

Master Class in Topological Social Choice

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There are two objectives. The first one is to give a genuinely elementary introduction to the formal tools and techniques used in this area of research for which no prerequisites are assumed. This will be done in the first two classes. The second objective is to present all of the major results together with a critical discussion. This will be done in the Choice Group Seminar.

Class 1: Topological methods in social choice I

Linear preferences and circles; simple mathematics of circles; social welfare functions; continuity; loops in circles; degree and homotopy; and the fundamental equation of topological social choice theory.

Class 2: Topological methods in social choice II

Contractibility; holes in sets; Fundamental Groups of circles and Cartesian products of circles; covering of a set; nerve of a cover; topological formulation of Arrow's theorem (time permitting).

Class 3: Choice Group Seminar Talk: Topological Social Choice Theory Major results; gaps and problems in the literature; discussion and conclusions.

Literature

No prior reading will be assumed. Indeed, the presentations should lower the barriers to entry into topological social choice theory. There are only two papers at this level of exposition in the literature. They are:

N. Baigent (2010): "Topological Theories of Social Choice" Forthcoming in *"Handbook of Social Choice and Welfare: volume 2"*, eds. Arrow, K.J., A.K. Sen and K. Suzumura, K.; published by Elsevier. (The master class will follow this survey quite closely.)

Heal, G.M. (1997): "Social Choice and Resource Allocation: a topological perspective", in G. Heal (ed), *Topological social choice*, Springer, Heidelberg, 87-97. (This book is also available as a special issue of *Social Choice and Welfare* (1997)).

There are three other surveys, all more or less challenging to those without any prior knowledge of algebraic topology. They are:

Mehta, P. (1997): Topological methods in social choice: an overview, in G. Heal (ed), *Topological social choice*, Springer, Heidelberg, 87-97. (This is cryptic, but is accessible to those who are familiar with homotopy and degree theory.)

Lauwers, L. (2000): Topological social choice, *Mathematical Social Sciences*, 40(1):1-39. (This is the most comprehensive and detailed survey, but requires a knowledge of homotopy and homology theory. It also contains a comprehensive bibliography.)

Lauwers, L. (2009): The topological approach to the aggregation of preferences, *Social Choice and Welfare*, 33:449-476. (This is slightly gentler than the previous survey and is more focused on major results. Parts are accessible with moderate background, but some parts are challenging.)

Not all mathematics books on topology cover the algebraic topology used in topological social choice theory. In the mathematics literature, look in subject indexes for homotopy, fundamental groups and, if you are more ambitious, homology theory. My own favorites are:

Armstrong, M.A. (1983): *Basic topology*, Springer-Verlag, New York. (This gives a particularly nice introduction to homology theory in chapter 5 and homology theory is covered in chapter 8.)

Henle, H. (1979): *A Combinatorial Introduction to Topology*, Dover edition, 1994. (This is the gentlest introduction to homology theory of which I am aware.)

Mauder, C.R.F. (1996): *Algebraic topology*, Dover Publications, New York. (This is a standard reference, but considerably more demanding than the previous two books.)

The seminal results in topological social choice theory were all obtained by Chichilnisky in the following papers:

Chichilnisky, G. (1979): On fixed point theorems and social choice paradoxes, *Economic Letters*, 3:347-351. (This requires a knowledge of Brower's fixed point theorem and retractions.)

Chichilnisky, G. (1980): Social choice and the topology of spaces of preferences, *Advances in Mathematics*, 37(2):165-176. (This is challenging.)

Chichilnisky, G. (1982): Social aggregation rules and continuity, *Quarterly Journal of Economics*, 97:337-352. (Parts of this are quite accessible.)

Chichilnisky, G. (1982a): The topological equivalence of the Pareto condition and the existence of a dictator, *Journal of Mathematical Economics*, 9:223-233.

Chichilnisky, G., (1982b): Structural instability of decisive majority rules. *Journal of Mathematical Economics*, 9:207-221.

Chichilnisky, G. (1983): Social choice and game theory: recent results with a topological approach, chapter 6 in Pattanaik, P.K. and M. Salles (eds), *Social choice and welfare*, North-Holland, Amsterdam, pp103-120.

Chichilnisky, G. (1993): On strategic control, *Quarterly Journal of Economics*, 37:165-176. (Parts of this are quite accessible.)

Chichilnisky, G. and G.M. Heal (1983): A necessary and sufficient condition for the resolution of the social choice paradox, *Journal of Economic Theory*, 31:68-87. (Parts of this are quite accessible.)

The topological proof of Arrow's theorem is given in:

Baryshnikov, Y.M. (1997): Topological and discrete social choice: in search of a theory, in G. Heal (ed), *Topological social choice*, Springer, Heidelberg, 53-63. (While parts of this are challenging, the section introducing the concept of the nerve are far more accessible than in Baryshnikov's earlier paper in *Advances in Mathematics* (1993)).