Indistinguishable States I: The Perfect Model Scenario

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Abstract

An accurate forecast of a nonlinear system will require an accurate estimation of the initial state. It is shown that even under the ideal conditions of a perfect model and infinite past observations of a deterministic nonlinear system, uncertainty in the observations makes exact state estimation is impossible. Consistent with the noisy observations there is a set of states indistinguishable from the true state. This implies that an accurate forecast must be based on a probability density on the indistinguishable states. This paper shows that this density can be calculated by first calculating a maximum likelihood estimate of the state, and then an ensemble estimate of the density of states that are indistinguishable from the maximum likelihood state. A new method for calculating the maximum likelihood estimate of the true state is presented which allows practical ensemble forecasting even when the recurrence time of the system is long. In a subsequent paper the theory and practice described in this paper are extended to an imperfect model scenario.