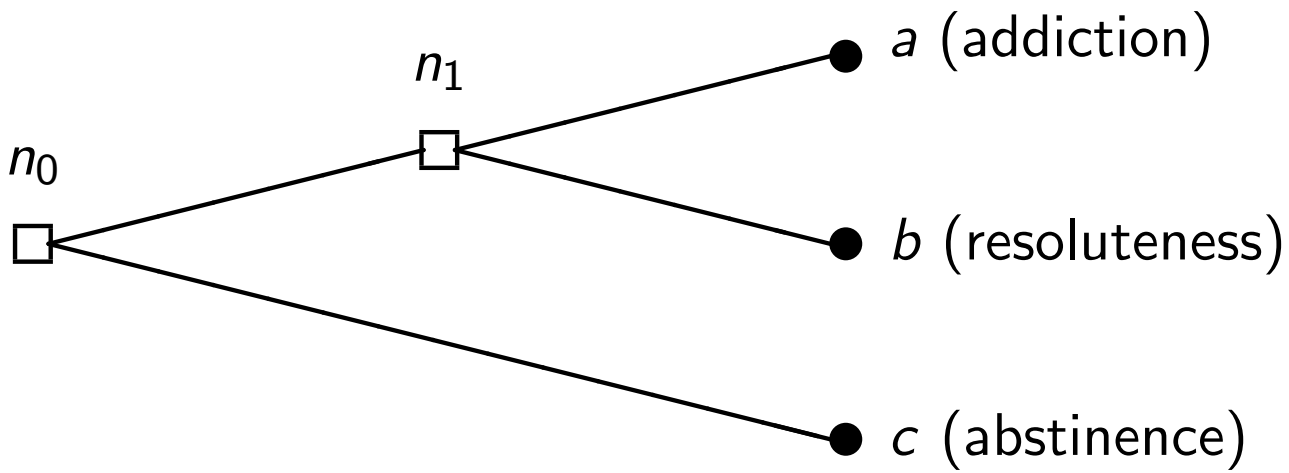
Naive versus Sophisticated Behaviour



This contrast between the preferences $b \succ_0 c \succ_0 a$ and $a \succ_1 b$ is an essential inconsistency.

Naive behaviour — at n_0 choose n_1,
planning to go to b later,
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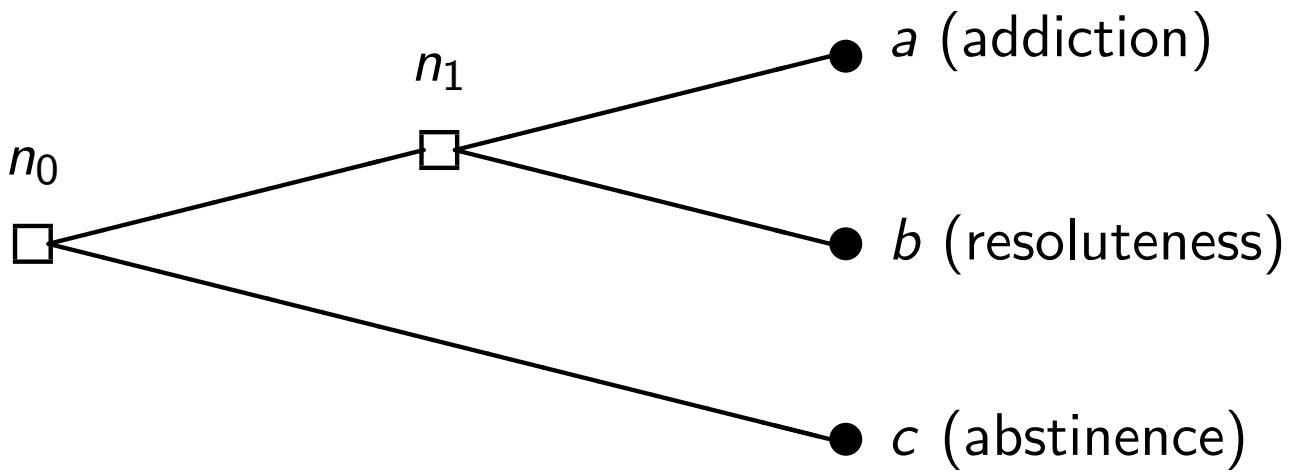

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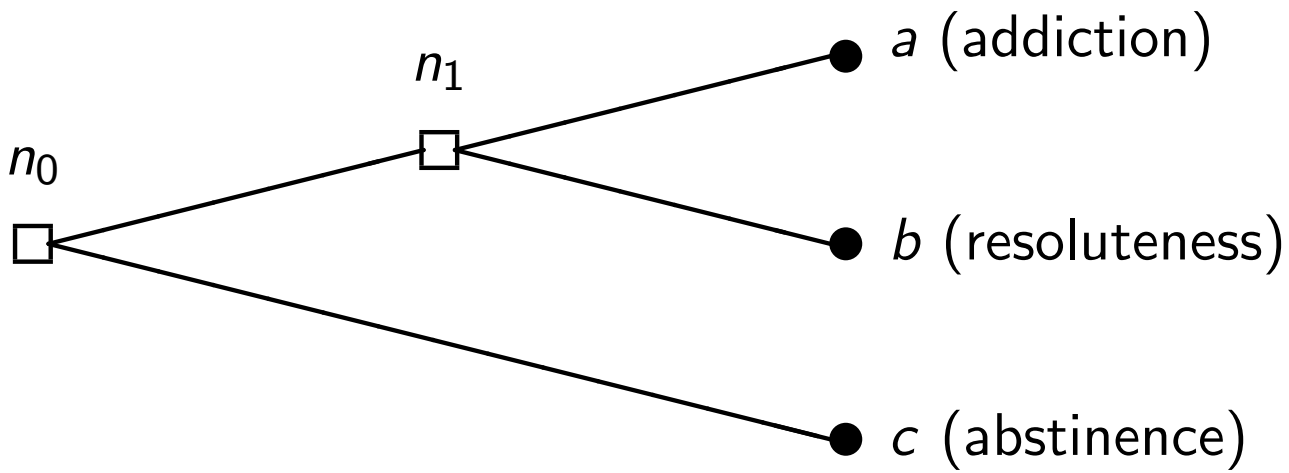

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Behaviour with Pre-Commitment

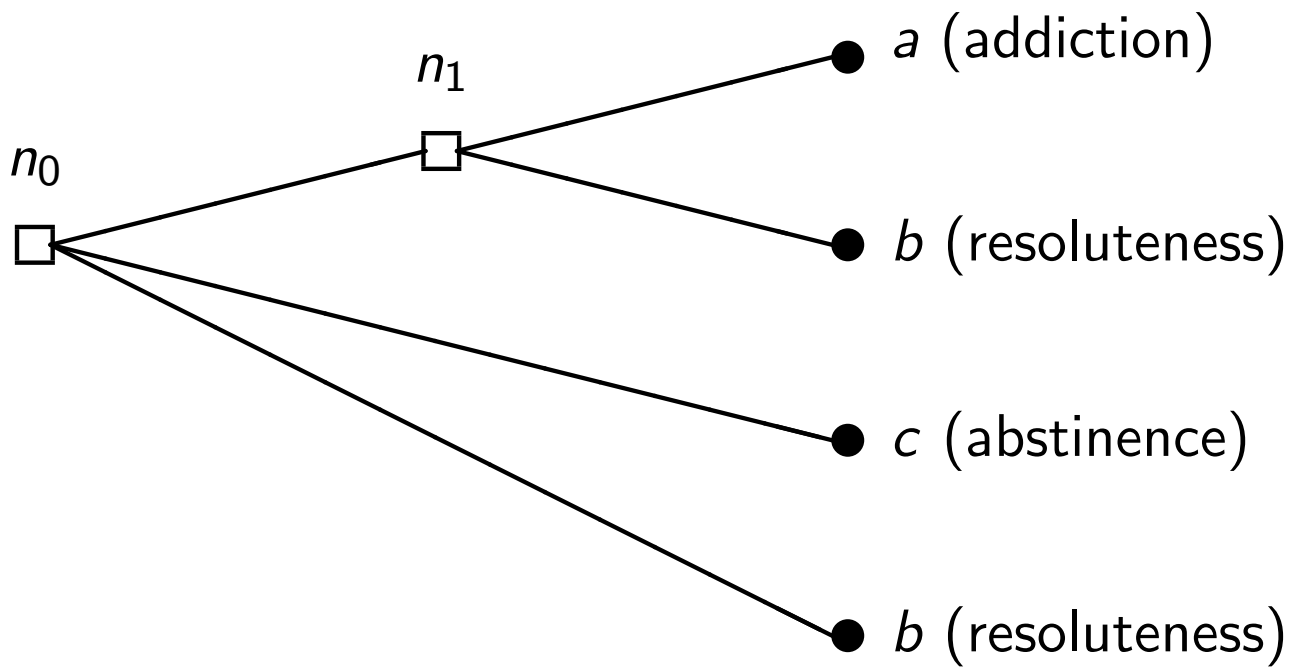

Figure: Tree with a pre-commitment device

Pre-commitment:
choose the best outcome b directly, without going to n_1.

Classical Example: Odysseus and the Sirens

Odysseus was advised to stop up his crew's ears with wax,
to have himself bound to the mast of his ship,
and to tell the crew to bind him even more firmly
should he try to escape and turn the ship around.

Outline

1. Introduction
2. A Classical Example
3. ZNK Paradigm
   - Zermelo, von Neumann, and Kolmogorov
   - Meta Stochastic Processes
4. Enlivened Trees
5. Normative Theory and Descriptive Models
6. Conclusions

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that Chess, starting from any (legal) position,
has a definite result with best play:
either White can force a win; or Black can;
or the game should be drawn.
Zermelo's proof works even though exhaustive analysis
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Except in greatly simplified "endgame" positions
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Thus, there must be a fixed state space
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Limitations of the ZNK Paradigm

The standard Zermelo/von Neumann/Kolmogorov (ZNK) paradigm is too limited to serve as a realistic model of actual economic behaviour in complex settings.

It reduces decision trees and extensive games to their normal form, in which each agent or player makes a single once and for all choice of strategy.

This imposes unrealistic demands on real decision makers' modelling and planning abilities, and on their imagination of what events could occur.

In particular, for the games of Chess and Go, there is no possibility of either player "outwitting" the other, since both are assumed to make perfect moves all the time.

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Beyond ZNK

The ZNK paradigm also rules out unforeseen contingencies.

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Anything else, such as an earthquake
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No fixed model will do
— not even models as rich as von Neumann's (or Kuhn's),
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Extreme Arrogance

Ich habe jede Möglichkeit von vorn herein einkalkuliert.
I have taken into account every possibility from the beginning on.
Quotation in the Kölnische Stadtmuseum (Cologne City Museum),
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Meta Stochastic Processes

Could one define a "meta" stochastic process
on the space of all possible models
involving a sequence of stochastic processes
in which the state space is continually being enriched
unpredictably?
The stochastic process model is based on
Kolmogorov's extension theorem in probability.
This theorem demonstrates that
any "consistent" family of probability laws
on finite Cartesian subproducts
of an arbitrary collection of measurable spaces
can be extended to a probability law
on the whole Cartesian product.

Meta Stochastic Processes: A Challenge

The theorem, however, depends on a significant assumption:
for example, that the probability distribution
on each single measurable space is tight
— i.e., the probability of any measurable set must equal
the supremum of the probabilities of all its compact subsets.
(See Aliprantis and Border (1999), which also includes a significant generalization due to Neveu (1965).)
It seems difficult to find a suitable topology
on the class of all potentially relevant sequences
of stochastic process models
which allows an interesting probability measure to exist.

Alephs . . .

Jorge Luis Borges *El Aleph* (1945),
begins with quotations from *Hamlet* II, 2

> *O God! I could be bounded in a nutshell,*
> *and count myself a King of infinite space . . .*

and from Thomas Hobbes' *Leviathan*, IV, 46

> *But they will teach us that Eternity is the Standing still of the Present Time, a* $Nunc\text{-}stans$ *(as the schools call it); which neither they, nor any else understand, no more than they would a* $Hic\text{-}stans$ *for an Infinite greatness of Place.*

Borges' Aleph I

Translation by Norman Thomas Di Giovanni
in collaboration with the author.

> *When I opened my eyes, I saw the Aleph . . . the only place on earth where all places are — seen from every angle, each standing clear, without any confusion or blending.*
>
> *The Aleph's diameter was probably little more than an inch, but all space was there, actual and undiminished. Each thing (a mirror's face, let us say) was infinite things, since I distinctly saw it from every angle of the universe.*

Borges' Aleph II

I saw the Aleph from every point and angle, and in the Aleph I saw the earth and in the earth the Aleph and in the Aleph the earth; I saw my own face and my own bowels; I saw your face; and I felt dizzy and wept, for my eyes had seen that secret and conjectured object whose name is common to all men but which no man has looked upon — the unimaginable universe.

I felt infinite wonder, infinite pity.

. . . for Cantor's $\mathcal{M}engenlehre$, [Aleph, or $\aleph$] is the symbol of transfinite numbers, of which any part is as great as the whole.

. . . and Universes

Indeed, is the class of all potentially relevant sequences of stochastic process models a "proper class", beyond the realm of set theory?
Even if it can be made a set,
or "von Neumann universe" with a hierarchy of sets,
can we assign it a meaningful σ-algebra of measurable sets and a meaningful probability measure?

Beyond Meta Stochastic Processes

Suppose one could define a meta stochastic process on the space of all potentially relevant stochastic processes. Even so, it would surely be vastly too complex for practical use.
More from Borges' *Aleph*:

> *Out on the street, going down the stairways inside Constitution Station, riding the subway, every one of the faces seemed familiar to me. I was afraid that not a single thing on earth would ever again surprise me; I was afraid I would never again be free of all I had seen. Happily, after a few sleepless nights, I was visited once more by oblivion.*

Yasunari Kawabata (1954) *The Master of Go*

The Japanese game of Go
is a two-person zero sum game of perfect information.
Orthodox game theory predicts it should be played perfectly, and so perfectly predictably. But, at the end of chapter 37:

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Outline

1. Introduction
2. A Classical Example
3. ZNK Paradigm
4. Enlivened Trees
   - Dead and Live Trees
   - Odyssey II
5. Normative Theory and Descriptive Models
6. Conclusions

A Dead Tree, I

Figure: Our Stanford neighbours' tree, commissioned by Peter and Juthica Stangl, carved by a local tree sculptress. View from our garden. Removed (2008) by new owners.

A Dead Tree, II

Figure: Sculpted tree in public park, Marple, Greater Manchester

One Enlivened "Tree" (The Stanford Mascot)

An Awakening Tree

Figure: Our oldest magnolia tree, growing in Stanford land, planted *circa* 1972.

An Enlivened Tree

Figure: The same tree a few weeks later, trying to produce new trees . . .

A Menacingly Enlivened Tree, Even After Pruning

Figure: Accompanying "A system overwhelmed by innovation" Clive Crook's article in the *Financial Times*, 13th October 2008

Kirke Tells Odysseus of the Sirens

First you will come to the Sirens who enchant all who come near them.

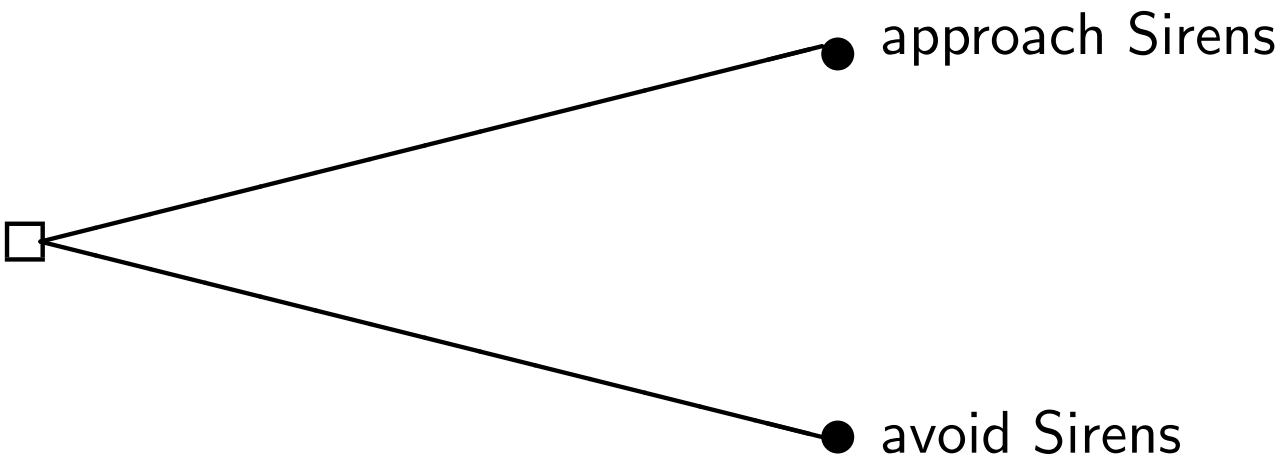

Figure: Naive sailor's perceived decision tree

Sirens Are Dangerous

. . . the Sirens . . . sit in a green field and warble him to death with the sweetness of their song. There is a great heap of dead men's bones lying all around, with the flesh still rotting off them.

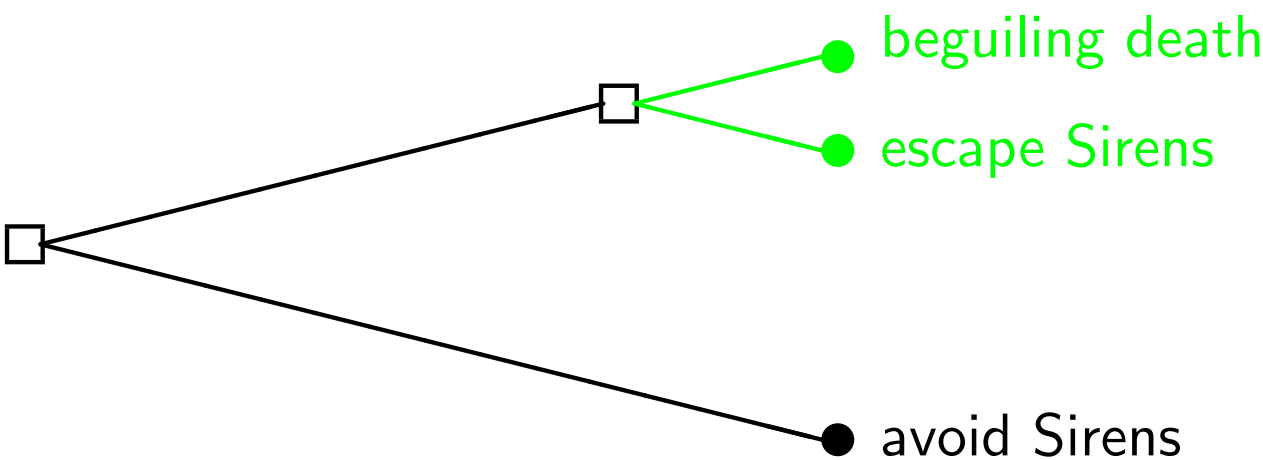

Figure: Naive sailor's actual decision tree

Getting Past the Sirens

Therefore pass these Sirens by, and stop your men's ears with wax that none of them may hear;

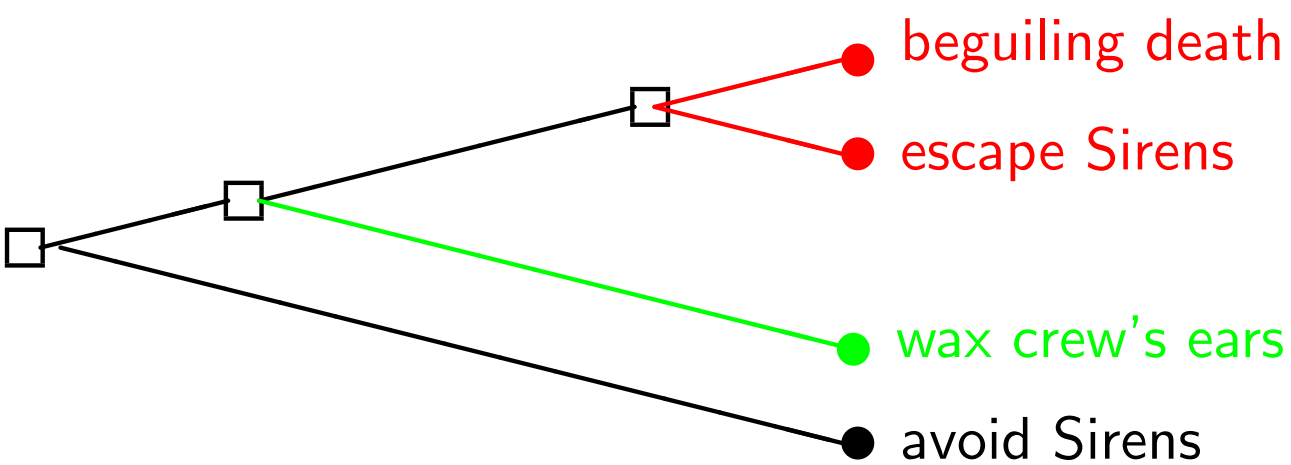

Figure: Sailor's decision tree with precommitment

Hearing the Sirens in Safety

but if you like you can listen yourself, for you may get the men to bind you as you stand upright on a cross-piece half way up the mast, and they must lash the rope's ends to the mast itself, that you may have the pleasure of listening.

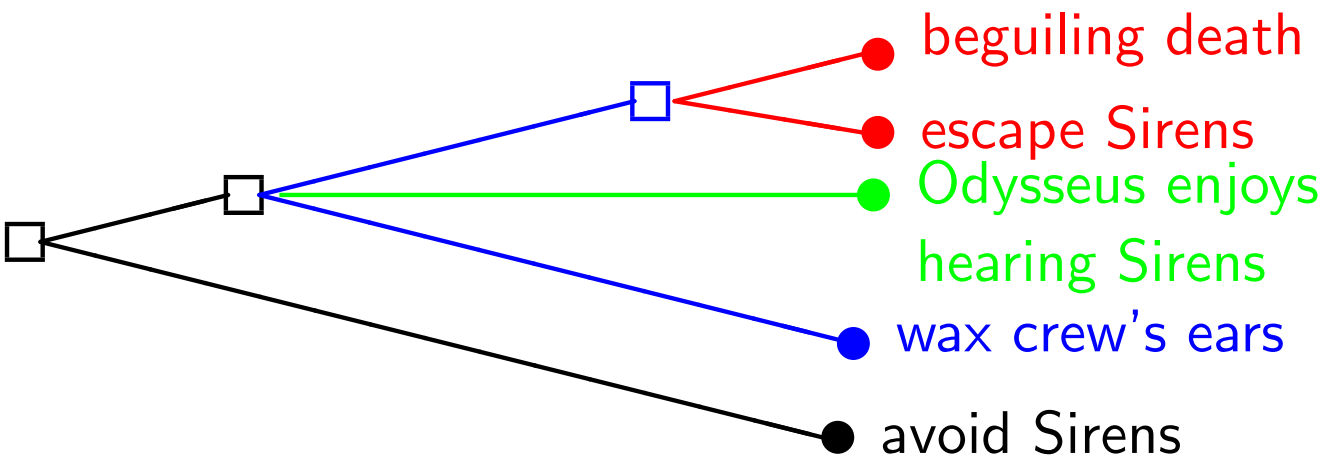

Figure: Odysseus' decision tree

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   - Normative Theory
   - Rational Boundedness
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An Initial Simple Tree

Let Y be a fixed consequence domain.
Assume an agent who wants to maximize the expected value of a von Neumann–Morgenstern utility function $v : Y \to \mathbb{R}$.
Consider a (dead) decision tree

- with an initial (decision) node n_0,
- where the agent chooses one $i \in I$
- that leads to a succeeding chance node n_1^i in $N_{+1}(n_0) = \{n_1^i \mid i \in I\}$,
- where chance determines each possible $j \in J_i$ with a known transition probability $\pi(j|i)$
- that leads to a succeeding terminal node n_2^{ij} in the set $N_{+1}(n_1^i) = \{n_2^{ij} \mid j \in J_i\}$,
- resulting in the final consequence probability distribution $\gamma(n_2^{ij}) \in \Delta(Y)$.

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Initial Evaluation

Given the known consequence probability distribution $\gamma(n_2^{ij}) \in \Delta(Y)$,
the expected utility of reaching
any terminal node n_2^{ij} $(i \in I,\ j \in J_i)$ is

$$w_2(n_2^{ij}) = \sum_{y \in Y} \gamma(n_2^{ij})(y)v(y) = \mathbb{E}_{\gamma(n_2^{ij})} v(y)$$

Working backwards, the expected utility of reaching
any preceding chance node $n_1^i \in N_{+1}(n_0)$
is $w_1(n_1^i) = \sum_{j \in J_i} \pi(j|i) w_2(n_2^{ij})$.
An optimal decision maximizes $w_1(n_1^i)$ with respect to $i \in I$,
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An Evolving Simple Tree

The agent can hardly make an unmodelled decision, so we assume that $N_{+1}(n_0)$ remains fixed.

We also assume that any enrichment of the tree is assumed to take place after a chosen decision node $n_1^i \in N_{+1}(n_0)$ has been reached. For each $i \in I$, contingent on reaching n_1^i, we postulate an expanded set of succeeding terminal nodes $N_{+1}^+(n_1^i) = \{n_2^{ij} \mid j \in J_i^+\}$ (where $J_i^+ \supseteq J_i$). We also have:

- revised transition probabilities $\pi^+(j|i)$;
- revised consequence probabilities $\gamma^+(n_2^{ij}) \in \Delta(Y)$.

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Revised Evaluation

Of course, the revised expected utility of reaching any terminal node n_2^{ij} $(i \in I, j \in J_i^+)$ is

$$w_2^+(n_2^{ij}) = \sum_{y \in Y} \gamma^+(n_2^{ij})(y)v(y) = \mathbb{E}_{\gamma^+(n_2^{ij})} v(y)$$

The revised expected utility of reaching any chance node $n_1^i \in N_{+1}(n_0)$ is therefore $w_1^+(n_1^i) = \sum_{j \in J_i^+} \pi^+(j|i) w_2^+(n_2^{ij})$.

Retrospective Evaluation

Ex post, the appropriate decision at initial node n_0
would have been to maximize $w_1^+(n_1^i)$ with respect to $i \in I$.
Ex ante, however, only the details of the original model
can be used, by definition.
What the agent can still do, however,
is to recognize that the original evaluation function $w_1(n_1^i)$
may be revised to an unknown evaluation function $w_1^+(n_1^i)$
that ranges over a function space of possible evaluation functions.
In other words, following some interesting ideas
like those in Hansen and Sargent's book *Robustness*,
we can apply a robust decision analysis.

Regret

But with a specific form of misspecification:
namely, in the evaluation function $w_1(n_1^i)$.
Most relevant is the extent of the expected *ex post* regret

$$r^+ = \max_{i \in I} w_1^+(n_1^i) - w_1^+(n_1^*)$$

from having the wrong model *ex ante*
and choosing $n_1^* \in \arg\max_{i \in I} \{w_1(n_1^i)\}$.

Goethe

Mephistopheles in Goethe's *Faust*, Part I:
"Grau, teurer Freund, ist alle Theorie
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with the (obviously incomplete) translation:
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Replace Bounded Rationality . . .

When it comes to describing actual behaviour,
many decision models in social science
do feature "bounded rationality", or "procedural rationality".
Usually, however, this involves
what Herbert Simon (1955, 1957) called "satisficing"
— making a decision that seems good enough
rather than optimal.

. . . with Rationally Bounded Models

The normative framework we propose, however,
suggests that satisficing behaviour should occur,
not within a given decision model,
but in how much detail to include within the model.
One possibility is that, given a rich feasible set of options,
at the first initial stage a decision maker is likely to select,
quite possibly at random,
only a small subset for later serious consideration.
Some options are much more likely to be selected than others,
and the selected options may well be correlated random variables.
Then, at a later second stage,
an optimal element of that subset is selected.
Cf. Manzini and Mariotti's "rational shortlist" method.

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Welfare Significance

Provided an agent uses some kind of enlivened bounded model, perhaps after suitable advice, in principle one can infer revealed preferences concerning the modelled consequences of the decisions that the agent seriously contemplated.
In general this will not be possible if preferences are changing.
So is this the last chance for revealed preference?

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What are the ethical implications of enlivened bounded models?
What about psychological or neurological biases?

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