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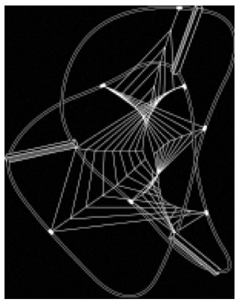

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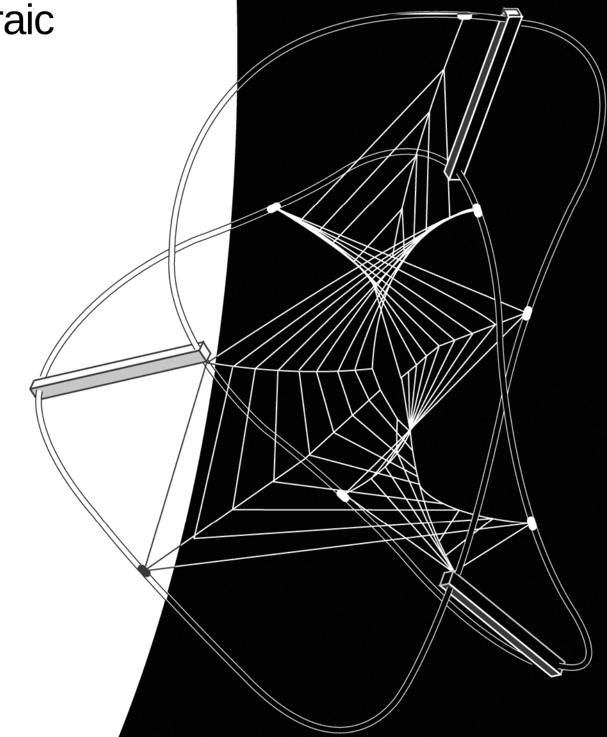

PP/08/10
ISSN 1750-3590

Einstein meets von Neumann: locality and operational independence in algebraic quantum field theory

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1 Main Claim

I will argue in this paper that Einstein and von Neumann meet in algebraic relativistic quantum field theory in the following metaphorical sense: algebraic quantum field theory was created in the late fifties/early sixties and was based on the theory of "rings of operators", which von Neumann established in 1935-1940 (partly in collaboration with J. Murray). In the years 1936-1949 Einstein criticized standard, non-relativistic quantum mechanics, arguing that it does not satisfy certain criteria that he regarded as necessary for any theory to be compatible with a field theoretical paradigm. I claim that algebraic quantum field theory does satisfy those criteria and hence that algebraic quantum field theory can be viewed as a theory in which the mathematical machinery created by von Neumann made it possible to express in a mathematically explicit manner the physical intuition about field theory formulated by Einstein.

The argument in favor of this claim has two components:

Historical: An interpretation of Einstein's (semi)formal wordings of his critique of non-relativistic quantum mechanics.

This interpretation results in mathematically explicit operational independence definitions, which, I claim, express independence properties of systems that are localized in causally disjoint spacetime regions. Einstein regarded these as necessary for a theory to comply with field theoretical principles.

Systematic: Presenting several propositions formulated in terms of algebraic quantum field theory that state that the operational independence conditions in question do in fact typically hold in algebraic quantum field theory.

*Work supported in part by the Hungarian Scientific Research Found (OTKA), contract number: K68043.

The structure of the paper is the following. After some historical comments in Section 2, I will quote from Einstein's famous 1948 *Dialectica* paper in Section 3 and I will isolate from the text three requirements Einstein thought a physical theory must satisfy (they are called **Spatio-temporality**, **Independence** and **Local operations**). After recalling some basic notions of algebraic quantum mechanics in Section 4, the main axioms of algebraic quantum field theory are recalled in Section 5, and it is argued here that **Spatio-temporality** is the very principle on which the key notion of algebraic quantum field theory (the definition of local net of observables) is based. Section 6 reviews several of the most important definitions in the hierarchy of independence concepts formulated in algebraic quantum mechanics and concludes with two propositions stating that the independence properties typically hold in algebraic quantum field theory. Section 7 interprets the **Local operations** requirement by identifying it with what is called "operational separability", definition of which is formulated in this section in terms of operations (understood as completely positive unit preserving linear maps). Linking operational separability to the independence condition called "operational independence", Section 7 concludes that the **Local operations** condition also is typically satisfied in algebraic quantum field theory. Section 8 raises the problem of relation of operational independence and operational separability in general (i.e. irrespectively of the quantum field theoretical context), and, distinguishing a stronger and a weaker version of operational separability, argues that the weaker version is strictly weaker than operational separability. I am aware that the historical aspect of the main claim is somewhat controversial because it rests on a particular interpretation of Einstein's wording of his criticism of standard, non-relativistic quantum mechanics. Section 9 will indicate other possible interpretations of Einstein's criticism and will qualify the main claim. The systematic part of the main claim seems to me meaningful however irrespective of its historical accuracy: The operational separability notions are intuitively-physically motivated and mathematically well-defined concepts, investigating them and their relation to other independence concepts raises non-trivial questions, showing the richness and beauty of algebraic quantum field theory.

2 Preliminary historical comments

Einstein and von Neumann played very different but crucial roles in establishing non-relativistic quantum mechanics: Einstein's Nobel prize-winning explanation of the photoelectric effect in 1905 was a decisive step in lending credibility to the quantum hypothesis, whereas von Neumann's three papers [43], [45], [44] and his subsequent book [46] clarified and summarized the mathematical foundations of quantum theory. In addition to their contribution to (mathematical) physics proper, both Einstein and von Neumann were deeply interested in interpretational-philosophical problems related to quantum mechanics: Einstein's dissatisfaction with quantum mechanics and his criticism of this theory as a complete description of physical reality is well-known and has been the subject of intensive scrutiny by historians and philosophers of physics. Von Neumann's foundational work also was deeply philosophical: his famous no-go theorem on hidden variables presented in [46], his idea of a non-classical (quantum) logic associated with quantum mechanics and the problems he had seen in this connection are classical examples of his philosophical

attitude and treatment of interpretational problems. [1]

Given their interest in philosophical-foundational issues related to quantum mechanics and the fact that from 1933 on both were members of the Institute for Advanced Study in Princeton, one would expect that they exchanged ideas and had discussions about the interpretation and foundations of quantum mechanics, especially around 1935-1936: this was the time when Einstein was working with Podolsky and Rosen on the EPR paper [14] and von Neumann was working on the theory of "rings of operators" (called today "von Neumann algebras") and on quantum logic [3] – both motivated by Hilbert space quantum mechanics. Curiously however Einstein and von Neumann did not seem to have discussions of the foundations of quantum mechanics. To be more precise, the only record I am aware of which shows specifically that Einstein and von Neumann did possibly talk about interpretational problems of quantum mechanics is von Neumann's letter to Schrödinger dated April 11, 1936, in which von Neumann writes:

> Einstein has kindly shown me your letter as well as a copy of the Pr. Cambr. Phil. Soc. manuscript. I feel rather more over-quoted than under-quoted and I feel that my merits in the subject are over-emphasized.
> (von Neumann to Schrödinger, April 11, 1936), [22][p. 211]

Von Neumann refers here to the second of Schrödinger's two papers on the problem of probabilistic correlations between spatially separated quantum systems [30], [31]. Einstein and Schrödinger corresponded about the EPR paper in the summer of 1935 (see Jammer's paper [18]) and Schrödinger's two papers were motivated by his correspondence with Einstein. Schrödinger was bothered by the EPR type correlations between spatially distant systems predicted by non-relativistic quantum mechanics and he seems to have thought that quantum field theory will also be problematic for this reason:

> Though in the mean time some progress seemed to have been made in the way of coping with this condition (quantum electrodynamics), there now appears to be a strong probability (as P. A. M. Dirac [Schrödinger's footnote: P.A.M. Dirac, Nature. 137 (1936), 298] has recently pointed out on a special occasion) that this progress is futile.
> [31][p. 451]

von Neumann did not share Schrödinger's concern:

> I think that the difficulties you hint at are "pseudo-problems". The "action at distance" in the case under consideration says only that even if there is no dynamical interaction between two systems (e.g. because they are far removed from each other), the systems can display statistical correlations. This is not at all specific for quantum mechanics, it happens classically as well.
> (von Neumann to Schrödinger, April 11, 1936) [22][p. 212]

To illustrate his point, von Neumann gives a simple example of a non-problematic correlation between spatially distant systems – the example shows that von Neumann regarded distant correlations non-problematic as long as one can give an explanation of them in

[1]Since the literature on both Einstein's and von Neumann's work on quantum mechanics is enormous, I make no attempt here to list even the most important works.

terms of common causes (see [23] for this interpretation of von Neumann's position). Whether the EPR correlations can be given an interpretation in terms of common causes is a subtle matter to which I return in Section 9 briefly; now I wish to point out that von Neumann reacted explicitly to Schrödinger's skeptical remark about the prospects of relativistic quantum field theory as well:

> And of course quantum electrodynamics proves that quantum mechanics and the special theory of relativity are compatible "philosophically" – quantum electrodynamic fails only because of the concrete form of Maxwell's equations in the vicinity of a charge.
> (von Neumann to Schrödinger, April 11, 1936), [22][p. 213]

So it seems that von Neumann thought that the real problem with relativistic quantum field theory was the presence of singularities caused by the assumption of pointlike charges and the related infinitely sharp localization of fields (both originating in classical field theory) rather than some irreconcilable "philosophical incompatibility" between quantum theory and principles of causality; however, von Neumann did not attempt to make the compatibility explicit by formulating postulates that a quantum field theory should satisfy in order to be acceptable "philosophically". This was done by Einstein in his critique of standard quantum mechanics as a complete theory.

3 Einstein's contrasting standard non-relativistic quantum mechanics with field theory in 1948

Einstein's dissatisfaction with quantum mechanics and his attempts to show that quantum mechanics is an incomplete theory is one of the best known and most analyzed facts in history and philosophy of quantum mechanics. Specifically, it is well known that Einstein gave his argument against the completeness of quantum mechanics (at least) four times after the publication of the EPR paper [14] in 1935. The first formulation is contained in his 1935 letter to Schrödinger which he wrote just a few weeks after the EPR paper had appeared (see ([18] and [17] for the details of the Einstein-Schrödinger correspondence)). This was followed by formulations in 1936 [12], 1948 [15] and 1949 [13]. All the formulations are informal and, while the core idea remains the same, they are slightly different and one can see a gradual shift towards what I claim is a formulation of several criteria that Einstein thought must be satisfied by a physical theory if it is to be compatible with a field theoretical paradigm. These criteria are most explicitly present in his 1948 *Dialectica* paper [15], where he writes:

> If one asks what is characteristic of the realm of physical ideas independently of the quantum theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, i.e. ideas are posited of things that claim a 'real existence' independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a spacetime continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these

things claim an existence independent of one another, insofar as these things 'lie in different parts of space'. Without such an assumption of mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four dimensional) space-elements the elementary things existing independently of one another that it takes as basic as well as the elementary laws it postulates for them.

For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the 'principle of local action', which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-)closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us."

[...]

Matters are different, however, if one seeks to hold on principle II – the autonomous existence of the real states of affairs present in two separated parts of space R_1 and R_2 – simultaneously with the principles of quantum mechanics. In our example the complete measurement on S_1 of course implies a physical interference which only effects the portion of space R_1. But such an interference cannot immediately influence the physically real in the distant portion of space R_2. From that it would follow that every measurement regarding S_2 which we are able to make on the basis of a complete measurement on S_1 must also hold for the system S_2 if, after all, no measurement whatsoever ensued on S_1. That would mean that for S_2 all statements that can be derived from the postulation of ψ_2 or ψ_2', etc. must hold simultaneously. This is naturally impossible, if ψ_2, ψ_2', are supposed to signify mutually distinct real states of affairs of S_2,...

[15] (translation taken from [17])

In the above passage Einstein formulates (informally) three requirements for a physical theory to be compatible with a field theoretical paradigm:

Spatio-temporality "... physical things [...] are conceived of as being arranged in a spacetime continuum..."

Independence "... essential for this arrangement of the things introduced in physics is that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space'."

Local operation "... an external influence on A has no *immediate* effect on B; this is known as the 'principle of local action'"; "... measurement on S_1 of course implies a physical interference which only effects the portion of space R_1. But such an interference cannot immediately influence the physically real in the distant portion of space R_2."

The above three requirements are not independent: **Independence** presupposes **Spatiotemporality** conceptually: only if physical things are assumed to be arranged in a spacetime continuum can one ask whether the things *so arranged* have the feature **Independence**; and it also is more or less clear that the **Local operations** requirement is an independence condition – independence of system S_2 of (measurement) operations carried out on system S_1.

It is true that standard (non-relativistic) Hilbert space quantum mechanics is *not* field theoretical in the sense that observables in non-relativistic quantum theory are *not* "conceived of as being arranged in a spacetime continuum": the observable quantities in quantum theory do *not* carry labels that would indicate their spatiotemporal location in a four dimensional spacetime continuum and; accordingly, quantum measurements and operations are also *not* conceived of in quantum mechanics as possessing spatiotemporal tags explicitly. Nor is Hilbert space quantum mechanics covariant with respect to a relativistic symmetry group. Thus quantum mechanics does *not* meet requirements of relativistic locality interpreted in the sense of a field theoretical paradigm. But algebraic quantum field theory does, or so I'll argue in the rest of this paper.

4 Some notions of algebraic quantum mechanics

In what follows, $\mathcal{A}$ denotes a unital C^*-algebra, $\mathcal{A}_1, \mathcal{A}_2$ are assumed to be C^*-subalgebras of $\mathcal{A}$ (with common unit). $\mathcal{N}$ denotes a von Neumann algebra; algebras $\mathcal{N}_1, \mathcal{N}_2$ are assumed to be von Neumann subalgebras of $\mathcal{N}$ (with common unit). $\mathcal{A}_1 \vee \mathcal{A}_2$ (respectively $\mathcal{N}_1 \vee \mathcal{N}_2$) denotes the C^*- (respectively von Neumann) algebra generated by $\mathcal{A}_1$ and $\mathcal{A}_2$ (respectively by $\mathcal{N}_1$ and $\mathcal{N}_2$). A W^*-algebra $\mathcal{N}$ is hyperfinite (or *approximately finite dimensional*) if there exist a series of finite dimensional full matrix algebras M_n ($n = 1, 2, \ldots$) such that $\cup_n M_n$ is weakly dense in $\mathcal{N}$. $\mathcal{B}(\mathcal{H})$ is the C^*-(and von Neumann) algebra formed by the set of *all* bounded operators on Hilbert space $\mathcal{H}$. $\mathcal{B}(\mathcal{H})$ is hyperfinite if $\mathcal{H}$ is a separable Hilbert space. For von Neumann algebra $\mathcal{N} \subseteq \mathcal{B}(\mathcal{H})$, $\mathcal{N}'$ stands for the commutant of $\mathcal{N}$ in $\mathcal{B}(\mathcal{H})$. $S(\mathcal{A})$ is the state space of C^*-algebra $\mathcal{A}$. The selfadjoint elements in a C^*-algebra are interpreted as representatives of physical observables, the elements of the state space $S(\mathcal{A})$ represent physical states. (For the operator algebraic notions see [19], [5] or [4].)

In what follows, T will denote a completely positive (CP) map on a C^*-algebra $\mathcal{A}$, such a T will also be assumed to preserve the identity: $T(I) = I$ (where I is the unit of $\mathcal{A}$). A (unit preserving) CP map is called a (non-selective) *operation.* An operation T on a von Neumann algebra $\mathcal{N}$ is called a *normal* operation if it is σ weakly continuous.

The dual T^* of an operation defined by

$$S(\mathcal{A}) \ni \phi \mapsto T^*\phi \doteq \phi \circ T \in S(\mathcal{A})$$

maps the state space $S(\mathcal{A})$ into itself. If T is a normal operation on the von Neumann algebra $\mathcal{N}$, then T^* takes normal states into normal states.

Operations are the mathematical representatives of physical operations: physical processes that take place as a result of physical interactions with the system. For a detailed description and physical interpretation of the notion of operation see [20].

Remark 1. In sharp contrast to states, operations defined on a subalgebra of an arbitrary C^*-algebra are *not*, in general, extendible to an operation on the larger algebra [2]. A C^*-algebra $\mathcal{B}$ is said to be *injective* if for any C^*-algebras $\mathcal{A}_1 \subset \mathcal{A}$ every completely positive unit preserving linear map $T_1 : \mathcal{A}_1 \to \mathcal{B}$ has an extension to a completely positive unit preserving linear map $T : \mathcal{A} \to \mathcal{B}$. A von Neumann algebra is injective (by definition) if it is injective as a C^*-algebra. It was shown in [2] that $\mathcal{B}(\mathcal{H})$ is injective. Hyperfiniteness of a von Neumann algebra entails injectivity in general, and a von Neumann algebra acting on a separable Hilbert space is injective if and only if it is hyperfinite [9], [10][Theorem 6] – this is why injectivity of the double cone algebras in algebraic quantum field theory (Proposition 4) will be important.

A classic result characterizing operations is

Proposition 1 (Stinespring's Representation Theorem). *$T\colon \mathcal{A} \to \mathcal{B}(\mathcal{H})$ is a completely positive linear map from C^*-algebra $\mathcal{A}$ into $\mathcal{B}(\mathcal{H})$ iff it has the form*

$$T(X) = V^*\pi(X)V \qquad\qquad X \in \mathcal{A}$$

where $\pi\colon \mathcal{A} \to \mathcal{B}(\mathcal{K})$ is a representation of $\mathcal{A}$ on Hilbert space $\mathcal{K}$ and $V\colon \mathcal{H} \to \mathcal{K}$ is a bounded linear map. If $\mathcal{A}$ is a von Neumann algebra and T is normal, then π is a normal representation.

A corollary of Stinespring theorem is

Proposition 2 (Kraus' Representation Theorem). *$T\colon \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is a normal operation iff there exists bounded operators W_i on $\mathcal{H}$ such that*

$$T(X) = \sum_i W_i^* X W_i \qquad\qquad \sum_i W_i^* W_i = I \tag{1}$$

The infinite sums are taken to converge in the σ-weak topology. W_i are sometimes called "Kraus operators".

It is important in Stinespring's theorem (and hence also in Kraus' theorem) that T takes its value in the set of all bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space. Stinespring's theorem does not hold for an arbitrary von Neumann algebra in place of $\mathcal{B}(\mathcal{H})$ because if it did then this would entail that operations defined on subalgebras are always extendible from the subalgebra to the superalgebra, which however is not the case (cf. Remark 1). To put it differently: Kraus' representation theorem does not hold for an arbitrary von Neumann algebra; hence general operations on a von Neumann algebra are *not* of the form (1).

A special case of operations are *measurements*: If one measures a (possibly unbounded) observable Q defined on the Hilbert space $\mathcal{H}$ with purely discrete spectrum λ_i and corresponding spectral projections P_i, then the "projection postulate" is described by the operation T_{proj} defined by

$$\mathcal{N} \ni X \mapsto T_{proj}(X) = \sum_i P_i X P_i \tag{2}$$

T_{proj} is a normal operation from $\mathcal{B}(\mathcal{H})$ into the commutative von Neumann algebra $\{P_i\}'$ consisting of operators that commute with each P_i:

$$\{P_i\}' = \{X \in \mathcal{B}(\mathcal{H}) \ : \ XP_i = P_iX \text{ for all } i\}$$

It is known that the projection postulate has limited applicability: Not every interaction with (operation on) a quantum system can be described by a T_{proj} of the form (2) (the Kraus operators need not be projections; an operation might not even have a Kraus representation at all). For instance, if the observable to be measured does not have a discrete spectrum then one cannot directly generalize formula (2) to obtain a CP map [11]. But one can generalize some of the characteristic features of T_{proj}:

A positive, linear, unit preserving map T from C^*-algebra $\mathcal{A}$ onto a C^*-subalgebra $\mathcal{A}_0$ of $\mathcal{A}$ whose restriction to $\mathcal{A}_0$ is equal to the identity map is called a *conditional expectation* from $\mathcal{A}$ onto $\mathcal{A}_0$. Such a map will be denoted by T_c. Conditional expectations are completely positive.

If for a state φ on $\mathcal{A}$, the T_c conditional expectation also preserves φ, i.e. $\varphi(T_c(X)) = \varphi(X)$ for all $X \in \mathcal{A}$, then T_c is called a *φ-preserving conditional expectation* from $\mathcal{A}$ onto $\mathcal{A}_0$, and it will be denoted by T_c^φ.

It is known that for a given $\mathcal{A}$, $\mathcal{A}_0$ and φ, a φ-preserving conditional expectation from $\mathcal{A}$ onto $\mathcal{A}_0$ does not necessarily exist. But if φ is a faithful normal state on von Neumann algebra $\mathcal{N}$ then there always exists a φ-preserving CP map T^φ from $\mathcal{N}$ into any subalgebra $\mathcal{N}_0$: the so-called Accardi-Cecchini φ-conditional expectation [1] (note that T^φ is *not* a conditional expectation, i.e. it is not a projection to $\mathcal{N}_0$; so "φ-preserving conditional expectation" and "φ-conditional expectation" are different concepts, although the terms are deceptively close).

5 Algebraic quantum field theory

The basic idea of algebraic quantum field theory is precisely what Einstein requires in **Spatio-temporality**: "... physical things [...] are conceived of as being arranged in a spacetime continuum...": In algebraic quantum field theory, observables, interpreted as selfadjoint parts of C^*-algebras, are assumed to be localized in regions V of the Minkowski spacetime M. The basic object in the mathematical model of a quantum field is thus the association of a C^*-algebra $\mathcal{A}(V)$ to (open, bounded) regions V of M, and *all* the physical content of the theory is assumed to be contained in the assignment $V \mapsto \mathcal{A}(V)$. In particular, relativistic covariance of the theory also is expressed in terms of the *net of algebras* $(\{A(V)\}, V \subset M)$, the net of algebras of local observables.

The net $(\{A(V)\}, V \subset M)$ is specified by imposing on it physically motivated postulates. Below we list these postulates.

(i) Isotony: $\mathcal{A}(V_1)$ is a C^*-subalgebra (with common unit) of $\mathcal{A}(V_2)$ if $V_1 \subseteq V_2$

(ii) Local commutativity (also called Einstein causality or microcausality): $\mathcal{A}(V_1)$ commutes with $\mathcal{A}(V_2)$ if V_1 and V_2 are spacelike separated.

Let

$$\mathcal{A}_0 \equiv \cup_V \mathcal{A}(V)$$

then $\mathcal{A}_0$ is a normed $*$-algebra, completion of which (in norm) is a C^*-algebra, called the *quasilocal algebra* determined by the net $(\{A(V)\}, V \subset M)$.

(iii) Relativistic covariance: there exists a continuous representation α of the identity-connected component of the Poincaré group $\mathcal{P}$ by automorphisms $\alpha(g)$ on $\mathcal{A}$ such

that

$$\alpha(g)\mathcal{A}(V) = \mathcal{A}(gV) \tag{3}$$

for every $g \in \mathcal{P}$ and for every V.

(iv) Vacuum: It also is postulated that there exists at least one physical representation of the algebra $\mathcal{A}$, that is to say, it is required that there exist a Poincaré invariant state ϕ_0 (vacuum) such that the spectrum condition ((v) below) is fulfilled in the corresponding cyclic (GNS) representation $(\mathcal{H}_{\phi_0}, \Omega_{\phi_0}, \pi_{\phi_0})$. In this representation of the quasilocal algebra the representation α is given as a unitary representation, and there exist the generators P_i, $(i = 0, 1, 2, 3)$ of the translation subgroup of the Poincaré group $\mathcal{P}$. The spectrum condition is formulated in terms of these generators as

(v) Spectrum condition:

$$P_0 \geq 0, \qquad P_0^2 - P_1^2 - P_2^2 - P_3^2 \geq 0 \tag{4}$$

Given a state ϕ one can consider the net in the representation π_ϕ determined by ϕ. If the particular state ϕ is not important, then the local von Neumann algebras $\pi_\phi(\mathcal{A}(V))''$ will be denoted by $\mathcal{N}(V)$.

It is a remarkable feature of the above axioms that (under some additional assumptions) they are very rich in consequences: they entail a number of non-trivial features of the net. We mention two sorts of consequences that will be referred to in what follows: one is the Reeh-Schlieder theorem:

Proposition 3 (Reeh-Schlieder Theorem)**.** *The vacuum state ϕ_0 (more generally, any state of bounded energy) is faithful on local algebras $\mathcal{A}(V)$ pertaining to open bounded spacetime regions V.*

The other type of result concerns the type and structure of certain local algebras. To state the proposition in this direction recall that a double cone region $D(x, y)$ of the Minkowski spacetime determined by points $x, y \in M$ such that y is in the forward light cone of x is, by definition, the interior of the intersection of the forward light cone of x with the backward light cone of y. A general double cone is denoted by D.

Proposition 4 ([6], [16] p. 225)**.** *The local von Neumann algebras $\mathcal{N}(D)$ associated with double cones D are hyperfinite.*

6 Independence

Einstein's **Independence** requirement demands that if V_1 and V_2 are spacelike separated regions then the systems represented by algebras $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ must be "independent". The **Local commutativity** postulate is such an independence requirement; however, as it turns out, there are many other, stronger, nonequivalent (but also non-independent) concepts of independence that one can formulate for two C^*-and W^*-algebras. Below we list a few from the rich hierarchy, the list is far from being complete (for an extensive review of the independence concepts see [33]).

Definition 1. A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{A}$ is called C^*-independent if for any state ϕ_1 on $\mathcal{A}_1$ and for any state ϕ_2 on $\mathcal{A}_2$ there exists a state ϕ on $\mathcal{A}$ such that

$$\begin{aligned} \phi(X) &= \phi_1(X) \quad \text{for any} \quad X \in \mathcal{A}_1 \\ \phi(Y) &= \phi_2(Y) \quad \text{for any} \quad Y \in \mathcal{A}_2 \end{aligned}$$

C^*-independence expresses that any two partial states (states on $\mathcal{A}_1$ and $\mathcal{A}_2$) are copossible as states of the larger system $\mathcal{A}$. The next independence condition is a strengthening of C^*-independence:

Definition 2. A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of a C^*-algebra $\mathcal{A}$ is called *C^*-independent in the product sense* if the map η defined by

$$\eta\colon \mathcal{A}_1 \vee \mathcal{A}_2 \rightarrow \mathcal{A}_1 \otimes \mathcal{A}_2 \tag{5}$$

$$\eta(XY) \doteq X \otimes Y \tag{6}$$

extends to an C^*-isomorphism of $\mathcal{A}_1 \vee \mathcal{A}_2$ with $\mathcal{A}_1 \otimes \mathcal{A}_2$, where $\mathcal{A}_1 \otimes \mathcal{A}_2$ denotes the tensor product of $\mathcal{A}_1$ and $\mathcal{A}_2$ with the minimal C^*-norm.

If the C^*-algebras happen to be von Neumann algebras then states can have stronger continuity properties and this makes it possible to formulate independence notions that fit the von Neumann algebra category more naturally and are in analogy of the above independence concepts:

Definition 3. Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ are called W^*-independent if for any *normal* state ϕ_1 on $\mathcal{N}_1$ and for any *normal* state ϕ_2 on $\mathcal{N}_2$ there exists a *normal* state ϕ on $\mathcal{N}$ such that

$$\begin{aligned} \phi(X) &= \phi_1(X) \quad \text{for any} \quad X \in \mathcal{N}_1 \\ \phi(Y) &= \phi_2(Y) \quad \text{for any} \quad Y \in \mathcal{N}_2 \end{aligned}$$

The analogue of Definition 2 in the von Neumann algebra setting is:

Definition 4. Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ are called W^*-independent in the product sense if for any normal state ϕ_1 on $\mathcal{N}_1$ and for any normal state ϕ_2 on $\mathcal{N}_2$ there exists a normal product state ϕ on $\mathcal{N}$, i.e. a normal state ϕ on $\mathcal{M}$ such that

$$\phi(XY) = \phi_1(X)\phi_2(Y) \quad \text{for any} \quad X \in \mathcal{N}_1, Y \in \mathcal{N}_2$$

The following notion is an even stronger independence property:

Definition 5. Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ are called W^*-independent in the spatial product sense if there exist faithful normal representations $(\pi_1, \mathcal{H}_1)$ of $\mathcal{N}_1$ and $(\pi_2, \mathcal{H}_2)$ of $\mathcal{N}_2$ such that the map

$$\mathcal{N} \ni XY \mapsto \pi_1(X) \otimes \pi_2(Y) \qquad X \in \mathcal{N}_1 \quad Y \in \mathcal{N}_2$$

extends to a spatial isomorphism of $\mathcal{N}_1 \vee \mathcal{N}_2$ with $\pi(\mathcal{N}_1) \otimes \pi(\mathcal{N}_2)$.

If $\mathcal{N}_1, \mathcal{N}_2$ are commuting von Neumann subalgebras of von Neumann algebra $\mathcal{N}$ then they are W^*-independent in the spatial product sense if and only if they satisfy the following so called *split property*:

Definition 6. $\mathcal{N}_1, \mathcal{N}_2$ have the split property if there exists a type I factor $\mathcal{N}$ such that

$$\mathcal{N}_1 \subset \mathcal{N} \subset \mathcal{N}_2'$$

The next four definitions formulate an idea of operational independence, these definitions were proposed recently in [28] and they are closely related to the **Local operation** requirement as we will see.

Definition 7. A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{A}$ is operationally C^*-independent in $\mathcal{A}$ if any two operations on $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively, have a joint extension to an operation on $\mathcal{A}$; i.e. if for any two completely positive unit preserving maps

$$\begin{aligned} T_1 &: \quad \mathcal{A}_1 \to \mathcal{A}_1 \\ T_2 &: \quad \mathcal{A}_2 \to \mathcal{A}_2 \end{aligned}$$

there exists a completely positive unit preserving map

$$T \colon \mathcal{A} \to \mathcal{A}$$

such that

$$\begin{aligned} T(X) &= T_1(X) \qquad \text{for all } X \in \mathcal{A}_1 \\ T(Y) &= T_2(Y) \qquad \text{for all } Y \in \mathcal{A}_2 \end{aligned}$$

The following is a natural strengthening of operational independence:

Definition 8. A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{A}$ is operationally C^*-independent in $\mathcal{A}$ in the product sense if any two operations on $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively, have a joint extension to an operation on $\mathcal{A}$ that factorize across the algebras $(\mathcal{A}_1, \mathcal{A}_2)$ in the sense that

$$T(XY) = T(X)T(Y) \qquad X \in \mathcal{A}_1,\ Y \in \mathcal{A}_2 \tag{7}$$

The corresponding von Neumann algebra versions of the above two operational independence notions are:

Definition 9. A pair $(\mathcal{N}_1, \mathcal{N}_2)$ of von Neumann subalgebras of a von Neumann algebra $\mathcal{N}$ is operationally W^*-independent in $\mathcal{N}$ if any two *normal* operations on $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively, have a joint extension to a *normal* operation on $\mathcal{N}$.

Definition 10. If normal extensions exist that factorize across the pair $(\mathcal{N}_1, \mathcal{N}_2)$ in the manner of eq. (7) then the pair $(\mathcal{N}_1, \mathcal{N}_2)$ is called operationally W^*-independent in $\mathcal{N}$ in the product sense.

Operational C^*-independence expresses that any operation (procedure, state preparation etc.) on system S_1 is co-possible with any such operation on system S_2 – if these systems are represented by C^*-algebras (similarly for W^*-algebras). For a more detailed motivation of operational independence see [28].

The characterization of the interdependence relations of the above independence notions is a highly non-trivial task and there are still a number of open problems related to this issue. We refer to [33] for a review of the main results known by 1990 on this problem. What is important from the perspective of the current paper is that local algebras in algebraic quantum field theory pertaining to spacelike separated spacetime regions *typically* do satisfy the above independence notions. We just recall, in the form of two propositions, the status of the above independence properties in algebraic quantum field theory:

Proposition 5. *If $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are two von Neumann algebras associated with* strictly *spacelike separated double cone regions D_1 and D_2 in a local net of von Neumann algebras satisfying the standard axioms then $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are* typically *independent in the sense of* all *of the Definitions 1.–10., where operational C^*-independence of $\mathcal{N}(D_1), \mathcal{N}(D_2)$ (and operational C^*-independence in the product sense) is understood to hold in the von Neumann algebra $\mathcal{N}(D)$ with D being a double cone containing the double cones D_1 and D_2 and operational W^*-independence (and operational W^*-independence in the product sense) is understood to hold in $\mathcal{N}(D_1)\overline{\otimes}\mathcal{N}(D_2)$.*

Remark 2. Note that hyperfiniteness of the double cone algebras (Proposition 4) is crucial in the above claim that operational C^*-independence of $\mathcal{N}(D_1), \mathcal{N}(D_2)$ holds in $\mathcal{N}(D)$, and it is *not* known whether $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are also operationally W^*-independent in $\mathcal{N}(D)$ – although they are operationally W^*-independent in $\mathcal{N}(D_1)\overline{\otimes}\mathcal{N}(D_2)$. The reason why this latter fact does not entail the former is that while operations on $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are extendible to $\mathcal{N}(D)$ by injectivity of $\mathcal{N}(D)$ ensured by hyperfiniteness of $\mathcal{N}(D)$ (Remark 1) it is unclear whether extendibility obtains under the additional requirement of the operations being normal.

"Typically" in Proposition 5 means: "in all physically non-pathological models" and possibly with a spacelike distance of the double cones that is above a certain threshold. In those physically non-pathological models the algebras associated with strictly spacelike separated double cones are split (Definition 6), which entails all the other independence properties. (See [35] for a detailed, non-technical discussion of the split property and for further discussion of what "(non-)pathological" means in this context.)

Proposition 6. *If $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are two von Neumann algebras associated with spacelike separated* tangent *double cone regions D_1 and D_2 in a local net of von Neumann algebras satisfying the standard axioms then $\mathcal{N}(D_1), \mathcal{N}(D_2)$ are independent in the sense of Definitions 1, and 3 but* not *independent in the sense of Definitions 4, 5, 8 and 10. It is* not *known whether operational C^*-or W^*-independence is violated by these von Neumann algebras.*

7 Local operations

I what follows, V_1, V_2 and V are assumed to be open bounded spacetime regions, with V_1 and V_2 spacelike separated and $V_1, V_2 \subseteq V$. Let T be an operation on $\mathcal{A}(V)$ and ϕ be a state on $\mathcal{A}(V)$. Then

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T) \tag{8}$$

is called a *local system*.

Given such a local system, let ϕ_1 and ϕ_2 be the restrictions of ϕ to $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$, respectively. Suppose T_1 is an operation on $\mathcal{A}(V_1)$. Carrying out this operation changes the state ϕ_1 into $T_1^*\phi_1$. By the requirement of isotony $\mathcal{A}(V_1)$ is a subalgebra of $\mathcal{A}(V)$, so the operation T_1 is an operation that is carried out on the elements of $\mathcal{A}(V)$ that are localized in region $V_1 \subset V$. Assume that T is an operation on $\mathcal{A}(V)$ that is an extension of T_1 from $\mathcal{A}(V_1)$ to $\mathcal{A}(V)$.[2] Then T changes the state ϕ into $T^*\phi$ and, since T is an extension of T_1, the restriction of $T^*\phi$ to $\mathcal{A}(V_1)$ coincides with $T_1^*\phi$. Since V_1 and V_2 are spacelike separated hence causally independent regions, one would like to have the extension T of T_1 be such that the change $\phi \mapsto T^*\phi$ caused by the operation T in state ϕ is restricted to $\mathcal{A}(V_1)$; that is to say, causally well behaving systems are the ones for which $T^*\phi(X) = \phi_2(X)$ for every $X \in \mathcal{A}(V_2)$. The next definition of *operational separatedness* formulates this idea precisely.

Definition 11. The local system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$ with an operation T on $\mathcal{A}(V)$ is defined to be *operationally separated* if both of the following two conditions are satisfied:

1. If T is an extension of an operation on $\mathcal{A}(V_1)$ then the operation conditioned state $T^*\phi = \phi \circ T$ coincides with ϕ on $\mathcal{A}(V_2)$, i.e.

$$\phi(T(A)) = \phi(A) \quad \text{for all} \quad A \in \mathcal{A}(V_2) \tag{9}$$

2. If T is an extension of an operation on $\mathcal{A}(V_2)$ then the operation conditioned state $T^*\phi = \phi \circ T$ coincides with ϕ on $\mathcal{A}(V_1)$, i.e.

$$\phi(T(A)) = \phi(A) \quad \text{for all} \quad A \in \mathcal{A}(V_1) \tag{10}$$

Given a local system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T_{proj})$ with the operation T_{proj} defined by the projection postulate (eq. (2)), the local commutativity requirement of algebraic quantum field theory entails that if the operation T_{proj} is defined by projections P_i in $\mathcal{A}(V_1)$ then T is the identity map on $\mathcal{A}(V_2)$ (and if T is defined by projections P_i in $\mathcal{A}(V_2)$ then it is the identity on $\mathcal{A}(V_1)$). This is well known and is sometimes referred to as the "No-signaling Theorem" and is typically cited as the motivation for the local commutativity (Einstein causality) axiom in algebraic quantum field theory. We state it explicitly:

Proposition 7 (No-signaling Theorem)**.** *The local system*

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T_{proj})$$

with T_{proj} describing the projection postulate (2) is operationally separated for every *state ϕ.*

Not every interaction with (operation on) a quantum system can be described by a T_{proj} of the form (2) however (see Section 4); hence it is not obvious that every local system in algebraic quantum field theory is operationally separated. In fact, one can show that operational separatedness fails in algebraic quantum field theory :

[2] Since operations defined on C^*-subalgebras of C^*-algebras are not necessarily extendible from the subalgebra to the larger algebra (see Remark 1), it is not obvious that an operation on $\mathcal{A}(V_i)$ $(i = 1, 2)$ can be extended to $\mathcal{A}(V)$; consequently the *assumption* here (and below) that T represents T_1 is not a redundant one.

Proposition 8 ([29]). *There exists a local system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$ which is not operationally separated.*

The above proposition shows that a No-signaling Theorem does not hold for spacelike separated local algebras for arbitrary operations. Furthermore, the failure is not exceptional or atypical because non-operationally separated local systems are not rare: as the argument in [29] shows, for every locally faithful state ϕ there exists an operationally non-separated local system and such states are very typical by the Reeh-Schlieder Theorem (Proposition 3). Thus it would seem that the **Local operations** requirement is violated in algebraic quantum field theory – contrary to the claim in the introductory section. But this conclusion would be too quick. One can argue [29], [24], [25] that the mere existence of operationally not separated local systems should not be interpreted as a genuine incompatibility of algebraic quantum field theory with the **Local operations** requirement because one can not expect a theory such as algebraic quantum field theory to exclude causally non-well-behaving local systems necessarily. But it is reasonable to demand that algebraic quantum field theory allow a locally equivalent and causally acceptable description of an operationally not separated local system. In other words, one can say that it may happen that the possible causal bad behavior of the local system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$ is due to the non-relativistically conforming choice of the operation T on $\mathcal{A}(V)$ representing an operation T_1 carried out in $\mathcal{A}(V_1)$, say, and there may exist another operation T' on $\mathcal{A}(V)$ that has the same effect on $\mathcal{A}(V_1)$ as that of T, (i.e. $T'(X) = T(X)$ for all $X \in \mathcal{A}(V_1)$) and such that the system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T')$ is causally well-behaving. This idea of reducibility of operational non-separatedness is formulated explicitly in the form of the following weakening of Definition 11 (see [29]):

Definition 12. The local system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$ is called *operationally C^*-separable* if it is operationally separated in the sense of Definition 11, or, if it is not operationally separated and T is an extension of an operation in either $\mathcal{A}(V_1)$ or in $\mathcal{A}(V_2)$ then the following is true:

1. If T is an extension of an operation in $\mathcal{A}(V_1)$, then there exists an operation $T'\colon \mathcal{A}(V) \to \mathcal{A}(V)$ such that $T'(X) = T(X)$ for all $X \in \mathcal{A}(V_1)$ and such that the system
$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T')$$
is operationally separated.

2. If T is an extension of an operation in $\mathcal{A}(V_2)$, then there exists an operation $T'\colon \mathcal{A}(V) \to \mathcal{A}(V)$ such that $T'(X) = T(X)$ for all $X \in \mathcal{A}(V_2)$ and such that the system
$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T')$$
is operationally separated.

Operational W^*-separability is defined analogously, by requiring the operations and the state ϕ to be normal.

Einstein's requirement of **Local operations** can now be interpreted as the requirement that local systems in algebraic quantum field theory should be operationally C^*-and W^*-separable in the sense of Definition 12. To answer the question of whether local systems in algebraic quantum field theory are operationally separable one can relate operational separability to operational independence:

Proposition 9 (Redei and Valente, [29]).

- *If the pair $(\mathcal{A}(V_1), \mathcal{A}(V_2))$ is operationally C^*-independent in $\mathcal{A}(V)$ then for every ϕ and every T the system $(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$ is operationally C^*-separable.*

- *If the pair $(\mathcal{N}(V_1), \mathcal{N}(V_2))$ is operationally W^*-independent in $\mathcal{N}(V)$ then for every normal state ϕ and every normal T the system $(\mathcal{N}(V), \mathcal{N}(V_1), \mathcal{N}(V_2), \phi, T)$ is operationally W^*-separable.*

Since operational C^*-independence in $\mathcal{A}(V)$ does hold for local algebras $\mathcal{A}(D_1)$ and $\mathcal{A}(D_2)$ associated with strictly spacelike separated double cone regions D_1, D_2, and double cone $D \supset D_1, D_2$ (Proposition 5), one concludes that algebraic quantum field theory *typically* satisfies the **Local operations** requirement – at least in the C^*-sense. As it was remarked (Remark 2) it is not clear at this point whether operational W^*-independence of the pair $(\mathcal{N}(D_1), \mathcal{N}(D_2))$ *in* $\mathcal{N}(D)$ also holds; so one cannot yet conclude in full generality that the **Local operations** condition also holds in algebraic quantum field theory typically but it should be clear from these results that algebraic quantum field theory is a theory that displays exactly the features that Einstein thought were necessary for a physical theory to be compatible with a field theoretical paradigm.

8 Are operational separability and operational independence equivalent?

Operational C^*-and W^*-independence was defined in Section 6 for a general pair of operator algebras, independently of algebraic quantum field theory. While in the previous section the notion of operational separability was defined for local systems in algebraic quantum field theory, it is clear that operational separability also can be defined for a general pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-or W^*-algebras: Keeping in mind that the state space of a C^*-algebra (and the normal state space of a W^*-algebra) is separating and that states (also: normal states) defined on subalgebras are always extendible, it is clear that the definition below is the reformulation of Definition 12 in terms of general algebras:

Definition 13. The pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{A}$ is operationally C^*-separable in $\mathcal{A}$ if every operation T_1 that has an extension to $\mathcal{A}$, also has an extension T' which is the identity map on $\mathcal{A}_2$, and every T_2 that has an extension to $\mathcal{A}$, also has an extension T' which is the identity map on $\mathcal{A}_1$. (Operational W^*-separability is defined similarly by assuming the algebras to be W^*-algebras and requiring the operations to be normal.)

The argument in [29] leading to Proposition 9 applies to the general case, so one has

Proposition 10. *Let $\mathcal{A}_1, \mathcal{A}_2$ be C^*-subalgebras of C^*-algebra $\mathcal{A}$ and $\mathcal{N}_1, \mathcal{N}_2$ be W^*-subalgebras of W^*-algebra $\mathcal{N}$. Then operational C^*-independence of $\mathcal{A}_1, \mathcal{A}_2$ in $\mathcal{A}$ entails operational C^*-separability of $\mathcal{A}_1, \mathcal{A}_2$ in $\mathcal{A}$, and operational W^*-independence of $\mathcal{N}_1, \mathcal{N}_2$ in $\mathcal{N}$ entails operational W^*-separability of $\mathcal{N}_1, \mathcal{N}_2$ in $\mathcal{N}$.*

While no counterexample is known, I conjecture that the converse of Proposition 10 does not hold: Operational C^*-and W^*-separability seem strictly weaker than operational C^*-and W^*-independence. To see why, consider the following strengthening of the definition of operational separability:

Definition 14. The pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{A}$ is *strongly* operationally C^*-separable in $\mathcal{A}$ if every operation T_1 has an extension T' which is the identity map on $\mathcal{A}_2$, and every T_2 has an extension T' which is the identity map on $\mathcal{A}_1$. (Strong operational W^*-separability is defined similarly by assuming the algebras to be W^*-algebras and requiring the operations to be normal.)

The difference between Definitions 14 and 13 is that strong operational separability requires that all operations on the subalgebras have extensions to the superalgebra whereas Definition 13 does not require this. Since operations are not extendible in general (Remark 1), requiring the existence of extensions in Definition 14 is a highly non-trivial demand and so strong operational separability seems much stronger than operational separability. In fact, it is easy to see that *strong* operational C^*-separability is *equivalent* to operational C^*-independence and that strong operational W^*-separability also is equivalent to operational W^*-independence (see [25] for details).

Since there are natural subclasses of operations, such as "measurements" and conditional expectations, one can further distinguish specific instances of the concepts of operational independence and operational separability by narrowing the admissible operations to these subclasses. One obtains this way the notions of operational C^*-and W^*-independence in the sense of measurements and conditional expectations and operational C^*-and W^*-separability in the sense of measurements and conditional expectations (see the paper [25] for details). The result is a hierarchy of operational independence and operational separability concepts with non-trivial logical interdependencies about which virtually nothing is known at this point.

9 Closing comments

The main message of this paper is that algebraic quantum field theory satisfies the criteria Einstein formulated in his critique of standard quantum mechanics as necessary for a physical theory to be compatible with a field theoretical paradigm. Here I wish to qualify this claim.

The first qualification is emphasizing a trivial point: The conditions Einstein required are just necessary but not sufficient. No claim is made here that satisfying the three requirements (**Spatio-temporality**, **Independence** and **Local operations**) is *sufficient* to accept a theory as field theory compatible. For instance, the time evolution of the system also must respect the causal structure of the spacetime but constraints on the dynamics are independent of the three other conditions discussed here.

The second qualification is more substantial: We have seen that both the **Independence** and the **Local operations** requirements can be given different formulations and that the answer to the question of whether an independence condition holds or not depends sensitively on the particular specification of the independence notion. Certain independence conditions (such as W^*-independence in the product sense, the split property and operational W^*-independence in the product sense – see Proposition 6) do *not* hold for certain causally disjoint local systems and it is not entirely clear how to interpret this fact.

One also can take the position that the "independence" Einstein refers to in the *Dialectica* paper is to be taken as lack of probabilistic correlation, (especially lack of EPR correlation, i.e. absence of entanglement) between casually disjoint local systems.

This is the interpretation Howard [17] and Clifton and Halvorson [8] subscribe to. Under this interpretation one must conclude (as Clifton and Halvorson do [8]) that, ironically, algebraic quantum field theory fares even *worse* than non-relativistic quantum mechanics because entanglement is even more endemic in algebraic quantum field theory than in non-relativistic quantum mechanics (Bell's inequality is violated even more dramatically in algebraic quantum field theory than in non non-relativistic quantum mechanics, see the papers [36], [37] [38] [39] [40] [32] [34], [7].) But one can argue, as von Neumann did in his reply to Schrödinger's entanglement papers that correlations between spacelike separated local systems does not in and by itself make algebraic quantum field theory unacceptable from a field theoretical point of view – as long as it is possible, in principle at least, to give a causal explanation (in terms of local common causes) of the spacelike correlations (see the paper [23] for an analysis of von Neumann's position from this perspective). Whether such local common cause explanations of the spacelike correlations predicted by algebraic quantum field theory is possible within the framework of algebraic quantum field theory, i.e. whether algebraic quantum field theory complies with Reichenbach's Common Cause Principle, is still an open question, only unsatisfactory results are known on the problem: one can show that there exists common causes for correlations between projections lying in von Neumann algebras associated with spacelike separated spacetime regions but the common causes are known to be localized in the *union* rather than in the *intersection* of the backward light cones of the regions that contain the correlated projections (see the papers [21], [26] and [27] for these results).

Yet another position one can take is that Proposition 8 *does* entail that the **Local operations** requirement is violated in algebraic quantum field theory hence algebraic quantum field theory is not really compatible with the field theoretical paradigm after all; that is to say, one can regard the definition of operational C^*-and W^*-separability (Definition 12) too weak. It would be interesting to explore how these definitions can be strengthened and to see whether physically well-motivated stronger definitions of operational separability are satisfied in algebraic quantum field theory.

References

[1] L. Accardi and C. Cecchini. Conditional expectations in von Neumann algebras and a theorem of Takesaki. *Journal of Functional Analysis*, 45(245-273), 1982.

[2] W. Arveson. Subalgebras of C^*-algebras. *Acta Mathematica*, 123:141–224, 1969.

[3] G.D. Birkhoff and J. von Neumann. The logic of quantum mechanics. *Annals of Mathematics*, 37:823–843, 1936. Reprinted in [41], No. 7.

[4] B. Blackadar. *Operator Algebras: Theory of C*-Algebras and von Neumann Algebras.* Encyclopaedia of Mathematical Sciences. Springer, 1. edition, 2005.

[5] O. Bratteli and D.W. Robinson. *Operator Algebras and Quantum Statistical Mechanics*, volume I. C^*-algebras and von Neumann algebras. Springer Verlag, Berlin, Heidelberg and New York, 1979.

[6] D. Buchholz, C. D'Antoni, and K. Fredenhagen. The universal structure of local algebras. *Commununications in Mathematical Physics*, 111:123–135, 1987.

[7] R. Clifton and H. Halvorson. Generic Bell correlation between arbitrary local algebras in quantum field theory. *Journal of Mathematical Physics*, 41:1711–1717, 2000.

[8] R. Clifton and H. Halvorson. Entanglement and open systems in algebraic quantum field theory. *Studies in History and Philosophy of Modern Physics*, 32:1–31, 2001.

[9] A. Connes. Classification of injective factors. Cases II_1, II_∞, III_λ, $\lambda \neq 1$. *The Annals of Mathematics*, 104:73–115, 1976.

[10] A. Connes. *Non-commutative Geometry.* Academic Press, San Diego and New York, 1994.

[11] E.B. Davies. *Quantum Theory of Open Systems.* Academic Press, London, 1976.

[12] A. Einstein. Physik und Realität. *Journal of the Franklin Institute*, 221:313–347, 1936.

[13] A. Einstein. Autobiographisches. In P.A. Schilpp, editor, *Albert Einstein: Philosopher-Scientist*, The library of living philosophers, pages 1–96. Open Court, Evanston, Illinois, 1949.

[14] A. Einstein, B. Podolsky, and N. Rosen. Can quantum mechanical description of physical reality be considered complete? *Physical Review*, 47:777–780, 1935. Reprinted in [47] pp. 138-141.

[15] A. Einsten. Quanten-Mechanik und Wirklichkeit. *Dialectica*, 2:320–324, 1948.

[16] R. Haag. *Local Quantum Physics.* Springer Verlag, 1992.

[17] D. Howard. Einstein on locality and separability. *Studies in History and Philosophy of Science*, 16:171–201, 1985.

[18] M. Jammer. The EPR problem in its historical development. In P.Lahti and P.Mittelstaedt, editors, *Symposium on the Foundations of Modern Physics. 50 years of the Einstein-Podolsky-Rosen Gedankenexperiment*, pages 129–149. World Scientific, Singapore, 1985.

[19] R.V. Kadison and J.R. Ringrose. *Fundamentals of the Theory of Operator Algebras*, volume I. and II. Academic Press, Orlando, 1986.

[20] K. Kraus. *States, Effects and Operations*, volume 190 of *Lecture Notes in Physics.* Springer, New York, 1983.

[21] M. Rédei. Reichenbach's Common Cause Principle and quantum field theory. *Foundations of Physics*, 27:1309–1321, 1997.

[22] M. Rédei, editor. *John von Neumann: Selected Letters*, volume 27 of *History of Mathematics*, Rhode Island, 2005. American Mathematical Society and London Mathematical Society.

[23] M. Rédei. Von Neumann on quantum correlations. In B . Demopoulos and I. Pitowsky, editors, *Physical Theory and its Interpretation: Essays in Honor of Jeffrey Bub*, pages 241–252. Springer, 2006.

[24] M. Rédei. Einstein's dissatisfcation with non-relativistic quantum mechanics and relativistic quantum field theory. *Philosophy of Science (Supplement)*, 2010. PSA 2008. Proceedings of the Biennial Conference of the Philosophy of Science Association. Forthcoming.

[25] M. Rédei. Operational separability and operational independence in algebraic quantum mechanics. *Foundations of Physics*, 2010. Forthcoming.

[26] M. Rédei and S.J. Summers. Local primitive causality and the common cause principle in quantum field theory. *Foundations of Physics*, 32:335–355, 2002.

[27] M. Rédei and S.J. Summers. Remarks on causality in relativistic quantum field theory. *International Journal of Theoretical Physics*, 46:2053–2062, 2007.

[28] M. Rédei and S.J. Summers. When are quantum systems operationally independent? *International Journal of Theoretical Physics*, 2009. Forthcoming. Preprint: http://arxiv.org/abs/0810.5294.

[29] M. Rédei and G. Valente. How local are local operations in local quantum field theory? *submitted*, 2009.

[30] E. Schrödinger. Discussion of probability relations between separated systems. *Proceedings of the Cambridge Philosophical Society*, 31:555–563, 1935.

[31] E. Schrödinger. Probability relations between separated systems. *Proceedings of the Cambridge Philosophical Society*, 32:446–452, 1936.

[32] S.J. Summers. Bell's inequalities and quantum field theory. In *Quantum Probability and Applications V.*, volume 1441 of *Lecture Notes in Mathematics*, pages 393–413. Springer, 1990.

[33] S.J. Summers. On the independence of local algebras in quantum field theory. *Reviews in Mathematical Physics*, 2:201–247, 1990.

[34] S.J. Summers. Bell's inequalities and algebraic structure. In S. Doplicher, R. Longo, J.E. Roberts, and L. Zsido, editors, *Operator Algebras and Quantum Field Theory*, volume 1441 of *Lecture Notes in Mathematics*, pages 633–646. International Press, distributed by the American Mathematical Society, 1997.

[35] S.J. Summers. Subsystems and independence in relativistic microphysics. *Studies in History and Philosophy of Modern Physics*, 40:133–141, 2009.

[36] S.J. Summers and R. Werner. The vacuum violates Bell's inequalities. *Physics Letters A*, 110:257–279, 1985.

[37] S.J. Summers and R. Werner. Bell's inequalities and quantum field theory, I. General setting. *Journal of Mathematical Physics*, 28:2440–2447, 1987.

[38] S.J. Summers and R. Werner. Bell's inequalities and quantum field theory, II. Bell's inequalities are maximally violated in the vacuum. *Journal of Mathematical Physics*, 28:2448–2456, 1987.

[39] S.J. Summers and R. Werner. Maximal violation of Bell's inequalities is generic in quantum field theory. *Commununications in Mathematical Physics*, 110:247–259, 1987.

[40] S.J. Summers and R. Werner. Maximal violation of Bell's inequalities for algebras of observables in tangent spacetime regions. *Annales de l'Institut Henri Poincaré – Physique théorique*, 49:215–243, 1988.

[41] A.H. Taub, editor. *John von Neumann: Collected Works*, volume IV. Continuous Geometry and Other Topics, New York and Oxford, 1961. Pergamon Press.

[42] A.H. Taub, editor. *John von Neumann: Collected Works*, volume I. Logic, Theory of Sets and Quantum Mechanics, New York and Oxford, 1961. Pergamon Press.

[43] J. von Neumann. Mathematische Begründung der Quantenmechanik. *Göttinger Nachrichten*, pages 1–57, 1927. Reprinted in [42] No. 9.

[44] J. von Neumann. Thermodynamik quantenmechanischer Gesamtheiten. *Göttinger Nachrichten*, pages 273–291, 1927. Reprinted in [42] No. 11.

[45] J. von Neumann. Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik. *Göttinger Nachrichten*, pages 245–272, 1927. Reprinted in [42] No. 10.

[46] J. von Neumann. *Mathematische Grundlagen der Quantenmechanik*. Springer Verlag, Berlin, 1932. English translation: Mathematical Foundations of Quantum Mechanics (Princeton University Press, Princeton) 1955.

[47] John A. Wheeler and Wojciech H. Zurek, editors. *Quantum Theory and Measurement*, Princeton, 1983. Princeton University Press.