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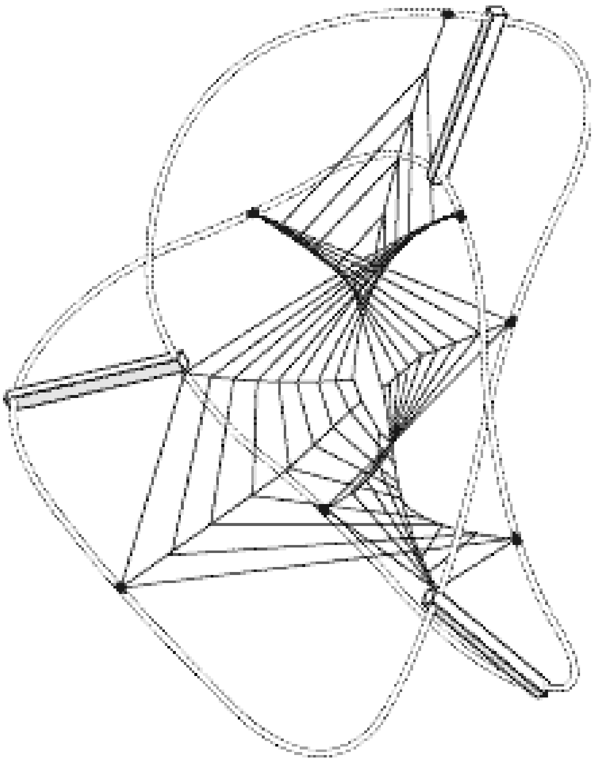
Philosophical Perspectives on Fuzzy Set-Theoretic Models of Causality

Jordi Cat



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Philosophical Perspectives on
Fuzzy Set-Theoretic Models of Causality

Jordi Cat*
Department of History and Philosophy of Science
Goodbody Hall 130
Indiana University
Bloomington, IN 47405
j.cat@indiana.edu
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Abstract

This paper examines a novel type of model of causality, namely, fuzzy set-theoretic causality. Over the last decade a number of models of this type have appeared in the scientific literature and have circulated widely from one scientific discipline to another. However, they have received no philosophical attention. In this paper I will discuss the value and limitations of this type of models and will read into its application several dimensions of philosophical significance.

1 Introduction

The purpose of this paper is to examine a novel type of model of causality, namely, fuzzy set-theoretic causality. It is a conceptual product of the tradition of research that has introduced fuzzy sets in connection to fuzzy logic and has inspired fuzzy technology. Over the last decade a number of causal models of the same type have appeared in the scientific literature and have circulated widely from one scientific discipline to another. However, they have received no philosophical attention. In this paper I will discuss the value and limitations of this type of models, and will read into it several dimensions of philosophical significance.

The broadest dimension of philosophical significance I bring to bear is that of the relation between science and philosophy. Causality is a notion common to scientific and philosophical discussion. Fuzzy-set causality is an example of a strong interaction between both. The fuzzy-set approach is also relevant to general issues in set theory and logic. But this is of less interest for the purpose of this paper. Of more interest to me is the difference that this approach makes to empirical research and how this relates to empirical method. Fuzzy-set causality illustrates also an aspect of mathematical science that I have discuss in detail elsewhere: mathematical formalisms can be endowed with conceptual contents beyond the purely formal and mathematical aside from a specific theory in which they are applied. That interpretation often plays a mediating and guiding role in their application. As a consequence such formalisms, including fuzzy-set causality, can circulate and extend their domain of application beyond one single theory and scientific discipline. In this regard fuzzy-set causality is an example of dynamical unification in the sciences. Finally, I will critically evaluate fuzzy causal models from a more normative and critical standpoint.

2 What is a fuzzy set?

Lotfi Zadeh introduced the notion of fuzzy sets to model vague predicates and informal reasoning in the form of fuzzy logic (Zadeh 1965). Let X be a classical set of objects, denoted by x, called the universe. Let C be a classical subset of X and m(x) be a function from X to the pair of values {0,1} such that if $x \in C$, m(x)= 1, and if $x \notin C$, m(x)= 0. In classical sets, then, elements of the universe either belong to a set or they don't, The set is characterized by a sharp boundary and is identical with its members. By contrast, a fuzzy set, F, is a subset characterized by the set of pairs

$$F = \{(x, m(x)), x \in X\},$$

where m(x) represents the degree of membership with possible values ranging over the real interval [0,1]. In each context of application of the predicate F, the fuzzy set F will be normalized if there exists one x, such that m(x)= 1.

There is no sharp boundary. The set is characterized by a membership function. In this way fuzziness formalizes a kind of deterministic uncertainty and ambiguity. For instance, if F represents the vague predicate TALL, the fuzzy set of tall individuals will include very tall individuals, with m closer to 1, and short individuals, with m closer to 0. As an interpretation of fuzzy logic, the membership function designates a degree of truth valuation for statements of the

form ‘x is F’. This provides the extensional semantics for multivalent logic.[1]

Zadeh himself and other researchers have made clear that the assignment of membership values, and the corresponding truth-values of propositions, is subjective and local (see Zadeh 1975, Dubois and Prade 1980, Haack 1996 and Ragin 2000). It is subjective in all cases because at least one assignment is subjective, namely, to a standard individual, relative to which other individuals get their membership degree. It is subjective because at least that assignment is not based on any intersubjective, formal or material method of assignment that would ensure consensus. It might, however, not be arbitrary. Instead, especially in the scientific case, it might be guided by background knowledge or assumptions and motivated by certain values. It is based on expert judgement. It is also local, because the assignment of membership degrees varies from context to context (small chihuahuas are addressed differently from small St. Bernards).

For fuzzy sets one can introduce an algebra of set-theoretic operations. The most basic ones introduced by Zadeh are:

*union: $m(F \cup G)(x) = \max(mF(x), mG(x))$
*intersection: $m(F \cap G)(x) = \min(mF(x), mG(x))$
*complement: -F is the set of x such that $m'(x) = 1 - mF(x)$,

where mF(x) is the degree of membership in F. -F is the complement of F. Unlike for classical sets, for fuzzy sets $F \cup -F \neq \Omega$, where Ω is the universe, and $F \cap -F \neq \emptyset$.

*subset: $F \subset G$ iff $mF(x) \leq mG(x)$, for all x.

If $\sum mF(x) = M(F)$ is the size of F, and $\sum mG(x) = M(G)$, the size of G,

$F \subset G$ iff $M(F) \leq M(G)$.

These operations form the basis for the models of causality I present and discuss in the next sections.

3 Fuzzy set-theoretic models of causality

A. The first two models are due to Bart Kosko (Kosko 1986 and 1990). They inspired the subsequent ones. In the first one, Kosko introduces a definition of causality between conditions represented by fuzzy sets. Given conditions (concepts) Ci, Cj such that $C_{i,j} = (Q_{i,j} \cup -Q_{i,j}) \cap M_{i,j}$, where Qi,j and -Qi,j are fuzzy quantity sets (e.g., stability) and their respective complements (e.g., instability), and Mi are modifiers (e.g., social),

Ci causally increases Cj iff $(Qi \cap Mi) \subset (Qj \cap Mj)$ and $(-Qi \cap Mi) \subset (-Qj \cap Mj)$,
Ci causally decreases Cj iff $(Qi \cap Mi) \subset (-Qj \cap Mj)$, and $(-Qi \cap Mi) \subset (Qj \cap Mj)$.

[1] For a critical discussion of fuzzy logic see Haack (1996). I have very little to say about the general significance of fuzzy sets and fuzzy logic. Fuzzy sets are not Cantorian collections, with elements as fundamental, but more like Russellian classes, based on criteria of membership. They can be taken to capture a prototype notion of categorization based on an individual as a standard of assigned membership.

With these definitions Kosko generalized R. Axelrod's 1970s Cognitive Maps in political science to Fuzzy Cognitive Maps represent causal reasoning as calculus of causal influence. The calculus is expressed in terms the algebra of adjacency matrices Cij, where map nodes Cij represent the sign of causal influence between Ci and Cj.

In general, the causal relation A causes B is defined in terms of subsethood, or set inclusion, $A \subset B$, The subsethood relation clearly models and generalizes the bivalent logical implication in the form of truth-functional material conditional, $A \rightarrow B$.

In the second model, Kosko introduces the notion of degree of subsethood formulated more generally, in terms of inclusion in the power set. It is meant as a fuzzy generalization of Zadeh's subsethood relation. The power set of B, 2(B) is set of all subsets of B. If $A \subset B$, $A \subset 2(B)$. He motivates a definition of degree of subsethood, S(A,B), as

$$S(A,B) = 1 - [\max(0, M(A) - M(B))]/M(A),$$

which he identifies with

$$S(A,B) = M(A \cap B)/M(A).$$

S(A,B) represents fuzzy causality relation between A and B. A and B now stand for properties or conditions involving all sorts of physical systems.

Kosko frames the causal model in a geometric formalism: Sets {xn} representing different predicates can be represented, instead of as functions (Zadeh), as points in an n-dimensional hypercube representing the power set. Each dimension representing the degree of membership of an element of a set with a unit distance representing the range of possible membership values. He believes that the association of the general set-theoretic results with geometric properties of the hypercube provides the causal models and other results with both an intuitive presentation and a more general foundation.

B. The third model is introduced in the social sciences by Charles Ragin (Ragin 2000). Ragin argues that many social factors or conditions are best represented by configurations of qualitative properties in the form of fuzzy sets. In addition, he believes that social causal knowledge for the purpose of explanation, intervention and prediction is expressed in terms of necessary and sufficient conditions. Necessary and sufficient conditions are interpreted set-theoretically in terms of subsethood. Generalization to fuzzy sets involves fuzzy subsethood and the measure of fuzzy subsethood is represented in terms of the membership function, m, for all instance in the set.

A is a **necessary** condition for B (or we have evidence that A is a necessary cause of B), if B is a subset of A: $mB(x) \leq mA(x)$ for all instances. Therefore, $\sum m(B) = M(B) \leq M(A) = \sum m(A)$.

A is a **sufficient** condition for B (or we have evidence that A is a sufficient cause of B) if A is a subset of B, that is, $M(A) \leq M(B)$.

For a combination of necessary causes, M(A) is $\min\{M(Ai)\}$. For a combination of sufficient causes, M (A) is $\max\{M(Ai)\}$. Necessary and sufficient conditions, $M(A) = M(B)$: define the diagonal in a (hyper) cube defined by each coordinate edge representing one condition. By inclusion, we can make necessity and sufficiency claims for individual instances in terms of m(A) and m(B).

C. The fourth model I want to consider, albeit in less detail, is due to Cathy Helgason and

Thomas Jobe (Helgason and Jobe 2003). It is meant as an alternative to population-statistics-based medicine. The latter, they object, assumes an artificial separation among variables. Their goal is to provide a perception-based representation of physiological interaction of elements in an individual patient , and to view the evolution over time provided initial conditions.

They adopt Kosko's hypercube formulation with sets as points. They take relevant causal knowledge to be knowledge of the evolution in the hypercube of the causal nexus of interacting conditions of an individual patient, from the initial set of conditions, A, to a later set, B, after a change with or without intervention. This is represented by a trajectory along the corresponding points in the hypercube of relevant variables.

In this model, causal knowledge is characterized in terms of quantities they call Formal Causal Ground and Clinical Causal Effect, which are defined in terms of necessary and sufficient conditions, viz., necessary and sufficient causal ground. These are characterized, in turn, in terms of fuzzy measure M and fuzzy subsethood relation, S:

Necessary Causal Ground, NCG = $[M(A \cap B)/M(A)]^2 = S(A,B)^2$
Sufficient Causal Ground, SCG = $[M(A \cap B)/M(B)]^2 = S(B,A)^2$.

From these quantities they define Formal Causal Ground as

FCG = (NCG).(SCG).

FGC is the degree of necessity and sufficiency of the initial conditions. The Clinical Causal Effect is

CCE = $\sqrt{[S(A,B).S(B,A)]}$.

CCE is the measure of clinical intervention.

4 Philosophical significance (1): Fuzzy empiricism and local subjective and enhanced coordination of theory and data

The philosophical problem of empiricism is the determination of the coordination of theory and data. Duhem and Neurath raised the issue of the indeterminacy in the coordination of precise technical terms of theory and vague concepts of perception, viz., practical facts and protocol sentences, respectively (Duhem 1906, Neurath 1932, Cat 1995 and Cartwright et al. 1996).

Ragin and other proponents of fuzzy causality note that in many scientific contexts the data is precise—GDP, income, etc., whereas what counts as the 'theory' is now formulated in vague terms: dangerous neighborhoods, financially secure households, rich countries, social stability, religious fundamentalism, poverty. Fuzzy sets defined for precise data or individuals in a population operationalize those theoretical categories. Membership functions provide a sort of bridge principle.

Membership function introduces two paradoxically inseparable aspects of fuzzy empiricism. On the one hand, the locality and subjectivity of the membership function is an irreducible aspect of fuzzy empiricism. On the other, the acceptance of degrees of membership enhances the pool of empirical data beyond the clear or prototypical cases, viz., with m = 1. Note that the degree of

membership in a set providing the degree of truth-value of propositions relating a member of the set and the property the set represents. This fact introduces a notion of approximation.

5 Philosophical significance (2): Fuzzy empiricism and fuzzy causality as example of the relative a priori

Duhem, Poincaré and others have argued that hypotheses cannot be tested individually. They are tested in conjunction with a bunch of auxiliary hypotheses required to generate the experimental prediction. The same is true for explanations or descriptions of phenomena. They are mediated by auxiliary criteria and assumptions.

What are these auxiliary preconditions? (1) assumptions from the theory of the instrument that will generate the predicted data; (2) background assumptions linking theory and relevant evidence such as standards of evidence and argumentation; (3) standards of approximation; (4) physical constitutive assumptions: Newton's law of universal gravitation is testable only in conjunction with the second law of motion F=ma; (5) representational assumptions about the preferred sort of models given certain cognitive goals: atoms, genes, fuzzy properties, geometric descriptions, etc.; (6) conceptual constitutive assumptions such as models of causality that link a theory of a phenomenon and causal language—for instance, statistical or probabilistic models of causality, causal graphs, fuzzy models of causality; (7) mathematical assumptions about mathematical formalism such as topological properties as preconditions of the differential calculus and set theory as precondition of probability theory; (8) cognitive values such as simplicity, symmetry, consistency, intuitiveness, tractability, etc.

All those assumptions share the property of being 'relatively a priori'. They are adopted prior to the adoption of the tested hypotheses or the description or the explanation of phenomena. Reichenbach, Carnap and Friedman would consider some of those more strongly as 'relativized a priori': as a priori constitutive conditions of conceptual possibility (Friedman 2001). They are valid for the definition of concepts in specific theories or frameworks. Fuzzy-set models of causality are a priori formal criteria that specify and facilitate the application of causal talk. They constitute a bridge principle that concretizes the notion of causality for application to phenomena represented through fuzzy sets.

6 Philosophical significance (3): Fuzzy causality as example of interpreted formalism

The standard picture of how scientific terms get their meaning is that in each theory an abstract symbolic calculus is interpreted relative to a particular kind of phenomena or models. By contrast, I believe that mathematics is often applied with a certain conceptual interpretation, or understanding, over and above any particular theory. These interpretations help us apply within different theories more abstract and general representations/criteria such as individual particle, determinism, locality, causal relation, information, etc. (see Cat 2002 for details). For instance, to Maxwell and Kelvin, differential equations constituted a canonical representation of contiguous action, whereas integral laws exposed action at a distance. Similarly, fuzzy-set models of causality, mediate the application of fuzzy set theory to the formal treatment of phenomena and they received a causal interpretation above any specific theory or domain of application.

7 Philosophical significance (4): Fuzzy causality as example of dynamical unification in science

Ideals of unification of science abound. Ideals of fundamental languages, concepts, laws or entities, or ideals of a unique method provide values, standards or projects that drive and evaluate research. They are not universal values shared among different scientific communities, even within the same discipline, such as physics (Dupré 1993, Galison and Stump 1996 and Cat 1998).

Real unification is often the outcome of a partial reduction of elements of different theories and hybrid models and inter-field theories (Sarkar 2000). Less noted is the actual unification that consists in the circulation of techniques and models within subdisciplines (i.e., renormalization and symmetry-breaking within physics) or among different sciences—statistical analysis, analogies such as genetic algorithms, cognitive maps, etc. The example of fuzzy-set causality is a clear one, based on its nature as interpreted formalism. Such formalisms, I have noted above, have the corresponding property of being more general than any particular theory or model in which they are applied to a domain of phenomena. This makes their circulation from science to science possible. It is an example of dynamical unification of the sciences.

8 Philosophical significance (5): Fuzzy causality as example of the interaction between science and philosophy

I believe that the respective identity and continuity of science and philosophy are inseparable from their respective history. And throughout their history science and philosophy have long been engaged in exchanging notions and developing in response to each other's practices and deliverances, each other's devices and desires. Until the 17th century any distinctions between scientific and philosophical disciplines are typically internal to the subject matter and mode of inquiry of philosophy. Subsequent developments in each discipline appear both to engage in interaction and to articulate and promote external distinction.

Lesser known scientists as well as figures such as Oersted, Faraday, Maxwell, Darwin, Helmholtz, Poincaré, Duhem, Bohr, Einstein, and others, have found in philosophical reflection guidance and ideas. They have also contributed conceptual insight. Newton himself couldn't separate science from philosophical pronouncements. Philosopher-scientists such as Descartes and Leibniz criticized and formulated scientific claims on philosophical grounds and Kant made the metaphysical foundation of the sciences central to philosophy. Philosophy would explore the conditions of possibility of objective knowledge by way of the conditions of intelligibility of scientific theories in areas such as arithmetic, geometry and mechanics. Later philosophical developments from the hands of Russell, Poincaré and logical empiricists made much use of contemporaneous scientific developments in mathematics and physics in order to qualify or rebut Kant's insights (see Friedman 2001). What can be said about the philosophical significance of Einstein's relativity theories can be matched by the intricate relation of Gestalt psychology to the history of 20th century scientific philosophy and philosophy of science (see Cat forthcoming). And philosophy of science is still considered a branch or subdiscipline of philosophy.

The topic of causality is one in the changing overlap and interaction of science and philosophy. Even for Kant the synthetic a priori status characterizes both scientific and causal judgements. Causal claims and causal reasoning pervade scientific knowledge and practice.

Philosophy has since its recognizable early moments addressed questions regarding the nature of causality and the reliability of methods to yield and ascertain causal claims. In philosophy of science attention has been paid to scientific claims and to causal assumptions that make intelligible and possible much scientific practice.[2]

What is relevant from this point of view to the case of fuzzy-set models of causality is that they have been both issued and applied by scientists themselves. This fact obliterates the distinction between science and philosophy in terms of a strict division of labor.

9 Philosophical significance: Conceptual criticism of fuzzy-set causal models

The point of conceptual criticism is the examination of limitations in these models so that they are developed and applied intelligently and valuably, and they don't become mere ad hoc labels. For this purpose I want to distinguish between two kinds of causal accounts: Conceptual, or ontological, viz., they explain what causality is, and epistemological, or methodological, viz., they offer methods for causal inferences, for hunting causes for the purpose of prediction, intervention or explanation.

The four models presented above represent causality as linking logical implication and empirical data. The logical implication is the one modeled set-theoretically by subsethood relations. The logical formulation of causal relations in terms of set theory has important limitations as an account of causation and as models of causal reasoning.

1) Lack of syntactic distinction between stable mechanisms and circumstantial conditions.

   (A implies B) implies that (not-A implies not-B),
   (smoking implies cancer) implies (not-cancer implies not-smoking).

   However, while the former might have causal implications, the latter lacks the same kind of causal support. Kosko's 1986 model, above, introduces the additional logical implications in terms of causal increase and decrease precisely to eliminate such possibilities. The other models don't.
2) The empirical truth-functional material implication is true even if A is false enough t(A implies B) = min{1, 1–t(A)+t(B)}. But only stronger, non-truth-functional forms of necessity or implication can make the implication false enough in that case.
3) In stronger, non-truth-functional forms of implication, the truth of A and B is insufficient to make the logical implication true. Causality is one.
   Empirical regularities or contiguous sequences are not sufficient for a stronger form of implication or necessity. This is a problem for conceptual accounts; the Humean reductionist project of understanding causation in terms of factual regularities fails to capture the element of necessity that distinguishes causal action. Of course, Humeans will disagree here. The problem for epistemological accounts is the limitation it implies on evidential relations. In the strongest case they aren't conclusive. The same problem extends to improved fuzzy cognitive maps, in epistemological and ontological versions, that include time lags, based on sequences or histories (Park and Kim 1995, Chaib-Draa

[2] For examples of such 'transcendental' approach see Bashkar (1975) and Cartwright (1989) and (2000).

and Desharnais 1998 and Helgason and Jobe 2003). The statistical version of this problem is that correlation is not causation.

Cartwright and other have argued that probabilistic models of causality fail to be reductive because in order to avoid Simpson-paradox-type difficulties, they need to be conditionalized upon relevant causal factors. But this requires prior causal -not just empirical- knowledge. Therefore, in the strongest formulation probabilistic models approximately measure but do not define causation (Cartwright 1983 and 1989).

At both at the epistemological and ontological level, provisions must be made to isolate the relevant causal candidates. All three models fail.

4) Material implication fails to distinguish causal relations from conceptual relations. Concepts can be defined extensionally or given extensional expression: Subsethood relations apply both to conceptual relations and to causal relations. Subsethood alone is not sufficient to isolate causal relations. For instance, being a mammal might be sufficient for having four limbs and yet the former doesn't cause the latter.
5) In cases of anti-correlations or conserved quantities, knowledge of the value of a property of one system is necessary and sufficient to know the value of the other. Is that causality? The controversy surrounding the case of EPR correlations in quantum mechanics illustrates this.
6) Necessary cause is different from only actual cause or only possible cause. Subsethood and material implication fail to tell the difference.
7) Problem of underdetermination of sufficient causes (similar to the problem of logical underdetermination of theories by empirical data): for any sufficient cause C, C&X is sufficient as well. At both at the epistemological and ontological level, provisions must be made to isolate the relevant causal candidates. All three models fail.
8) Problem of overdetermination: if at least two causes are sufficient, neither will prove necessary. There is the additional problem of spurious causation: if either shooter, A or B, can kill a prisoner D (D true), and C acts by ordering A to shoot (C true), in the measured relation either C or B are sufficient for the death of D. If both A and B fail to act (A and B false), the measured relation still entails that C is true and D is true/dead: C's action giving orders caused D. The problem is a limitation also in probabilistic models of causality, conceptually and epistemologically.
9) As stated, the logical models fail to distinguish causal connection from common cause. Provisions might be introduced.
10) All three models fail to make provisions for INUS conditions in complex cases: causes can be true causes while being neither necessary nor sufficient: insufficient but necessary parts of sufficient but unnecessary causal complexes. Example: A such that [(A&B) or (C&D)] causes E. This problem is particularly egregious in Ragin's methodological account, for he states that 'this type of causation may be the most common form of social causation' (Ragin 2000, p. 93).
11) The three models are vulnerable to the problem of reversibility: increase of A implies increase of B implies that decrease of A causes decrease of B, which is not always the case.
12) The three models obscure research into the intervening mechanisms that might actually explain the subsethood relation—in lieu of 'laws of nature'. Attention to the intervening causes of factor would involve a change in fuzzy cognitive maps and the associated adjacency matrices. Intervening mechanisms and shielding constraints—such as a pipe or

a blood vessel—might be worth taking into account as they may present degrees of failure (leaks) and may be required in order to understand and intervene (prevention).

13) Additional problems arise from the relation of fuzzy sets to probabilities. Kosko argues that degree of subsethood S(A,B) is set-theoretically equivalent to conditional probability:

$$S(A,B) = M(A \cap B)/M(A) = P(B \mid A).$$

If so, the FC model based on S is as vulnerable to all the know objections to causality defined in terms of high P(B | A) (see Salmon 1984, Cartwright 1989 and Pearl 2000).
The Helgason-Jobe model presumes to replace probability-based statistical knowledge as inadequate and irrelevant to the medical goal at hand. Yet, required prior knowledge might come from statistical models: in the pre-selection of relevant causal conditions for the transformation of variables into fuzzy membership functions. The set-theoretic structure of the model generates or maps onto a formal probabilistic structure: does that mean that the model can relate to probabilities or express them but only under a propensity interpretation about one individual? If so, this is inconsistent with the statistical or population interpretation of the probabilistic information required for constructing the fuzzy membership assignments.

10 Conclusions.

The previous discussion raises two metaquestions: are fuzzy necessary and sufficient conditions necessary and sufficient (even in the fuzzy sense) for causality? Is the notion of causality an example of a higher-order fuzzy category?[3] The discussion also sheds doubt on such suggestions. An alternative approach to accommodating the usefulness of fuzzy sets would be to apply other models of causality to conditions or variables represented by fuzzy categories without the difficulties and weaknesses introduced by the set-theoretic models of logical implication. The next step would be to add aspects of the alternative models to the set-theoretic model in those domains, no matter how idealized, in which the latter seems relevant.

The different causal models may be related as conceptual accounts or as cooperating or competing local epistemological models. The first task in order to address the suggestions above is to figure out in detail where and how successfully fuzzy and other models apply; and then to know how they can relate to one another. Aside from issues of limits of application, causal models in general, and fuzzy-set models of causality in particular have doubtless philosophical significance in all the aspects of science they illustrate: from the complexity of empiricism to the vagaries of the interaction between scientists and philosophies.

[3] I'm indebted to Lotfi Zadeh for discussions of this question.

References

Bhaskar, R. (1975) *A Realist Theory of Science*, London: Humanities Press.

Cartwright, N. (1983) *How the Laws of Physics Lie*. Oxford: Clarendon Press.

Cartwright , N. (1989) *Nature's Capacities and Their Measurement*. Oxford: Oxford University Press.

Cartwright, N. (2000) *The Dappled World: Studies on the Boundaries of Science*, Cambridge: Cambridge University Press.

Cat , J. (1995) 'The Popper-Neurath Debate and Neurath's Attack on Scientific Method', *Stud. Hist. Phil. Sci* **26**, No. 2, 219-250.

Cat, J. (1998) 'The Physicists' Debates on Unification in Physics at the End of the 20th Century', in *Hist. Stud. Phys. Sci*. **28**, n.2, 253-299.

Cat, J. (2002) 'Representations as Theories: How and Why Mathematical Representations Make a Difference to Physical Theory', Indiana University ms.

Cat, J. (forthcoming) 'Switching Gestalts on Gestalt Psychology: On the Relation between Science and Philosophy.'

Chaib-Draa, B. and J. Deshairness (1998) 'A Relational Model of Cognitive Maps'*, Int. J. Human-Computer Studies* **49**, 181-200.

Dubois, D. and H. Prade (1980) *Fuzzy Sets and Systems*, New York: Academic Press.

Dupré, J. (1993) *The Disorder of Things*, Cambridge, MA: Harvard University Press.

Friedman, M. (2001) *Dynamics of Reason*, Stanford: CSLI.

Galison, P. and D. Stump, eds. (1996) *The Disunity of Science*, Chicago: Chicago University Press.

Helgason, C, and T. Jobe (2003) 'Perception-Based Reasoning and Fuzzy Cardinality Provide Direct Measures of Causality Sensitive to Initial Conditions in the Individual Patient', *Int. J. Comp. Cognition* **1**, n.2, 79-104.

Haack, S. (1996) *Deviant Logic, Fuzzy Logic*, Chicago: University of Chicago Press.

Kosko, B. (1986) 'Fuzzy Cognitive Maps', *Int. J. Man-Machine Studies* **24**, 65-75.

Kosko, B. (1990) 'Fuzziness vs. Probability', *Int. J. General Systems* **17**, 211-240.

Pearl, J. (2000) *Causality*, Cambridge: Cambridge University Press.

Ragin, C. (2000) *Fuzzy-Set Social Science*, Chicago: University of Chicago Press.

Salmon, W.C. (1984) *Scientific Explanation and the Causal Structure of the World*, Princeton: Princeton University Press.

Sarkar, S. (2000) *Genetics and Reductionism*, Cambridge: Cambridge University Press.

Zadeh, L. (1965) 'Fuzzy Sets', *Information and Control* **8**, 338-353.

Zadeh, L. (1975) 'Fuzzy Logic and Approximate Reasoning', *Synthese* **30**, 407-425.