Extended abstract of the EPSA lecture given at the 14th CLMPS, Nancy 19-26 July 2011, by Philippe MONGIN
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WHAT THE BAYESIAN DECISION THEORIST COULD TELL THE BAYESIAN PHILOSOPHER

A pervasive doctrine of both science and philosophy, Bayesianism is not conceived of in the same way in all the fields it permeates, and this has contributed to make its definition elusive. A glance at epistemology, philosophy of science, decision theory, statistics, and computer science, testifies to differences both in content and emphasis. By and large, there are two main Bayesian tenets, i.e., that (i) uncertainties should be captured in terms of probability functions, and (ii) the incoming information in terms of conditioning these functions, but the emphasis on each tenet varies with the field, and there is a third tenet that is clearly optional, i.e., that (iii) decisions are to be made in accordance to the expected utility (EU) rule and its axiomatic foundations. This last claim is essential to all Bayesian decision theorists and statisticians, but by no means to all Bayesian philosophers. Arguments for (i) and (ii) can be classified as *pragmatic* or *non-pragmatic*, depending on whether or not they go through (iii) to reach their aim. Philosophers like Earman (1992) or Maher (1993) have defended pragmatic arguments borrowed from, or based on, decision theory, while others, like Howson and Urbach (1993), have rejected these arguments and exclusively relied on non-pragmatic ones. Still others, like Joyce (1999), have a relatively liberal view of the two styles of justification of probability and conditioning.

The present lecture is concerned with this major philosophical conflict, but it would be too bold to sketch a solution within its scope, and we are content with clarifying a particular set of pragmatic arguments, leaving the other pragmatic and non-pragmatic arguments aside. We focus on a severe complication of EU theory, *state-dependence*, which is famous among Bayesian decision theorists but regrettably much lesser known to Bayesian philosophers - hence the title for the lecture. We first explain the difficulties that state-dependence raises for Bayesian decision theory, second dramatize them and show that they also threaten Bayesian philosophy, and third and last, after reviewing alternative suggestions, account for some decision-theoretic work that aims at removing them. The conclusion favours the pragmatic against the non-pragmatic camp within the limits of the chosen set of arguments.

To implement the first part of the programme, we revisit the masterpiece of Bayesian decision theory, i.e., Savage's *Foundations of Statistics* (1972), in a somewhat unusual way. Instead of centring on the "sure-thing principle" (P2), as most commentators have done, we emphasize postulates (P3) and (P4), explaining how they capture the *state-independence* assumption that the agent's preferences do not vary across states or events of the world. We also explain why Savage needs this assumption in order to translate the agent's preferences between his possible acts into a satisfactory EU representation, i.e., one in which the probability measure is uniquely well-defined. It is easy to argue that the opposite assumption of *state-dependence* captures a natural psychological attitude and no irrationality at all. Hence Savage fails short of his programme, which was to apply the requisites of pratical rationality, and nothing else, in order to recover the three tenets (i), (ii), and (iii). At this stage, we provisionally side with those non-pragmatic philosophers who have interpreted Savage's results disparagingly.

The second part of the lecture begins by arguing that Bayesian epistemologists and philosophers of science cannot escape the difficulties of state-dependence. Epistemic agents, and typically scientists, in effect make choices under uncertainty, and Savage's system was meant to instruct pragmatic Bayesian philosophers on how they should formalize these choices. However, as we claim, neither (P3) nor perhaps (P4) should be assumed in the epistemic or scientific context. We emphasize that state-dependence blocks the derivation not only of (i) but also of (ii) - a crucial point about Bayesian revision that has often escaped notice. As we further argue, there are four main options to circumvent the difficulties. Option 1 is to cut one's losses and restrict the application domain of Savage's theory to the cases covered by (P3) and (P4). Option 2, which is Savage's, consists in arguing that decision problems can always be given a state-independent form by an appropriate redefinition of states, consequences and acts. Option 3 is to move to Jeffrey's (1983) competing system. Option 4 exploits some technical results of post-Savagian EU theory, which require more preference comparisons on the agent's part, and by suitably defined new postulates, manage to accommodate state-dependence while delivering a uniquely well-defined probability measure.

The third and last part of the lecture critically discusses these four proposals. Option 1 is described as being sound and realistic, but low-profile. It can fit the needs of insurance or financial economists, who can bear with (P3) and (P4) in various applications, but not really those of pragmatic Bayesian philosophers, who are after universally valid norms of rationality. Option 2 is discussed in terms of Savage's own examples in the *Foundations* and rebutted as being spurious. We also discard Option 3 on the ground that Jeffrey's system avoids the problem of state-dependence simply because it lacks the capacity of expressing it. Option 4 is our

favourite one, and we give a flavour of how post-Savageans, like Karni (1985, 1996) and collaborators (including this writer, see Karni and Mongin, 2000), have attempted to overcome the problem. Essentially, they require the agent to express preferences *across* states of the world, hence counterfactual preferences, on top of those expressed between possible courses of actions, which are the only ones considered by Savage and classical decision theorists. Both the old and the new sets of preferences are subjected to postulates, like (P2), which can standardly be interpreted in terms of practical rationality, and they are connected between themselves by a specific coherence condition, which can arguably be interpreted along the same line. The result is that (P3) and (P4) are dispensed with, but owing to the added material, a EU representation nonetheless exists with the desirable property that the probability measure is uniquely well-defined. That is to say, the new work obtains (i) from (iii) without Savage's unpalatable restrictions; it can be checked that (ii) also follows. In spite or because of its complexity, this work provides the best scheme that seems available to date for a pragmatic foundation of probability and conditioning. At this stage of the lecture, we side with the pragmatic camp against the other, and even if not all Bayesian philosophers would agree with us, we hope to have convinced them that contemporary Bayesian decision theory can be an inspiring source.

Earman, J. (1992), *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory*, Cambridge, Mass., The MIT Press.

Howson, C. and P. Urbach (1993), *Scientific Reasoning*, Open Court.

Jeffrey, R. (1965), *The Logic of Decision*, Chicago, Chicago University Press (2nd ed., 1983).

Joyce, J.M. (1999), *The Foundations of Causal Decision Theory*, Cambridge, Cambridge University Press.

Karni, E. (1985), *Decision Making Under Uncertainty. The Case of State-Dependent Preferences*, Cambridge, Mass., Harvard University Press.

Karni, E. (1996), "Probabilities and Beliefs", *Journal of Risk and Uncertainty*, 13, p. 249--262.

Karni, E. et P. Mongin (2000), "On the Determination of Subjective Probability by Choice, *Management Science*, 46, p. 233--248.

Maher, P.(1993), *Betting on Theories*, Cambridge, Cambridge University Press.

Savage, L.J. (1954), *The Foundations of Statistics*, New York, Dover Publications (2nd ed. 1972).