



Gradient descent assimilation for the point-vortex model

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Data assimilation is concerned with incorporating (noisy) observations into (imperfect) models that describe the underlying dynamics of the system, in order to infer the properties of the current state, by ensuring that the assimilated trajectories are consistent with both the observations and model dynamics. For many physical systems, particularly in oceanography, observations are usually available in the form of Lagrangian (particle trajectory) data that are augmented into models describing the flow fields. The incorporation of Lagrangian data into models of flow presents several challenges concerning the potential complexity of the Lagrangian trajectories in relatively simple flow fields, for example the appearance of nonlinear effects that are triggered by the exponential rate of separation of tracer trajectories in the region of saddle points [1]. As such, standard linear-based data assimilation methods, such as the Kalman filter, can fail.

A nonlinear approach known as gradient descent assimilation [2] is presented, in which analysis trajectories are found by minimising a cost function in an extended state space. The gradient descent approach is demonstrated in the context of assimilating Lagrangian tracer trajectories in two-dimensional flows of point-vortex systems. The point-vortex model plays an important role as a simplified version of many physical systems, including Bose-Einstein condensates, certain plasma configurations and inviscid turbulence, in which the model dynamics are described by a relatively simple system of nonlinear ODEs, which can exhibit regular or chaotic motion for the 2-point vortex or 3-point vortex system respectively. A set of tracer advection equations augment the point vortex model equations, allowing the observed tracer positions to update the state information about the unobserved vortex positions. The gradient descent approach to the two-point vortex system has been successfully demonstrated for the case of both full and partial observations in a wide variety of test cases.

[1] K. Ide, L. Kuznetsov and C. K. R. T. Jones. Lagrangian data assimilation for point vortex systems, *Journal of Turbulence*, **3**, 053 (2002).

[2] K. Judd, L. A. Smith and A. Weisheimer. Gradient free descent: Shadowing and state estimation using limited derivative information, *Physica D*, **190**, 153-166 (2004).