Derivatives and the mitigation of systemic risk:
a qualitative analysis

Hykel Hosni

CPNSS - London School of Economics

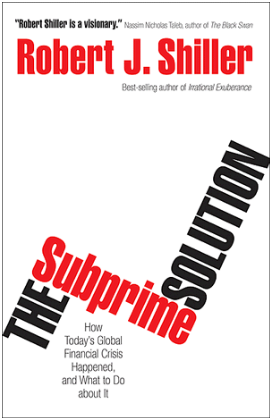
8 May 2014

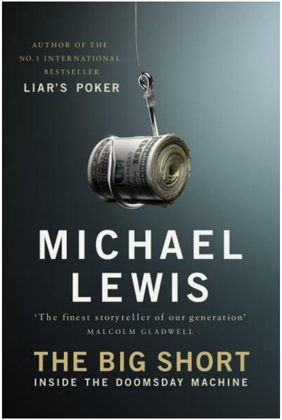
Weapons of financial mass destruction

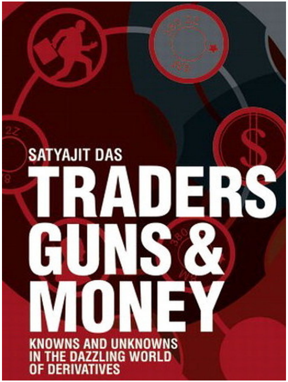
I view derivatives as time bombs, both for the parties that deal in them and the economic system.(. . .) In my view, derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.

Warren Buffett, *Berkshire Hathaway Annual Report* 2002

Prophets and commentators



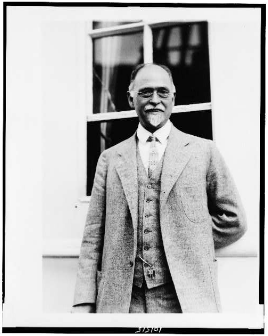




The Economist published a Post-crisis reading list on Apr 29th 2010
http://www.economist.com/node/15810590

But it all started very differently

The evils of speculation are particularly acute when, as generally happens with the investing public, the forecasts are not made independently. A chief cause of crises, panics, runs on banks, etc., is that risks are not independently reckoned, but are a mere matter of imitation. [...] Where, on the other hand, speculation is based on independent knowledge, its utility is enormous. It operates both to reduce risk by utilizing the special knowledge of speculators, and also to shift risk from those who lack this knowledge to those who possess it. [...] Risk is one of the direst economic evils, and all of the devices which aid in overcoming it—whether increased guarantees, safeguards, foresight, insurance or legitimate speculation—represent a great boon to humanity.



Irving Fisher, *The nature of capital and income*. Macmillan, 1906 (296-300)

Outline

1. Some (informal and incomplete) background
2. Can we design socially harmless derivatives?
3. "Encounters with philosophy"

Outline

1. Some (informal and incomplete) background
2. Can we design socially harmless derivatives?
3. "Encounters with philosophy"

Derivatives

Financial instruments (a.k.a. *securities*) whose monetary value depends on another asset.

EXAMPLE

A *call option* is the right –but not the obligation– to buy a stock share at time t for a price p (a.k.a. *strike price*) which was agreed at time $s < t$.

In general the value of a security at time t depends on a well-specified *state of the world* $\omega \in \Omega$ and can be written as $V_t(\omega)$.

- define a probability space (Ω, P)
- $V_t(\omega)$ is then a random variable on this space
- $E[V_t]$ and $E_s[V_t]$ are then meaningful
- how to choose P in such a way as to give a price to securities?

Black-Scholes (1973)

No arbitrage + log of prices follow a Brownian motion + technical assumptions lead to the existence of a unique price for call options.

The principle of no-arbitrage

An arbitrage opportunity is the possibility to make a profit in a financial market without risk and without net investment of capital. The principle of no-arbitrage states that a mathematical model of a financial market should not allow for arbitrage possibilities

Risk and uncertainty

- No-arbitrage makes risk an intrinsic feature of financial (and economic) practice
- are we able to understand the consequences of this?

The seemingly improbable in finance

The seemingly improbable happens all the time in financial markets. [In 1997] the Dow Jones had fallen 7.7 % in one day (probability: one in 50 billion). In July 2003, the index recorded three steep falls within seven trading days (probability: one in four trillion). And on October 19, 1987 [...] index fell 29.2 %. The probability [...] based on the standard reckoning of financial theorists was less than one over 10^{50} – **odds so small they have no meaning**. *It is a number outside the scale of nature*

Hudson, R., & Mandelbrot, B. (2004). *The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward*. Basic Books.

2007 Credit crunch

David Viniar [CFO at Goldman Sachs] announced that flagship GEO hedge fund had lost 27% since the start of the year [which he explained by saying] "We were seeing things that were 25-standard deviation moves, several days in a row.(FT, 13/8/07)" One commentator wryly noted:

> *That Viniar. What a comic. According to Goldman's mathematical models, August, Year of Our Lord 2007, was a very special month. Things were happening that were only supposed to happen once in every 100,000 years.* **Either that [. . .] or Goldman's models were wrong** *(Bonner, 2007).*

Dowd, K., Cotter, J., Humphrey, C., & Woods, M. (2008). "How Unlucky Is 25-Sigma". *The Journal of Portfolio Management*, 34(4), 76–80.

"Risk" and "Uncertainty"

Common usage

Uncertainty arises because at time s the value that some commodity of interest x will take at time t is unknown. This makes transactions on x at s **risky**

There exists a true state of nature ω which is unknown at s and will be known ad t

Market-based definition (Knight 1921)

An economic decision is **risky** if there exists a complete insurance market for it and **uncertain** otherwise

The distinction is contingent on the properties of the reference markets.

Derivatives as risk-management devices

Commonly held views about market-based risk sharing

- An efficient allocation of the unavoidable risks generated by the business activity is vital for the economic system.
  - capital is a limited resource and so it must be efficiently allocated to cover the highest risk-adjusted productive investments
- Risk allocation performed by the financial system has rapidly evolved over the past few decades
- Market-based allocation of risk is primarily accomplished through the use of derivative products

Pros and cons

Great boon

- efficient risk management
  - diversification of credit-risk
  - belief in market efficiency
- price discovery
  - “wisdom of the crowd”
- increased liquidity in the markets and in the general economy

WoFMD

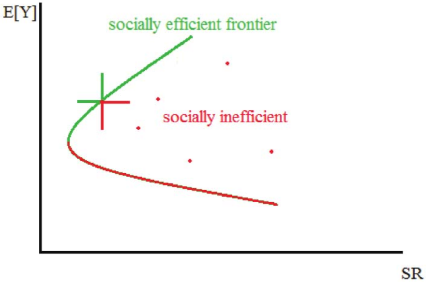
- transfer of risk to the (largely) unregulated market
- systemic risk
  - lack of transparency (total exposure and concentration)
  - contagion
- moral hazard
  - “too big to fail”
  - “betting somebody else’e money”

Acharya, V. V, Brenner, M., Engle, R., Lynch, A., & Richardson, M. (2009). “Derivatives - The Ultimate Financial Innovation” in Acharya, V and Richardson, M. *Restoring Financial Stability: How to Repair a Failed System*, Wiley.

Outline

Corsi, F., Hosni, H., & Marmi, S., *Risk allocation: The double face of financial derivatives*. http://eprints.lse.ac.uk/50971/

We define a criterion of efficiency for derivatives with goal of mitigating the downside of systemic risk.



The Model set up

We define a stylised economy characterised by

- REAL SECTOR a collection of risky investment projects;
- INDIVIDUALS $i = 1, ..., N$ endowed with initial capital (equity) C_i;
- PORTFOLIO RETURNS $I_i r_i$ with:
  - $r_i \sim N(\mu_i, \sigma_i^2)$ return per unit of capital of portfolio of agent i (ROI)
  - I_i amount invested in the portfolio of agent i.

Choice of individual leverage

For a given portfolio, each individual decides how much to leverage his positions on the basis of his own risk preferences.

Assume each agent has a CARA utility function with individual absolute risk aversion α_i. Hence each individual i maximizes,

$$\max_{\lambda_i} \quad \lambda_i \mu_i - \frac{\alpha_i}{2} \lambda_i^2 \sigma_i^2 \, ,$$

The first-order condition for this utility maximization is

$$\mu_i - \alpha_i \lambda_i \sigma_i^2 = 0$$

so that the OPTIMAL LEVERAGE for individual i, λ_i^* is

$$\lambda_i^* = \frac{I_i^*}{C_i} = \frac{\mu_i}{\alpha_i \sigma_i^2}$$

Imprecise Estimation

In practice, true values of μ_i and σ_i are not known. Let

- $\hat{\mu}_i$ the estimate made by agent i of the unknown expected return μ_i
- $\hat{\sigma}_i$ the estimates of the variance σ_i.

Hence, the desired leverage can be written as

$$\widehat{\lambda_i^*} = \frac{\hat{\mu}_i}{\alpha_i \hat{\sigma}_i^2} = \lambda_i^* \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i} = \lambda_i^* e_i \tag{1}$$

where

$$e_i - 1 = \frac{\sigma_i^2}{\hat{\sigma}_i^2} \frac{\hat{\mu}_i}{\mu_i} - 1$$

is the ESTIMATION ERROR of the agent i in evaluating the expected mean and risk of his position.

Individual default probability

Default

The event that loss on risky asset positions exceeds initial capital C_i. (i.e. risk capacity)

Agent's i probability of default is given by

$$p_i = P(l_i r_i < -C_i).$$

Distance to default

Since $r_i \sim N(\mu_i, \sigma_i^2)$ the return of investing I_i in such a portfolio is

$$I_i r_i \sim N(I_i \mu_i, I_i^2 \sigma_i^2)$$

Therefore the probability of default can be written as

$$P(I_i r_i < -C_i) = 1 - \Phi\left(\frac{C_i + I_i \mu_i}{I_i \sigma_i}\right) \simeq 1 - \Phi\left(\frac{C_i}{I_i \sigma_i}\right)$$

where Φ is the cumulative distribution of a Gaussian, i.e. as a decreasing function of the so called

Distance to Default

$$DD_i^* = \frac{C_i}{I_i \sigma_i} = (\lambda_i \cdot \sigma_i)^{-1}$$

i.e. the "# of standard deviations a company is away from its default threshold"

Distance to default with estimation errors

With no estimation errors, the portfolio variance per unit of capital, is $\lambda_i^2 \sigma_i^2$
Taking into account estimation errors on the optimal leverage:

$$V\left[\hat{\lambda}_i^* r_i\right] = V\left[\lambda_i^* e_i r_i\right] = \lambda_i^{*2}\left(E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i]\sigma_i^2\right). \qquad (2)$$

- $E[e_i] = 1$ and $V[e_i] = 0$ recovers σ with no estimation errors
- while the higher $E[e_i]$ and $V[e_i]$ the higher the variance

Therefore, the individual DD in presence of estimation error becomes:

$$DD_i = \left(\lambda_i \sqrt{E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i]\sigma_i^2}\right)^{-1}. \qquad (3)$$

Composition of individual DD_i's

- the degree of leverage λ_i,
- the true riskiness of the portfolio σ_i,
- statistical properties of the estimation error e_i

Systemic Risk

Informal definition (Brownlees and Engle (2012))

The sum of the expected capital shortage over the whole population of financial institutions i.e. the total amount of capital that the government would have to provide in order to bailout the financial system in case of distress.

In our framework, Systemic Risk (SR) is defined as

$$SR \equiv \sum_{i=1}^{N} ES_i$$

where ES_i is the expected (capital) shortfall defined by

$$ES_i = -E\left[I_i r_i | I_i r_i < -C_i\right]$$

i.e. the expected loss given the default of the financial institution i

Define the *aggregated return* for the whole economy as

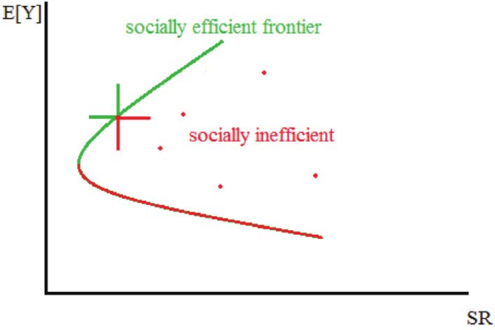
$$Y = \sum_{i=1}^{N} I_i r_i$$

Say that an *allocation is* **socially efficient** if, minimizes SR for any level of $E[Y]$, i.e.

$$\min SR \quad \text{subject to} \quad E[Y] = E[Y_0]$$

Hence, the socially efficient frontier will be a curve in the plane SR–$E[Y]$ which *maximizes the "Aggregate Sharpe ratio"* $E[Y]/SR$ for any level of $E[Y]$.

Socially harmless derivatives



Harmless derivatives allow us to move from an interior and inefficient point in the aggregate SR–$E[Y]$ plane to allocations with a greater "Aggregate Sharpe ratio" $E[Y]/SR$.

Under normality the expected shortfall, corresponds the first moment of a truncated Normal distribution:

$$ES_i = -E\left[I_i r_i | I_i r_i < -C_i\right] = -C_i E\left[-\hat{\lambda}_i^* r_i | \hat{\lambda}_i^* r_i < -1\right] = C_i \cdot DD_i^{-1} \cdot M(DD_i) \quad (4)$$

where $M(z)$ is the inverse Mills ratio defined as

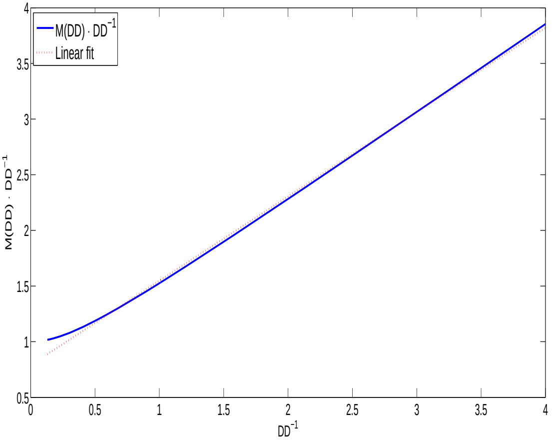
$$M(z) = \frac{\phi(z)}{1 - \Phi(z)}. \quad (5)$$

with $\phi(z)$ and $\Phi(z)$ the pdf and cdf function of the standard Normal.

The product $DD_i^{-1} \cdot M(DD_i)$ is:

- an increasing function of DD_i^{-1}
- virtually linear in DD_i^{-1}

Expected Shortfall factor $DD^{-1} \cdot M(DD)$



Plot of $DD_i^{-1} \cdot M(DD_i)$ as a function of DD_i^{-1} and its linear fit obtained by standard OLS (giving intercept 0.79 and slope 0.76)

Properties, ctd

Therefore, each individual expected shortfall is well described as:

$$ES_i \propto C_i \cdot DD_i^{-1} = C_i \lambda_i \sqrt{E[e_i]^2 \sigma_i^2 + \mu_i^2 V[e_i] + V[e_i]\sigma_i^2}. \quad (6)$$

Thus, the expected shortfall of individual i increases with:

- the degree of leverage λ_i,
- the true riskiness of the portfolio σ_i,
- the positive bias of the estimation error, i.e.
  - the underestimation of risk ($\hat{\sigma}_i < \sigma_i \Rightarrow E[e_i] > 1$)
  - the overestimation of the expected return ($\mu_i < \hat{\mu}_i \Rightarrow E[e_i] > 1$),
  - the variance of the estimation error $V[e_i]$.

In plain words

Harmless derivatives arise when

- idiosyncratic risks of the single projects are optimally diversified (reducing σ_i's);
- remaining non–diversifiable risk is optimally estimated ($V[e] = 0$ and $E[e] = 1$), and
- optimally shared among the investors according to their risk capacities (minimization of the aggregate leverage of the economy).

Hence three natural desiderata for socially harmless derivatives are

1. gains in information should be provided throw transparency and price discovery,
2. pre-existing risks should be re-allocated only to more capitalized, diversified or informed subjects,
3. no new risks should be created.

Is anyone trading in socially harmless derivatives?

Case 1: Plain vanilla derivatives

Plain vanilla interest swap

In this swap a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a predetermined number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time. **Is it socially harmless?**

YES if transferred to individuals with larger DD, i.e. to individuals with either

- higher risk capacity (lower leverage λ_i),
- better portfolio diversification (lower σ_i),
- or better information (smaller estimation error $e_i - 1$)

NO

in the presence of moral hazard of if no pre-existing risk is reallocated.

Case 2: Asset Backed Securities

Securities whose cash flows are derived from, and collateralized by, a specified pool of receivables or other financial assets. Those underlying assets are typically represented by illiquid and risky assets which individually possess a very low rating score.

Securitization

- pooling various types of contractual debts (mortgages, loans or credit card debt) and selling them to various investors
- the pool is sliced into *risk classes*
- "senior" tranches protect the "junior" tranches which are the most exposed to suffering losses in the pool.
- by pooling and splitting senior tranches usually make it to AAA-ratings
- prior to the 2007-8 crisis, many banks and other investors to hold large amount of AAA-tranches of a vast variety of ABS products

Are ABS socially harmless?

NO

In our model, ABS's expose financial markets to greater *SR* through:

1. an increase in the variance of estimation error $V[e]$ (lack of transparency);
2. a positive bias in e, i.e. $E[e] > 1$ (misspricing);
3. an increase in individual leverages λ_i (moral hazard);
4. an increase in the correlation of the probability of *ES*

Hence, even if ABS might increase $E[Y]$, it also tends to increase *SR*.

Case 3: Credit Default Swaps

Credit Default Swaps are "insurance" contracts against default events involving three parties

1. the protection seller who sells protection to
2. the protection buyer against a credit event
3. a third party issuing a debt.

- In case of a default the buyer is compensated for the loss generated by the failure of the third party.
- If the protection buyer does not hold the underlying security, CDS are said to be *naked*

Are CDS socially harmless?

YES

- When credit risk is allocated to more capitalized, diversified or informed subjects as it can either decrease SR or increase $E[Y]$;
  - in this case SR is reduced by reducing $V[e]$ hence CDS can also allow for efficient price discovery in the creditworthiness of the issuer

NO

- when CDS transfer credit risk to more leveraged and systemically important institutions
  - this generally leads SR to increase;
- when CDS magnify the underlying credit risk by compounding it with the counterparty risk of the protection sellers
  - this generally increases SR;

RETROSPECTIVELY

Uncertainty in finance is pervasive and *very probably unquantifiable* (to the degree required for good financial decision-making)

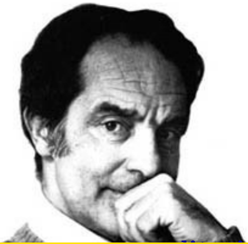
MAIN IDEA

Efforts to measure uncertainty can complemented by tackling the problem of designing derivatives which cannot be misused too badly i.e. *socially harmless derivatives*

1. providing the "great boon to humanity" whilst
2. mitigating the negative impact of derivatives as weapons of financial mass destruction

The best that you can expect is to avoid the worst

(I. Calvino, *If on a Winter's night a traveler*, 1979)



Outline

We statisticians, with our specific concern for uncertainty, are even more liable than other practical men to encounter philosophy, whether we like it or not.

Savage, L. J. (1967). "Difficulties in the theory of personal probability". *Philosophy of Science*, 34(4), 305–310.

1. Small probabilities

The probability of such an event was estimated to be one over 10^{50} *"***odds so small they do not have a meaning***. It is a number outside the scale of nature"*

Things were happening that were only supposed to happen once in every 100,000 years. **Either that [. . .] or Goldman's models were wrong**

Decision modelling

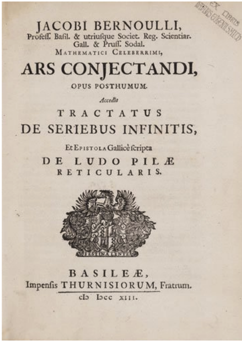
How small should a probability be before we are entitled to neglect it?

Decision making

How to make decisions with (non-negligible) *small* probabilities?

A problem as old as probability

That is morally certain whose probability nearly equals the whole certainty, so that a morally certain event cannot be perceived not to happen: on the other hand, that is morally impossible which has merely as much probability as renders the certainty of failure moral certainty. Thus, if one thing is considered morally certain which has 999/1000 certainty, another thing will be morally impossible which has only 1/1000 certainty. [. . .] Because it is only rarely possible to obtain full certainty, **necessity and custom demand that what is merely morally certain be taken as certain**. *It would therefore be useful if fixed limits were set for moral certainty by the authority of the magistracy —if it were determined, that is to say, whether 99/100 certainty is sufficient or 999/1000 is required*



Philosophical views

Cournot's principle

The meaning of small probability is that a small probability event will not happen

Lévy's praise That is the only connection between probability and the empirical world

Borel's rendering That is the only law of chance, but distinguishes degrees of practical impossibility
(human: 10^-6, earhtly: 10^-15, cosmic: 10^{-50} and universal 10^{-1000})

de Finetti's cricitism "Borel himself, and other capable writers, fail to avoid this misrepresentation when they give the status of a principle -Cournot's principle- to the confusion (or the attempt at a forced identification) between 'small probabilities', which, by convention, could be termed 'almost impossibility', and 'impossibility' in the true sense" (de Finetti 1974)

2. Basel regulations and coherence

The goal is to set capital requirements as a percentage of a bank's assets, with the value of the assets weighted in line with their riskiness (calculated internally or via standardised models)

Basel 3 will be phased in through 2019 and introduce two important changes related to systemic risk

1. banks are required to maintain top-quality capital equivalent to 7 per cent of their risk-weighted assets
2. Banks should move from a the *incoherent* risk measure known as Value-At-Risk to the *coherent ES expected shortfall* measure (i.e. the expected loss given the default of the institution)

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). "Coherent Measures of Risk". *Mathematical Finance*, 9, 203–228.

Monetary risk measures

- let $X(\omega)$ be the future value of financial position (i.e. a random variable on the state space Ω at a future date)
- denote $\mathcal{A} \subseteq \mathcal{X}$ the space of *acceptable* positions in the (position) space $\mathcal{X}$
- write $X \geq Y$ if $X(\omega) \geq Y(\omega),\ \forall\omega \in \Omega$
- if $Y \in \mathcal{A}$ a sufficient condition for X to be is that $X \geq Y$
- then define

$$\rho(X) = \inf\{m \mid X + m \in \mathcal{A}\}$$

i.e. the smallest amount of capital m such that the position X becomes acceptable when this amount is added and invested (risk-free)

ADEH axiomatisation

- Translation invariance $\rho(X + m) = \rho(X) - m$,for all constants m
- Monotonicity if $X \leq Y$ then $\rho(X) \geq \rho(Y)$
- Positive Homogeneity $\rho(\lambda X) = \lambda \rho(X)$, for all $\lambda \geq 0$
- Subadditivity $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). "Coherent Measures of Risk". *Mathematical Finance*, 9, 203–228.

Coherence and sub-additivity

Subadditivity promotes diversification and the decentralization of risk-management

Consider two trading desks with positions leading to losses L_1 and L_2. Imagine that a risk manager wants to ensure that $\rho(L)$, the risk of the overall loss $L = L_1 + L_2$, is smaller than some number M. If ρ which is subadditive, the risk manager may choose bounds M_1 and M_2 such that $M_1 + M_2 \leq M$ and impose on each of the desks the constraint that $\rho(L_i) \leq M_i$. Then, by subadditivity $\rho(L) \leq M_1 + M_2 \leq M$.

Desirable gambles

For non-empty Ω, a *gamble* is a bounded real-valued function on Ω. $X(\omega)$ is intuitively interpreted as the amount of (monetary) utility which depends on the uncertain outcome ω as a consequence of buying X

Let $\mathcal{G}(\Omega)$ be the set of all gambles on Ω.Then for all $\lambda \in \mathbb{R}$ and $X \in \mathcal{G}(\Omega)$, λX is the gamble defined by $(\lambda X)(\omega) = \lambda X(\omega)$. For all $X, Y \in \mathcal{L}(\Omega)$, $X + Y$ is the gamble defined by $(X + Y)(\omega) = X(\omega) + Y(\omega)$.

Desirable gambles

(D0) If $\sup X = \sup\{X(\omega)|\omega \in \Omega\} < 0$ then X is not desirable.

(D1) If $\inf X = \inf\{X(\omega)|\omega \in \Omega\} > 0$ then X is desirable.

(D2) If X is desirable and λ is a positive real number then λX is desirable.

(D3) If X and Y are each desirable then $X + Y$ is desirable.

Walley, P. (1991). *Statistical Reasoning with Imprecise Probabilities*. Wiley.

Coherence condition for lower previsions

A lower prevision $\underline{P}$ defined on an arbitrary subset $\mathcal{S} \subseteq \mathcal{G}(\Omega)$ (interpreted as the sup price μ for which $X - \mu$ is desirable) is *coherent* if

$$\sup \left\{ \sum_{j=1}^{n} (X_j - \underline{P}(X_j)) - m(X_0 - \underline{P}(X_0)) \right\} \geq 0,$$

for every non-negative integers m, n and for every $X_0, X_1, \ldots, X_n \in \mathcal{S}$.

Theorem (Walley 1991)

A lower prevision $\underline{P}$ defined on an linear subspace of $\mathcal{G}(\Omega)$ is coherent if and only if it satisfies:

(P1) $\underline{P}(X) \geq \inf X$ when $X \in \mathcal{G}(\Omega)$

(P2) $\underline{P}(\lambda X) = \lambda \underline{P}(X)$ when $X \in \mathcal{G}(\Omega)$ and $\lambda > 0$

(P3) $\underline{P}(X + Y) \geq \underline{P}(X) + \underline{P}(Y)$ when $X, Y \in \mathcal{G}(\Omega)$

Analogies between analogies

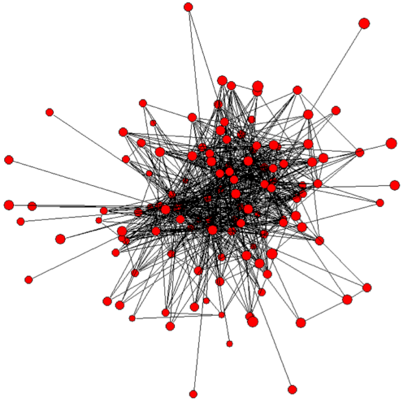
Can we gain a better understanding of decision-making under *severe* uncertainty by exploiting / developing the analogy between

- The theory of coherent lower previsions and
- Coherent (and more generally, *convex*) risk measures?

A bridge

- Convex risk measures can be represented as particular maxmin expected utility functions
  - Foellmer, H., Schied, A. (2002): *Convex measures of risk and trading constraints*. Finance Stoch. 6(4), 429–447 Convex risk measures can be represented as particular maxmin expected utility functions
- which in turn have a characterisation as variational preference relations
  - Maccheroni, F., Marinacci, M., Rustichini, A. (2006): "Ambiguity aversion, robustness, and the variational representation of preferences". *Econometrica* 74(6), 1447–1498

Causality and agency in systemic risk



Brazilian interbank network,

December 2007.

- Modelling of how much the financial system can be harmed by the failure of the institutions via weighed directed graphs
- Challenges
  - incorporate *agency* of institutions
  - understanding of causality
  - unreliable observables (e.g. balance sheets)

Cont, R., Moussa, A. & Santos, E. (2010) "Network structure and systemic risk in banking systems" in: J.P. Fouque & J. Langsam (Eds.) *Handbook of Systemic Risk*, Cambridge University Press.

Philosophy of social or natural science?

- Finance is strongly driven by data
  - tools from the philosophy of natural sciences
- However strong feedback mechanisms take place owing to a fundamental *epistemic component* (Keynes's beauty contest, etc.)
  - this requires tools from the philosophy of social sciences
  - normative/descriptive/prescriptive distinctions