HOW TO QUANTIFY THE PREDICTABILITY OF A CHAOTIC SYSTEM?

E.A. Sienkiewicz 1 | L.A. Smith 1 | E.L. Thompson 2

1 Centre for the Analysis of Time Series, London School of Economics | 2 Oxford Centre for Industrial and Applied Mathematics, Oxford University, UK.

ABSTRACT

Models are tools that describe reality in form of mathematical equations. For example General Circulation Models (GCM) represent actual climate system and are used to investigate major climate processes and help us better understand certain dependencies amongst climate variables. Global forecasts help foresee severe weather anywhere on the planet and save many lives, although meteorology is unreliable in long run. A model is only an approximate representation of nature, which is reflected by model error. In addition, small uncertainties in the initial conditions usually bring up errors in the final forecasts. We can handle initial condition uncertainty but not model error. This study examines how to quantify predictability of complex models with an eye towards experimental design.

1. EXPERIMENT TO DISCUSS MODEL ERROR AND CHAOS.

We carry out the experiment, which aims to distinguish model error and chaos. The research is based on a classical Henon Map (model) and simple nonlinear two-dimensional chaotic map, including a seasonal cycle - Henon Map described in "What might we learn from climate forecasts" (L.A. Smith, 2002), which constitutes system.

How to quantify the predictability of a chaotic system?

2. ANALYSIS OF THE EXPERIMENT. RELATIVE ENTROPY.

To check the properties of the distributions we use relative entropy. It is a measure of the difference between the two probability distributions P and Q expressed by the equation below.

\[ D_{RE}(P || Q) = \sum_{i} P(i) \log\left(\frac{P(i)}{Q(i)}\right). \]

Where: i represents number of bins. In our case there are 2^16 bins - a probability distribution of the model (MS1, MS2) Q - is a probability distribution of the system (SS1, SS2).

We expect the RE for sets developed on model and system to be at zero at t=0 since the distributions are identical, and to converge close to zero after a large number of time steps, because both attractors are similar. The interesting part should be in the middle, where the model error results in very large relative entropy, which means an overconfident forecast. For sets developed under the same dynamics we anticipate that the RE should be close to zero since they move around the same attractor.

The small difference between distributions is a result of initial uncertainty.

3. HOW DO THESE RESULTS RELATE TO WEATHER MODELS?

In SHORT TERM (step 1-7) model is useful and it gives a good prediction.

It tells where the points are going to be, and the relative entropy shows that. It is a good prediction, because it is constrained. It is analogous to a weather forecast of "between 3-6 degrees for tomorrow’s temperature, when the actual value is 5 degrees".

In MEDIUM TERM (step 8-20) the degree by which system distribution differs from the model distribution is the biggest. One could compare it to a forecast of "between 7 degrees and 10 degrees for tomorrow’s temperature", when the actual value is 5 degrees. In this case it would be actually wrong prediction (not just inaccurate), and the relative entropy, gives a very bad score for that.

In the LONG RUN (step 20+) model looks like climatology and it is not useful. It is going a good prediction but it would not be any more useful than looking at old statistics. It’s like a forecast of "between minus 20 and plus 15 degrees" for tomorrow’s temperature.

4. BIGGER PICTURE.

In the experiment we noticed that, as we decrease the number of members in the ensemble or cut down the noise level, the forecast changes and RE reflects this. We can expect longer time step till we lose predictability. We always observe such improvement? The problem is that, curves presented in Figure 3 change a lot depending on where on attractor are. The small difference between distributions is a result of initial uncertainty. According to Newton the bigger picture is that, curves presented in Figure 3 change a lot depending on where on attractor are, and the relative entropy, gives a very bad score for that.

5. CONCLUSIONS.

Predictions in meteorology are limited in time because of (i) initial uncertainty are highly sensitive to initial conditions and (ii) the presence of model error reflecting the fact that model is only an approximate representation of nature.

An interesting feature of chaotic systems is that as we go to the different places on attractor, the time till we lose the predictability varies by location of the ensemble of initial conditions. We can apply different techniques to change the way we make the forecast on the system i.e. by increasing the size of the ensemble or by cutting down the noise level, but this does not guarantee any improvement.

References: