Revising Conditional Preferences

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1 Project

Develop theory of preference revision analogous to Bayesian theory of belief revision in that:

- it is formal and subjectivist in character,
- it plays both a normative and descriptive role and
- its imperatives are hypothetical.

Belief Revision	*Preference/Desire Revision*
Classical Conditioning	Desirability Conditioning
Jeffrey Conditioning	Generalised Conditioning
Adams Conditioning	?

2 Bolker-Jeffrey Framework

Prospects

$\Omega = \{A, B, ...\}$ is a Boolean algebra containing the necessary prospect, T, but with the impossible prospect, F, removed.

$X \vee Y$ = the join/disjunct of any two prospects

XY = the meet/conjunct of any two prospects

$\neg X$ = the complement/negation of X.

Preferences

Unconditional: $\succeq$ (prior) and $\succeq^*$(posterior).

Conditional: $\succeq_A$(prior) and $\succeq_A^*$(posterior).

Assumed to be transitive, but not necessarily complete.

2.1 States of Mind

The state of mind of a maximally opinionated agent is represented by a pair of real-valued functions $\langle p, v\rangle$ on Ω where:

- p is a probability measure of her degrees of belief,

- v is a desirability measure of her degrees of preference satisfying:

1. Normality: $v(T) = 0$

2. Averaging: If $XY = F$:

$$v(X \vee Y) = \frac{v(X).p(X) + v(Y).p(Y)}{p(X) + p(Y)}$$

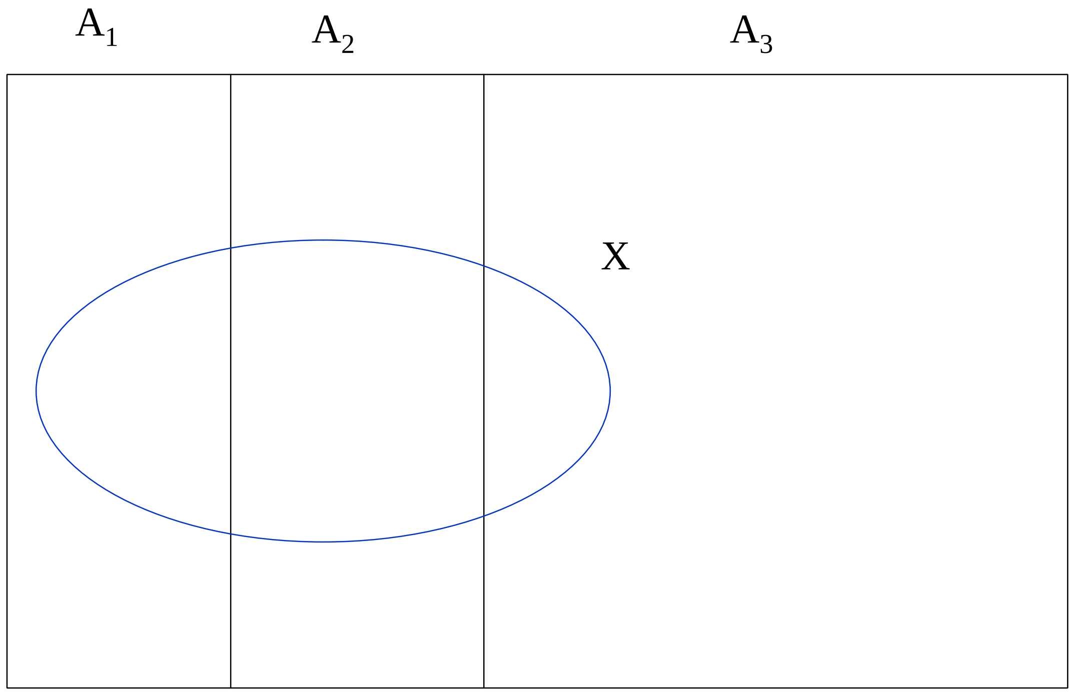

Figure 1: Weighted averaging

$$
\begin{aligned}
p(X) &= \sum p(X|A_i).p(A_i) \\
v(X) &= \sum (v(X|A_i) + v(A_i)).p(A_i|X)
\end{aligned}
$$

3 Changes of Mind

A state of mind $S = \{\langle p_i, v_i \rangle\}$ *explains* or *rationalises* an agent's preferences whenever, for all X, Y in the domain of $\succeq$, it is the case that, for all $\langle p_i, v_i \rangle \in S$:

$$X \succeq Y \Rightarrow v_i(X) \geq v_i(Y)$$

Changes in preference are explained or rationalised by changes in the state of mind of the agent; either in her beliefs or her desires or both.

In ‘perturbation-propagation’ models the change in an agent’s state of mind is viewed as a two-step process:

Stage 1: The agent changes her attitude(s) to a particular prospect or, more generally, some set of prospects.

Stage 2: She adjusts her attitudes to all other possibilities in order to restore consistency.

Formally a theory of attitude revision is a mapping from prior states of mind, $< p, v >$, to posterior states of mind, $< p^*, v^* >$, as a function of the changes induced by stage 1.

Stage 1 changes are summarised by a constraint on the agent's posterior state of mind and not themselves modelled.

The family of conditioning models are characterised by a single notion of minimal change applied to different constraints.

4 Generalised Conditioning

G**eneralised conditioning** on a partition $\{A_i\}$ applies when the effect of experience is exhaustively described by a redistribution $\alpha\beta$ of probability and desirability across the A_i:

$$
\begin{aligned}
p^*(X) &= \sum_i p(X|A_i).p^*(A_i) \\
v^*(X) &= \sum_i [v(X|A_i) + v^*(A_i)].p^*(A_i|X)
\end{aligned}
$$

Note that classical Bayesian conditioning is the special case in which probability is entirely loaded on one of the A_i.

Under what conditions is $\langle p^*, v^* \rangle$ the uniquely rational state of mind to arrive at after revision when the effect of experience is correctly described by $\alpha\beta$?

Rigidity of Conditional Preference wrt $\{A_i\}$:

$$X \succeq_{A_i} Y \Leftrightarrow X \succeq^*_{A_i} Y$$

Theorem 1 *If the pairs* $\langle p, v \rangle$ *and* $\langle p^*, v^* \rangle$ *satisfy the Rigidity condition with respect to* $\{A_i\}$*, then* $\langle p^*, v^* \rangle$ *is obtained from* $\langle p, v \rangle$ *by generalised conditioning on* $\{A_i\}$ *(and vice versa).*

Theorem 2 *Assume that* Ω *is countable. Let* $\langle p, v\rangle$ *and* $\langle p^*, v^*\rangle$ *be respectively an agent's prior and posterior states of mind. Then there exists some partition of* Ω *such that* $\langle p^*, v^*\rangle$ *is obtained from* $\langle p, v\rangle$ *by generalised conditioning on this partition.*

Theory says nothing about how redistribution of probability and desirabilitty across the relevant partition is achieved. But the information delivered by experience may be much less rich.

5 Revising Conditional Preferences

Conditional attitudes can change in response to experience:

1. *Experimentation*: Trying out different wines in combination with say Roquefort can give new conditional preferences for wines given blue cheese servings.

2. *Testimony*: The guide book advises that in a particular hotel, the rooms overlooking the park are much smaller than those overlooking the road. My prior conditional preference for a room on the park over one over the street, conditional on choosing that hotel to stay in, is reversed as a result.

In salient cases these changes in conditional attitudes, given some prospect, have no direct impact on unconditional attitudes to the prospect.

Condition 3 *Invariance wrt* $\{A_i\}$*:*

$$v^*(A_i) = v(A_i), p^*(A_i) = p(A_i)$$

Let $\{A_i\}$ and $\{B_j\}$ be two epistemically compatible partitions of the set of prospects. Suppose that as a result of experience the agent adopts new conditional probabilities and desirabilities, $p^*(B_j|A_i)$ and $v^*(B_j|A_i)$, for the B_i given the A_i. Let:

$$k_{ij} = v^*(B_j|A_i) - v(B_j|A_i)$$

Then assuming Rigidity of Conditional Preference wrt $\{A_iB_j\}$ and Invariance wrt $\{A_i\}$:

$$p^*(X) = \sum_{ij} p(XA_iB_j).\frac{p^*(B_j|A_i)}{p(B_j|A_i)}$$

$$v^*(X) = \sum_{ij}(v(XA_iB_j) + k_{ij}).p^*(A_iB_j|X)$$

Case 1: Changing conditional probability Conditional probabilities for the B_i given A changes, but not the relative conditional desirabilities. Then:

$$v^*(A) = \sum_j v(AB_j).p^*(B_j|A) + k$$

where k is just a renormalisation term.

Example: I learn that drinking red wine, but not white, reduces the chances of a heart attack. As a result I come to prefer drinking red wine to white

General law: A rise/fall in the conditional probability, given A, of some prospect B, should, *ceteris paribus*, lead to a rise/fall in the desirability of A iff $AB > A\neg B$.

Case 2: Changing conditional desirability Conditional desirabilities for the B_i given A changes, but not the conditional probabilities. Then:

$$v^*(A) = v(A) + \sum_j k_j . p^*(B_j|A)$$

Example: Discovering that strawberries taste even better if eaten with cream, may lead one to value cream more highly and to purchase it more often.

General law: A rise/fall in the conditional desirability, given A, of some epistemically possible prospect B, should, *ceteris paribus*, lead to a rise/fall in the desirability of B.

Case 3: Independence Suppose that the B_i are probabilistically independent of A and A'. Then:

$$v^*(A) = \sum_j v^*(A|B_j).p(B_j)$$

Example: If you are told that there is lots to do at a seaside town if it rains, your conditional desirability for visiting the town, given rain, rises. Hence so too does the desirability of visiting.

General law: A rise/fall in the conditional desirability, given A, of some epistemically possible prospect B, should, *ceteris paribus*, lead to a rise/fall in the desirability of A.

6 A Problem for Decision Theory

	Music	*No Music*
Go to Party	Dance the night away	Boring conversation
Don't Go	Quiet night at home	Quiet night at home

Alice and Bob are soulmates. Alice must decide whether (G) to go the party, a prospect whose desirability depends for her on whether there will be (M) music or not. Alice enjoys dancing, but especially (B) with Bob. Assume that the probability of music is independent of whether Alice decides to go. Then, according to both causal and evidential decision theory:

$$\begin{aligned} v(G) &= v(G|M).p(M) + v(G|\neg M).p(\neg M) \\ v(G|M) &= v(GB|M).p(B|GM) + v(G\neg B|M).p(\neg B|GM) \end{aligned}$$

Alice enjoys dancing, but especially (B) with Bob. So:

$$v(GB|M) > v(G\neg B|M)$$

	Music	*No Music*
Go to Party	Dance the night away	Boring conversation
Don't Go	Quiet night at home	Quiet night at home

Alice doesn't expect Bob to be at the party and so she should be disinclined to go. However, since they are soulmates and think the same way, $p(B|G)$ is close to 1. Hence:

$$
\begin{aligned}
v(G|M) &\approx v(GB|M) \\
v(G|\neg M) &\approx v(GB|\neg M)
\end{aligned}
$$

So if the probability of music is high enough the desirability of going to the party will exceed that is not.