The Bootstrap and Lyapunov Exponents in Deterministic Chaos

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Abstract

Inasmuch as Lyapunov exponents provide a necessary condition for chaos in a dynamical system, confidence bounds on estimated Lyapunov exponents are of great interest. Estimates derived either from observations or from numerical integrations are limited to trajectories of finite length, and it is the uncertainties in (the distribution of) these finite time Lyapunov exponents which are of interest. Within this context a bootstrap algorithm for quantifying sampling uncertainties is shown to be inappropriate for multiplicative-ergodic statistics of deterministic chaos. This result remains unchanged in the presence of observational noise. As originally proposed, the algorithm is also inappropriate for general nonlinear stochastic processes, a modified version is presented which may prove of value in the case of stochastic dynamics. A new approach towards quantifying the minimum duration of observations required to estimate global Lyapunov exponents is suggested and is explored in a companion paper.